Computer algebra independent integration tests

1_Algebraic_functions/1.2_Trinomial_products/1.2.3General/1.2.3.4(fx)^m(d+ex^n)^q(a+

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Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

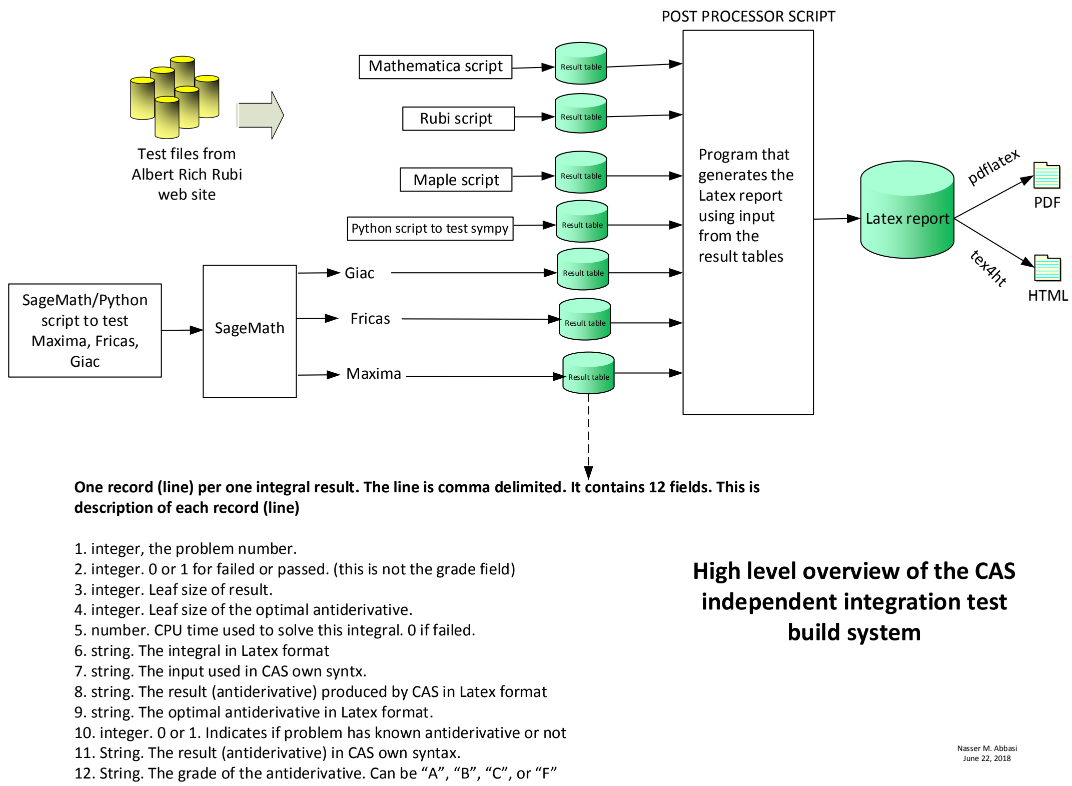
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (156)	% 0. (0)
Rubi in Sympy	% 78.85 (123)	% 21.15 (33)
Mathematica	% 93.59 (146)	% 6.41 (10)
Maple	% 87.82 (137)	% 12.18 (19)
Maxima	% 38.46 (60)	% 61.54 (96)
Fricas	% 70.51 (110)	% 29.49 (46)
Sympy	% 39.74 (62)	% 60.26 (94)
Giac	% 69.87 (109)	% 30.13 (47)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

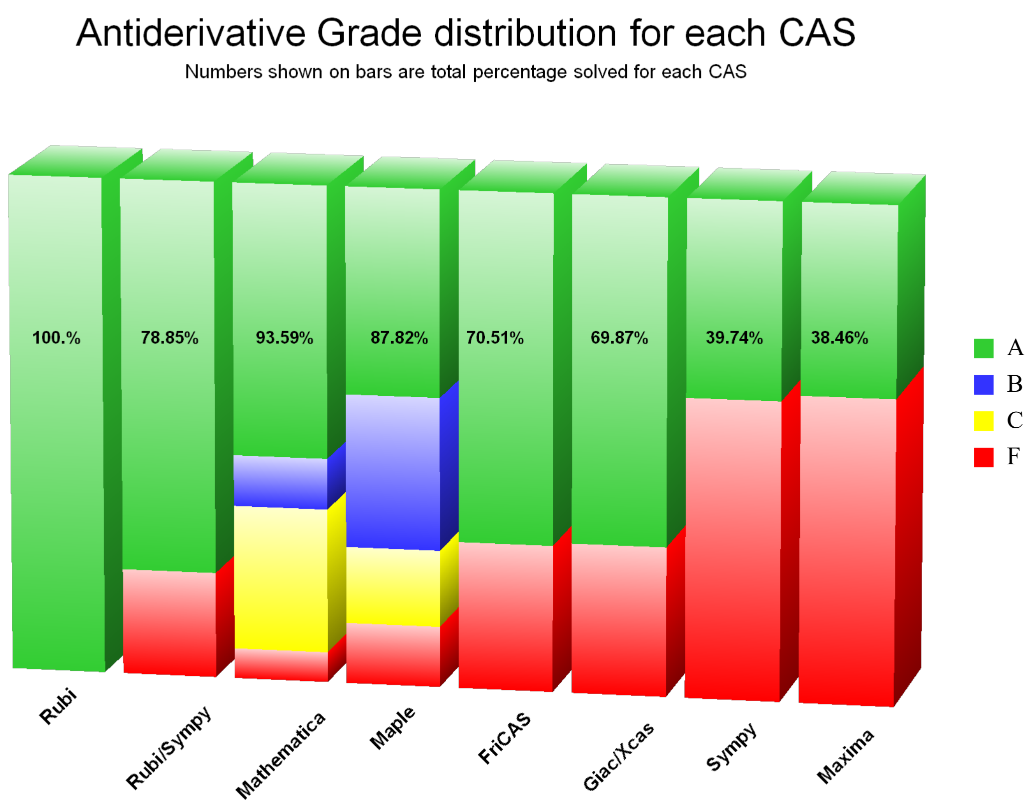
Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented.

For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

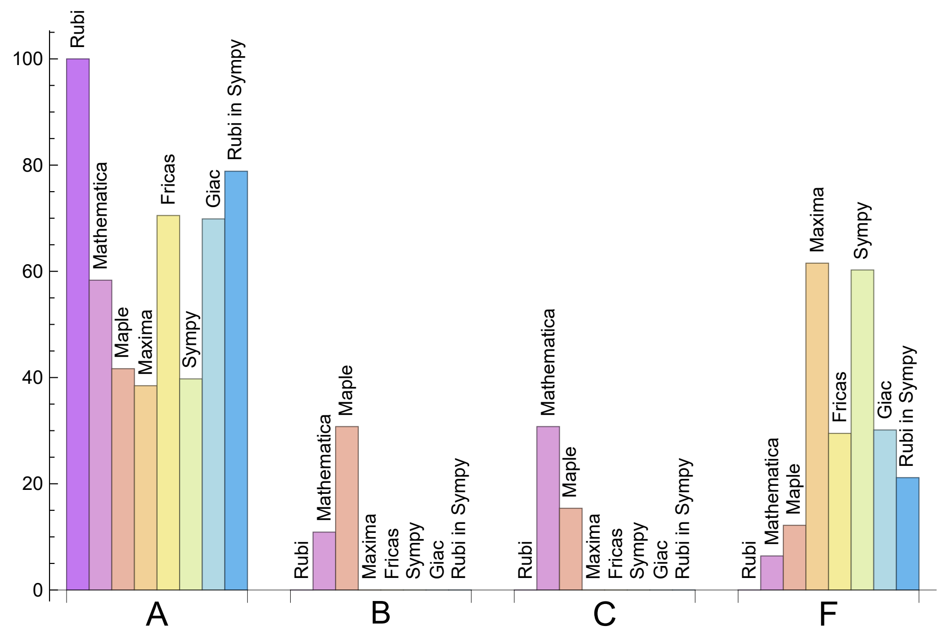
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Rubi in Sympy	78.85	0.	0.	21.15
Mathematica	58.33	10.9	30.77	6.41
Maple	41.67	30.77	15.38	12.18
Maxima	38.46	0.	0.	61.54
Fricas	70.51	0.	0.	29.49
Sympy	39.74	0.	0.	60.26
Giac	69.87	0.	0.	30.13

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.86	208.74	0.98	153.5	1.
Rubi in Sympy	46.97	139.14	0.87	48.	0.87
Mathematica	1.06	560.7	2.22	80.	0.96
Maple	0.03	2589.11	116.41	70.	1.24
Maxima	0.78	220.17	10.67	48.	1.39
Fricas	2.85	1058.09	6.33	44.	1.37
Sympy	6.16	226.48	8.77	44.	0.95
Giac	0.28	168.2	1.82	58.	1.33

1.8 list of integrals that has no closed form antiderivative

{86, 155, 156}

1.9 list of integrals not solved by each system

Not solved by Rubi {}

Not solved by Rubi in Sympy {1, 2, 3, 4, 5, 9, 10, 14, 15, 16, 18, 19, 20, 61, 62, 63, 69, 70, 71, 72, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 92, 142, 143}

Not solved by Mathematica {90, 91, 92, 145, 146, 147, 148, 149, 150, 151}

Not solved by Maple {87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154}

Not solved by Maxima {6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 25, 26, 27, 28, 29, 30, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 96, 100, 104, 129, 133, 137, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154}

Not solved by Fricas {14, 15, 18, 19, 35, 36, 37, 38, 39, 40, 41, 42, 49, 51, 67, 68, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154}

Not solved by Sympy {12, 13, 14, 15, 16, 17, 18, 19, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 96, 100, 104, 108, 109, 110, 111, 112, 116, 117, 118, 119, 120, 124, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156}

Not solved by Giac {14, 15, 16, 17, 18, 19, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 47, 49, 51, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 110, 118, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {79, 152}

Mathematica {35, 36, 37, 38, 39, 40, 41, 42, 79, 80, 81, 85, 152, 153, 154}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	164	169	224	1	187	234	0
normalized size	1	1.	1.01	1.04	1.37	0.01	1.15	1.44	0.
time (sec)	N/A	0.384	0.08	0.003	0.74	0.225	0.187	0.273	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	135	136	182	1	151	190	0
normalized size	1	1.	1.	1.01	1.35	0.01	1.12	1.41	0.
time (sec)	N/A	0.256	0.071	0.001	0.738	0.237	0.164	0.27	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	104	103	138	1	117	147	0
normalized size	1	1.	1.01	1.	1.34	0.01	1.14	1.43	0.
time (sec)	N/A	0.203	0.047	0.001	0.742	0.235	0.15	0.27	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	70	93	1	75	103	0
normalized size	1	1.	1.	0.96	1.27	0.01	1.03	1.41	0.
time (sec)	N/A	0.132	0.037	0.001	0.743	0.233	0.121	0.268	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	49	1	39	58	0
normalized size	1	1.	1.	0.88	1.17	0.02	0.93	1.38	0.
time (sec)	N/A	0.061	0.015	0.001	0.744	0.228	0.087	0.266	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	176	313	0	231	175	279	175
normalized size	1	1.	0.94	1.66	0.	1.23	0.93	1.48	0.93
time (sec)	N/A	0.411	0.297	0.008	0.	0.29	3.723	0.285	49.891

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	199	345	0	385	206	306	202
normalized size	1	1.	0.93	1.62	0.	1.81	0.97	1.44	0.95
time (sec)	N/A	0.449	0.492	0.014	0.	0.292	6.965	0.279	64.426

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	209	362	0	552	246	340	233
normalized size	1	1.	0.86	1.5	0.	2.28	1.02	1.4	0.96
time (sec)	N/A	0.525	0.49	0.015	0.	0.263	22.238	0.28	66.666

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	126	260	0	1	619	177	0
normalized size	1	1.	0.95	1.97	0.	0.01	4.69	1.34	0.
time (sec)	N/A	0.441	0.108	0.008	0.	0.613	46.176	0.27	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	93	175	0	1	434	128	0
normalized size	1	1.	0.96	1.8	0.	0.01	4.47	1.32	0.
time (sec)	N/A	0.256	0.113	0.005	0.	0.353	29.424	0.272	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	99	0	1	287	95	63
normalized size	1	1.	0.99	1.38	0.	0.01	3.99	1.32	0.88
time (sec)	N/A	0.168	0.086	0.004	0.	0.295	13.78	0.274	23.899

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	80	106	0	1	0	103	73
normalized size	1	1.	1.03	1.36	0.	0.01	0.	1.32	0.94
time (sec)	N/A	0.27	0.056	0.01	0.	0.424	0.	0.275	36.073

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	130	191	0	1	0	173	105
normalized size	1	1.	1.16	1.71	0.	0.01	0.	1.54	0.94
time (sec)	N/A	0.403	0.081	0.013	0.	0.826	0.	0.283	54.575

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	723	723	88	70	0	0	0	0	0
normalized size	1	1.	0.12	0.1	0.	0.	0.	0.	0.
time (sec)	N/A	3.97	0.078	0.031	0.	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	718	718	88	67	0	0	0	0	0
normalized size	1	1.	0.12	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	3.448	0.082	0.007	0.	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	634	634	59	49	0	19340	0	0	0
normalized size	1	1.	0.09	0.08	0.	30.5	0.	0.	0.
time (sec)	N/A	1.887	0.047	0.005	0.	4.371	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	634	634	61	47	0	18004	0	0	661
normalized size	1	1.	0.1	0.07	0.	28.4	0.	0.	1.04
time (sec)	N/A	1.65	0.048	0.004	0.	1.88	0.	0.	174.044

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	653	653	85	70	0	0	0	0	0
normalized size	1	1.	0.13	0.11	0.	0.	0.	0.	0.
time (sec)	N/A	2.606	0.075	0.01	0.	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	655	655	89	68	0	0	0	0	0
normalized size	1	1.	0.14	0.1	0.	0.	0.	0.	0.
time (sec)	N/A	2.514	0.078	0.01	0.	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	38	50	59	42	50	0
normalized size	1	1.	1.	0.83	1.09	1.28	0.91	1.09	0.
time (sec)	N/A	0.124	0.025	0.004	0.817	0.257	0.286	0.269	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	25	32	38	32	32	29
normalized size	1	1.	1.	0.81	1.03	1.23	1.03	1.03	0.94
time (sec)	N/A	0.083	0.012	0.004	0.817	0.272	0.269	0.271	14.248

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	43	49	37	43	34
normalized size	1	1.	1.	0.85	1.1	1.26	0.95	1.1	0.87
time (sec)	N/A	0.087	0.013	0.004	0.816	0.26	0.298	0.274	13.659

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	44	35	51	58	41	47	41
normalized size	1	1.	1.07	0.85	1.24	1.41	1.	1.15	1.
time (sec)	N/A	0.115	0.02	0.008	0.819	0.27	0.34	0.288	15.463

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	45	25	32	41	36	32	32
normalized size	1	1.	1.45	0.81	1.03	1.32	1.16	1.03	1.03
time (sec)	N/A	0.102	0.02	0.006	0.817	0.254	0.376	0.275	14.516

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	47	46	0	886	31	867	340
normalized size	1	1.	0.11	0.11	0.	2.12	0.07	2.07	0.81
time (sec)	N/A	1.124	0.019	0.009	0.	0.272	0.482	0.293	131.602

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	48	44	0	1447	32	1	340
normalized size	1	1.	0.13	0.12	0.	3.79	0.08	0.	0.89
time (sec)	N/A	0.778	0.02	0.008	0.	0.282	0.492	0.286	125.338

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	46	41	0	884	24	853	337
normalized size	1	1.	0.12	0.11	0.	2.34	0.06	2.26	0.89
time (sec)	N/A	0.635	0.019	0.007	0.	0.277	0.478	0.286	101.496

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	55	44	0	1415	22	1	352
normalized size	1	1.	0.13	0.11	0.	3.44	0.05	0.	0.86
time (sec)	N/A	0.682	0.018	0.006	0.	0.276	0.489	0.285	105.392

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	57	44	0	873	26	860	352
normalized size	1	1.	0.14	0.11	0.	2.12	0.06	2.09	0.86
time (sec)	N/A	0.693	0.017	0.007	0.	0.286	0.483	0.315	103.32

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	47	46	0	1463	31	1	338
normalized size	1	1.	0.11	0.11	0.	3.52	0.07	0.	0.81
time (sec)	N/A	0.697	0.02	0.01	0.	0.282	0.549	0.295	108.539

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	47	46	0	930	32	867	342
normalized size	1	1.	0.11	0.11	0.	2.22	0.08	2.07	0.82
time (sec)	N/A	0.854	0.02	0.009	0.	0.28	0.562	0.298	129.123

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	37	33	43	49	37	43	34
normalized size	1	1.	1.03	0.92	1.19	1.36	1.03	1.19	0.94
time (sec)	N/A	0.091	0.014	0.003	0.827	0.259	0.285	0.279	11.439

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	55	35	51	58	41	47	41
normalized size	1	1.	1.41	0.9	1.31	1.49	1.05	1.21	1.05
time (sec)	N/A	0.117	0.021	0.008	0.82	0.276	0.328	0.272	16.682

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	55	35	0	58	41	47	41
normalized size	1	1.	1.41	0.9	0.	1.49	1.05	1.21	1.05
time (sec)	N/A	0.114	0.016	0.006	0.	0.253	0.326	0.273	18.013

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	279	1070	0	0	400	0	379
normalized size	1	1.	0.7	2.7	0.	0.	1.01	0.	0.96
time (sec)	N/A	0.774	0.582	0.203	0.	0.	25.683	0.	46.905

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	249	1010	0	0	257	0	337
normalized size	1	1.	0.7	2.84	0.	0.	0.72	0.	0.95
time (sec)	N/A	0.602	0.524	0.045	0.	0.	13.534	0.	39.809

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	219	956	0	0	124	0	294
normalized size	1	1.	0.69	3.03	0.	0.	0.39	0.	0.93
time (sec)	N/A	0.504	0.476	0.044	0.	0.	6.816	0.	33.445

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	194	907	0	0	119	0	253
normalized size	1	1.	0.7	3.26	0.	0.	0.43	0.	0.91
time (sec)	N/A	0.37	0.606	0.044	0.	0.	6.305	0.	27.432

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	197	934	0	0	119	0	262
normalized size	1	1.	0.68	3.23	0.	0.	0.41	0.	0.91
time (sec)	N/A	0.402	0.456	0.06	0.	0.	59.45	0.	42.235

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	224	1005	0	0	0	0	286
normalized size	1	1.	0.72	3.25	0.	0.	0.	0.	0.93
time (sec)	N/A	0.438	0.674	0.07	0.	0.	0.	0.	43.763

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	262	1095	0	0	0	0	328
normalized size	1	1.	0.75	3.14	0.	0.	0.	0.	0.94
time (sec)	N/A	0.642	0.838	0.074	0.	0.	0.	0.	50.55

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	296	1182	0	0	0	0	371
normalized size	1	1.	0.76	3.04	0.	0.	0.	0.	0.95
time (sec)	N/A	0.737	1.032	0.078	0.	0.	0.	0.	57.496

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	88	67	0	17662	0	0	432
normalized size	1	1.	0.2	0.15	0.	40.79	0.	0.	1.
time (sec)	N/A	2.402	0.104	0.007	0.	4.467	0.	0.	169.92

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	99	0	1	287	95	63
normalized size	1	1.	0.99	1.38	0.	0.01	3.99	1.32	0.88
time (sec)	N/A	0.177	0.098	0.004	0.	0.395	24.314	0.276	24.648

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	59	51	0	19648	0	0	379
normalized size	1	1.	0.16	0.14	0.	52.39	0.	0.	1.01
time (sec)	N/A	0.948	0.064	0.005	0.	16.728	0.	0.	119.001

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	179	340	0	2072	0	1	189
normalized size	1	1.	0.97	1.85	0.	11.26	0.	0.01	1.03
time (sec)	N/A	0.421	0.286	0.023	0.	0.398	0.	0.998	54.637

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	61	47	0	12872	0	0	379
normalized size	1	1.	0.16	0.13	0.	34.33	0.	0.	1.01
time (sec)	N/A	0.804	0.066	0.009	0.	1.842	0.	0.	113.172

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	80	106	0	1	0	105	73
normalized size	1	1.	1.03	1.36	0.	0.01	0.	1.35	0.94
time (sec)	N/A	0.278	0.057	0.009	0.	0.862	0.	0.282	36.743

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	85	72	0	0	0	0	391
normalized size	1	1.	0.22	0.18	0.	0.	0.	0.	1.
time (sec)	N/A	1.458	0.089	0.01	0.	0.	0.	0.	160.037

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	89	365	0	3742	0	1	201
normalized size	1	1.	0.45	1.83	0.	18.8	0.	0.01	1.01
time (sec)	N/A	0.697	0.074	0.025	0.	0.835	0.	0.495	84.805

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	86	68	0	0	0	0	394
normalized size	1	1.	0.22	0.17	0.	0.	0.	0.	1.
time (sec)	N/A	1.386	0.099	0.009	0.	0.	0.	0.	153.716

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	46	34	0	261	170	281	530
normalized size	1	1.	0.17	0.12	0.	0.94	0.61	1.01	1.91
time (sec)	N/A	0.662	0.025	0.01	0.	0.263	0.652	0.279	80.625

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	43	49	37	43	34
normalized size	1	1.	1.	0.85	1.1	1.26	0.95	1.1	0.87
time (sec)	N/A	0.092	0.02	0.005	0.826	0.253	0.329	0.273	14.054

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	55	46	0	1223	27	342	481
normalized size	1	1.	0.15	0.13	0.	3.45	0.08	0.96	1.35
time (sec)	N/A	0.717	0.024	0.01	0.	0.282	4.618	0.283	102.214

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	44	39	0	59	42	42	42
normalized size	1	1.	0.88	0.78	0.	1.18	0.84	0.84	0.84
time (sec)	N/A	0.08	0.026	0.015	0.	0.248	0.289	0.286	20.258

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	57	44	0	1223	26	342	495
normalized size	1	1.	0.16	0.12	0.	3.45	0.07	0.96	1.39
time (sec)	N/A	0.506	0.021	0.001	0.	0.28	4.641	0.306	78.633

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	44	35	51	58	41	51	41
normalized size	1	1.	1.07	0.85	1.24	1.41	1.	1.24	1.
time (sec)	N/A	0.117	0.021	0.009	0.822	0.251	0.386	0.303	16.454

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	47	38	0	284	168	284	512
normalized size	1	1.	0.17	0.14	0.	1.01	0.6	1.01	1.83
time (sec)	N/A	0.536	0.023	0.013	0.	0.274	0.696	0.312	84.449

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	49	70	0	188	76	348	76
normalized size	1	1.	0.55	0.79	0.	2.11	0.85	3.91	0.85
time (sec)	N/A	0.189	0.024	0.007	0.	0.268	0.698	0.349	40.017

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	47	46	0	1197	32	348	318
normalized size	1	1.	0.13	0.12	0.	3.24	0.09	0.94	0.86
time (sec)	N/A	0.612	0.022	0.014	0.	0.299	4.69	0.294	77.103

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	283	662	0	1	0	398	0
normalized size	1	1.	1.01	2.36	0.	0.	0.	1.42	0.
time (sec)	N/A	1.207	0.405	0.018	0.	28.978	0.	0.291	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	218	512	0	1	0	302	0
normalized size	1	1.	1.	2.35	0.	0.	0.	1.39	0.
time (sec)	N/A	0.794	0.308	0.012	0.	17.944	0.	0.296	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	178	388	0	1	0	250	0
normalized size	1	1.	1.01	2.2	0.	0.01	0.	1.42	0.
time (sec)	N/A	0.535	0.32	0.011	0.	5.411	0.	0.314	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	132	275	0	1	0	201	129
normalized size	1	1.	0.89	1.85	0.	0.01	0.	1.35	0.87
time (sec)	N/A	0.399	0.211	0.009	0.	1.627	0.	0.301	80.285

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	107	169	0	1	0	171	112
normalized size	1	1.	0.86	1.36	0.	0.01	0.	1.38	0.9
time (sec)	N/A	0.323	0.129	0.007	0.	0.528	0.	0.299	62.253

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	105	168	0	1	0	170	112
normalized size	1	1.	0.85	1.37	0.	0.01	0.	1.38	0.91
time (sec)	N/A	0.28	0.135	0.007	0.	0.546	0.	0.297	60.69

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	159	152	285	0	0	0	221	136
normalized size	1	1.01	0.96	1.8	0.	0.	0.	1.4	0.86
time (sec)	N/A	0.553	0.34	0.011	0.	0.	0.	0.294	99.804

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	194	412	0	0	0	284	178
normalized size	1	1.	1.01	2.13	0.	0.	0.	1.47	0.92
time (sec)	N/A	0.714	0.286	0.017	0.	0.	0.	0.302	137.446

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	252	562	0	0	0	377	0
normalized size	1	1.	1.	2.23	0.	0.	0.	1.5	0.
time (sec)	N/A	0.913	0.377	0.018	0.	0.	0.	0.3	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	338	943	0	1	0	763	0
normalized size	1	1.	0.99	2.75	0.	0.	0.	2.22	0.
time (sec)	N/A	1.804	0.658	0.02	0.	118.759	0.	0.303	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	269	765	0	1	0	643	0
normalized size	1	1.	0.98	2.79	0.	0.	0.	2.35	0.
time (sec)	N/A	1.104	0.545	0.016	0.	50.476	0.	0.3	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	207	580	0	1	0	556	0
normalized size	1	1.	0.84	2.36	0.	0.	0.	2.26	0.
time (sec)	N/A	0.834	0.407	0.014	0.	17.632	0.	0.284	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	159	389	0	1	0	455	177
normalized size	1	1.	0.82	2.01	0.	0.01	0.	2.35	0.91
time (sec)	N/A	0.559	0.406	0.013	0.	6.045	0.	0.273	146.522

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	148	328	0	1	0	441	167
normalized size	1	1.	0.81	1.79	0.	0.01	0.	2.41	0.91
time (sec)	N/A	0.508	0.441	0.012	0.	5.23	0.	0.277	103.401

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	151	386	0	1	0	447	177
normalized size	1	1.	0.8	2.04	0.	0.01	0.	2.37	0.94
time (sec)	N/A	0.648	0.384	0.013	0.	2.751	0.	0.29	150.937

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	249	246	589	0	0	0	528	0
normalized size	1	1.	0.99	2.38	0.	0.	0.	2.13	0.
time (sec)	N/A	0.849	0.477	0.019	0.	0.	0.	0.297	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	287	791	0	0	0	657	0
normalized size	1	1.	0.99	2.72	0.	0.	0.	2.26	0.
time (sec)	N/A	1.202	0.658	0.023	0.	0.	0.	0.284	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	370	993	0	0	0	792	0
normalized size	1	1.	0.99	2.67	0.	0.	0.	2.13	0.
time (sec)	N/A	1.887	0.779	0.026	0.	0.	0.	0.29	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	981	981	10904	11938	0	0	0	0	0
normalized size	1	1.	11.12	12.17	0.	0.	0.	0.	0.
time (sec)	N/A	10.135	14.785	0.169	0.	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	778	778	7531	9182	0	0	0	0	0
normalized size	1	1.	9.68	11.8	0.	0.	0.	0.	0.
time (sec)	N/A	4.665	14.221	0.074	0.	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	636	636	5350	6302	0	0	0	0	0
normalized size	1	1.	8.41	9.91	0.	0.	0.	0.	0.
time (sec)	N/A	2.514	13.577	0.06	0.	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	550	3390	4361	0	0	0	0	0
normalized size	1	1.	6.16	7.93	0.	0.	0.	0.	0.
time (sec)	N/A	1.735	13.161	0.059	0.	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	955	955	758	3023	0	0	0	0	0
normalized size	1	1.	0.79	3.17	0.	0.	0.	0.	0.
time (sec)	N/A	7.743	13.112	0.06	0.	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	929	929	4893	3553	0	0	0	0	0
normalized size	1	1.	5.27	3.82	0.	0.	0.	0.	0.
time (sec)	N/A	7.193	13.119	0.066	0.	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1287	1287	6206	4957	0	0	0	0	0
normalized size	1	1.	4.82	3.85	0.	0.	0.	0.	0.
time (sec)	N/A	13.636	13.607	0.071	0.	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.245	0.647	0.	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	249	0	0	0	0	0	320
normalized size	1	1.	0.7	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.507	0.686	0.113	0.	0.	0.	0.	65.938

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	189	0	0	0	0	0	228
normalized size	1	1.	0.72	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.348	0.335	0.119	0.	0.	0.	0.	46.79

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	136	0	0	0	0	0	136
normalized size	1	1.	0.82	0.	0.	0.	0.	0.	0.82
time (sec)	N/A	0.216	0.148	0.105	0.	0.	0.	0.	27.471

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	0	0	0	0	0	0	162
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.504	0.096	0.131	0.	0.	0.	0.	88.172

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	0	0	0	0	0	0	258
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.763	0.195	0.131	0.	0.	0.	0.	155.763

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.054	0.879	0.131	0.	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	201	46548	19	1	1326	1	12
normalized size	1	1.	12.56	2909.25	1.19	0.06	82.88	0.06	0.75
time (sec)	N/A	0.035	0.273	0.007	0.767	0.264	0.967	0.267	3.672

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	233	46552	1674	1	1384	1	14
normalized size	1	1.	12.94	2586.22	93.	0.06	76.89	0.06	0.78
time (sec)	N/A	0.066	0.287	0.004	0.793	0.277	0.916	0.275	4.946

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	233	46552	1674	1	1394	1	14
normalized size	1	1.	12.94	2586.22	93.	0.06	77.44	0.06	0.78
time (sec)	N/A	0.07	0.287	0.004	0.78	0.266	0.924	0.277	4.991

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	260	2042	0	1751	0	1	17
normalized size	1	1.	11.3	88.78	0.	76.13	0.	0.04	0.74
time (sec)	N/A	0.069	0.727	0.099	0.	0.334	0.	0.326	14.23

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	201	47685	22	1	1326	1	12
normalized size	1	1.	11.17	2649.17	1.22	0.06	73.67	0.06	0.67
time (sec)	N/A	0.035	0.302	0.007	0.77	0.259	0.956	0.269	4.022

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	233	47688	1677	1	1384	1	14
normalized size	1	1.	11.65	2384.4	83.85	0.05	69.2	0.05	0.7
time (sec)	N/A	0.066	0.304	0.004	0.787	0.259	0.947	0.275	5.445

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	233	47688	1677	1	1394	1	14
normalized size	1	1.	11.65	2384.4	83.85	0.05	69.7	0.05	0.7
time (sec)	N/A	0.07	0.315	0.004	0.791	0.259	0.948	0.274	5.487

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	260	2046	0	1754	0	1	17
normalized size	1	1.	10.4	81.84	0.	70.16	0.	0.04	0.68
time (sec)	N/A	0.072	0.661	0.097	0.	0.329	0.	0.327	15.756

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	172	155	18	1	175	208	10
normalized size	1	1.	11.47	10.33	1.2	0.07	11.67	13.87	0.67
time (sec)	N/A	0.013	0.009	0.004	0.771	0.255	0.267	0.262	3.189

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	182	157	211	1	182	211	12
normalized size	1	1.	10.71	9.24	12.41	0.06	10.71	12.41	0.71
time (sec)	N/A	0.02	0.009	0.004	0.765	0.253	0.278	0.263	13.301

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	186	157	211	1	185	211	12
normalized size	1	1.	10.94	9.24	12.41	0.06	10.88	12.41	0.71
time (sec)	N/A	0.021	0.01	0.004	0.771	0.269	0.283	0.269	12.684

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	230	0	255	0	255	15
normalized size	1	1.	1.	10.95	0.	12.14	0.	12.14	0.71
time (sec)	N/A	0.062	0.054	0.057	0.	0.316	0.	0.27	12.011

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	10	12	15	15	10	15	10
normalized size	1	1.	0.91	1.09	1.36	1.36	0.91	1.36	0.91
time (sec)	N/A	0.008	0.005	0.002	0.773	0.279	1.164	0.265	3.699

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	20	14	20	14
normalized size	1	1.	1.	0.94	1.18	1.18	0.82	1.18	0.82
time (sec)	N/A	0.011	0.01	0.002	0.768	0.271	1.593	0.291	4.943

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	20	14	20	14
normalized size	1	1.	1.	0.94	1.18	1.18	0.82	1.18	0.82
time (sec)	N/A	0.011	0.011	0.002	0.772	0.282	1.989	0.266	4.995

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	26	24	31	26	0	26	15
normalized size	1	1.	1.37	1.26	1.63	1.37	0.	1.37	0.79
time (sec)	N/A	0.067	0.049	0.033	0.826	0.297	0.	0.266	12.138

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	15	15	19	473	0	19	15
normalized size	1	1.	0.94	0.94	1.19	29.56	0.	1.19	0.94
time (sec)	N/A	0.009	0.018	0.001	0.763	0.328	0.	0.275	3.666

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	475	475	0	0	17
normalized size	1	1.	1.	0.94	26.39	26.39	0.	0.	0.94
time (sec)	N/A	0.012	0.02	0.002	0.858	0.372	0.	0.	4.957

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	475	475	0	22	17
normalized size	1	1.	1.	0.94	26.39	26.39	0.	1.22	0.94
time (sec)	N/A	0.013	0.021	0.001	0.877	0.363	0.	0.359	5.006

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	562	532	0	28	20
normalized size	1	1.	1.	0.96	24.43	23.13	0.	1.22	0.87
time (sec)	N/A	0.072	0.082	0.095	1.214	0.363	0.	0.282	12.267

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	12	14	18	18	10	18	10
normalized size	1	1.	0.92	1.08	1.38	1.38	0.77	1.38	0.77
time (sec)	N/A	0.009	0.008	0.002	0.751	0.274	1.185	0.265	4.012

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	23	23	14	23	14
normalized size	1	1.	1.	0.95	1.21	1.21	0.74	1.21	0.74
time (sec)	N/A	0.01	0.01	0.001	0.744	0.254	1.597	0.291	5.4

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	23	23	14	23	14
normalized size	1	1.	1.	0.95	1.21	1.21	0.74	1.21	0.74
time (sec)	N/A	0.01	0.011	0.002	0.759	0.256	1.938	0.267	5.427

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	29	26	34	28	0	28	15
normalized size	1	1.	1.38	1.24	1.62	1.33	0.	1.33	0.71
time (sec)	N/A	0.073	0.048	0.033	0.853	0.312	0.	0.267	12.904

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	16	17	22	478	0	22	15
normalized size	1	1.	0.89	0.94	1.22	26.56	0.	1.22	0.83
time (sec)	N/A	0.011	0.021	0.001	0.757	0.322	0.	0.275	4.02

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	481	481	0	0	15
normalized size	1	1.	1.	0.95	24.05	24.05	0.	0.	0.75
time (sec)	N/A	0.012	0.027	0.001	0.84	0.334	0.	0.	5.426

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	481	481	0	24	15
normalized size	1	1.	1.	0.95	24.05	24.05	0.	1.2	0.75
time (sec)	N/A	0.014	0.028	0.001	0.875	0.335	0.	0.363	5.482

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	566	536	0	31	19
normalized size	1	1.	1.	0.96	22.64	21.44	0.	1.24	0.76
time (sec)	N/A	0.076	0.091	0.096	1.228	0.348	0.	0.281	13.299

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	9	9	14	14	8	15	8
normalized size	1	1.	0.9	0.9	1.4	1.4	0.8	1.5	0.8
time (sec)	N/A	0.008	0.006	0.002	0.741	0.256	1.071	0.263	3.172

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	15	14	23	18	12	20	15
normalized size	1	1.	0.94	0.88	1.44	1.12	0.75	1.25	0.94
time (sec)	N/A	0.009	0.01	0.007	0.751	0.282	1.234	0.264	11.581

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	15	14	23	18	12	20	15
normalized size	1	1.	0.94	0.88	1.44	1.12	0.75	1.25	0.94
time (sec)	N/A	0.01	0.011	0.007	0.746	0.259	1.296	0.265	10.867

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	63	23	0	23	36
normalized size	1	1.	1.	1.2	4.2	1.53	0.	1.53	2.4
time (sec)	N/A	0.071	0.018	0.03	0.766	0.295	0.	0.265	13.224

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	14	177	18	109	87	18	14
normalized size	1	1.	0.93	11.8	1.2	7.27	5.8	1.2	0.93
time (sec)	N/A	0.008	0.034	0.028	0.745	0.259	17.628	0.267	3.184

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	16	197	109	109	0	20	15
normalized size	1	1.	0.94	11.59	6.41	6.41	0.	1.18	0.88
time (sec)	N/A	0.011	0.049	0.028	0.764	0.279	0.	0.268	13.029

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	16	197	109	109	0	20	15
normalized size	1	1.	0.94	11.59	6.41	6.41	0.	1.18	0.88
time (sec)	N/A	0.011	0.059	0.021	0.767	0.274	0.	0.269	13.166

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	127	203	826	142	0	27	19
normalized size	1	1.	6.05	9.67	39.33	6.76	0.	1.29	0.9
time (sec)	N/A	0.063	0.079	0.086	0.798	0.354	0.	0.272	11.025

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	21	0	38	0	80	15
normalized size	1	1.	0.95	1.05	0.	1.9	0.	4.	0.75
time (sec)	N/A	0.013	0.03	0.004	0.	0.3	0.	0.269	3.991

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	24	45	45	0	92	19
normalized size	1	1.	0.96	0.96	1.8	1.8	0.	3.68	0.76
time (sec)	N/A	0.014	0.034	0.005	0.856	0.278	0.	0.272	5.556

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	24	45	45	0	92	19
normalized size	1	1.	0.96	0.96	1.8	1.8	0.	3.68	0.76
time (sec)	N/A	0.014	0.033	0.008	0.849	0.276	0.	0.281	5.63

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	40	53	51	0	36	20
normalized size	1	1.	0.96	1.48	1.96	1.89	0.	1.33	0.74
time (sec)	N/A	0.069	0.087	0.089	1.035	0.285	0.	0.288	12.312

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	21	23	0	43	0	89	15
normalized size	1	1.	0.95	1.05	0.	1.95	0.	4.05	0.68
time (sec)	N/A	0.011	0.031	0.004	0.	0.281	0.	0.266	4.385

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	26	50	50	0	101	19
normalized size	1	1.	0.96	0.96	1.85	1.85	0.	3.74	0.7
time (sec)	N/A	0.013	0.038	0.005	0.831	0.287	0.	0.273	6.069

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	26	50	50	0	101	19
normalized size	1	1.	0.96	0.96	1.85	1.85	0.	3.74	0.7
time (sec)	N/A	0.014	0.038	0.007	0.844	0.279	0.	0.275	6.138

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	28	45	58	57	0	39	20
normalized size	1	1.	0.97	1.55	2.	1.97	0.	1.34	0.69
time (sec)	N/A	0.069	0.088	0.094	1.05	0.307	0.	0.287	13.298

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	24	0	35	46	55	14
normalized size	1	1.	0.89	1.26	0.	1.84	2.42	2.89	0.74
time (sec)	N/A	0.01	0.032	0.006	0.	0.299	1.723	0.264	3.517

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	31	47	42	85	65	17
normalized size	1	1.	1.	1.29	1.96	1.75	3.54	2.71	0.71
time (sec)	N/A	0.012	0.044	0.005	0.906	0.28	61.081	0.268	4.972

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	31	47	42	0	65	17
normalized size	1	1.	1.	1.29	1.96	1.75	0.	2.71	0.71
time (sec)	N/A	0.013	0.045	0.005	0.923	0.281	0.	0.271	5.019

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	155	54	49	0	35	19
normalized size	1	1.	0.92	5.96	2.08	1.88	0.	1.35	0.73
time (sec)	N/A	0.126	0.084	0.14	1.107	0.288	0.	0.281	15.945

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	318	0	0	0	0	0	182
normalized size	1	1.	1.62	0.	0.	0.	0.	0.	0.93
time (sec)	N/A	0.549	1.538	0.047	0.	0.	0.	0.	50.056

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	5363	0	0	0	0	0	0
normalized size	1	1.	14.34	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.73	6.592	0.06	0.	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	816	816	20515	0	0	0	0	0	0
normalized size	1	1.	25.14	0.	0.	0.	0.	0.	0.
time (sec)	N/A	9.342	7.917	0.082	0.	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	36	46	46	0	46	48
normalized size	1	1.	1.	0.77	0.98	0.98	0.	0.98	1.02
time (sec)	N/A	0.102	0.04	0.005	0.768	0.274	0.	0.297	30.678

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	0	0	0	0	0	0	199
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.81
time (sec)	N/A	1.15	0.112	0.11	0.	0.	0.	0.	110.144

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	0	0	0	0	0	0	173
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.82
time (sec)	N/A	1.151	0.151	0.077	0.	0.	0.	0.	91.364

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	0	0	0	0	0	0	167
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.81
time (sec)	N/A	0.862	0.137	0.076	0.	0.	0.	0.	78.156

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	0	0	0	0	0	0	167
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.62	0.067	0.075	0.	0.	0.	0.	100.184

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	0	0	0	0	0	0	243
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.93
time (sec)	N/A	1.436	0.086	0.108	0.	0.	0.	0.	121.827

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	0	0	0	0	0	0	168
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.79
time (sec)	N/A	1.071	0.092	0.077	0.	0.	0.	0.	91.44

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	0	0	0	0	0	0	168
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.8
time (sec)	N/A	1.053	0.091	0.076	0.	0.	0.	0.	91.395

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F(-2)	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	498	498	1615	0	0	0	0	0	442
normalized size	1	1.	3.24	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	1.387	17.638	0.09	0.	0.	0.	0.	140.812

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	922	0	0	0	0	0	279
normalized size	1	1.	2.85	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.825	2.487	0.079	0.	0.	0.	0.	86.806

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	534	0	0	0	0	0	129
normalized size	1	1.	3.38	0.	0.	0.	0.	0.	0.82
time (sec)	N/A	0.328	0.92	0.079	0.	0.	0.	0.	34.908

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.126	0.108	0.	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.264	0.122	0.	0.	0.	0.	0.

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [83] had the largest ratio of [0.4231]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	22	0.045
2	A	2	1	1.	22	0.045
3	A	2	1	1.	22	0.045
4	A	2	1	1.	22	0.045
5	A	2	1	1.	20	0.05
6	A	8	8	1.	22	0.364
7	A	8	8	1.	22	0.364
8	A	8	8	1.	22	0.364
9	A	7	6	1.	25	0.24
10	A	6	6	1.	25	0.24
11	A	5	5	1.	25	0.2

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
12	A	7	6	1.	25	0.24
13	A	7	6	1.	25	0.24
14	A	14	8	1.	25	0.32
15	A	14	8	1.	25	0.32
16	A	13	7	1.	23	0.304
17	A	13	7	1.	22	0.318
18	A	14	8	1.	25	0.32
19	A	14	8	1.	25	0.32
20	A	7	6	1.	23	0.261
21	A	4	4	1.	23	0.174
22	A	5	5	1.	23	0.217
23	A	7	6	1.	23	0.261
24	A	5	4	1.	23	0.174
25	A	15	9	1.	23	0.391
26	A	15	9	1.	23	0.391
27	A	14	8	1.	23	0.348
28	A	13	7	1.	21	0.333
29	A	13	7	1.	20	0.35
30	A	14	8	1.	23	0.348
31	A	15	9	1.	23	0.391
32	A	5	5	1.	21	0.238
33	A	7	6	1.	21	0.286
34	A	8	7	1.	18	0.389
35	A	6	4	1.	24	0.167
36	A	5	4	1.	24	0.167
37	A	4	4	1.	24	0.167
38	A	3	3	1.	24	0.125
39	A	3	3	1.	24	0.125
40	A	3	3	1.	24	0.125
41	A	4	4	1.	24	0.167
42	A	5	4	1.	24	0.167
43	A	8	5	1.	25	0.2
44	A	5	5	1.	25	0.2
45	A	7	4	1.	25	0.16

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
46	A	4	3	1.	23	0.13
47	A	7	4	1.	22	0.182
48	A	7	6	1.	25	0.24
49	A	8	5	1.	25	0.2
50	A	5	4	1.	25	0.16
51	A	8	5	1.	25	0.2
52	A	20	7	1.	23	0.304
53	A	5	5	1.	23	0.217
54	A	21	7	1.	23	0.304
55	A	4	3	1.	21	0.143
56	A	19	6	1.	20	0.3
57	A	7	6	1.	23	0.261
58	A	20	7	1.	23	0.304
59	A	11	8	1.	23	0.348
60	A	21	9	1.	23	0.391
61	A	7	6	1.	25	0.24
62	A	7	6	1.	25	0.24
63	A	7	6	1.	23	0.261
64	A	7	6	1.	22	0.273
65	A	7	6	1.	25	0.24
66	A	7	7	1.	25	0.28
67	A	7	6	1.01	25	0.24
68	A	7	6	1.	25	0.24
69	A	7	6	1.	25	0.24
70	A	7	6	1.	25	0.24
71	A	7	6	1.	25	0.24
72	A	7	6	1.	23	0.261
73	A	7	6	1.	22	0.273
74	A	7	6	1.	25	0.24
75	A	8	7	1.	25	0.28
76	A	7	6	1.	25	0.24
77	A	7	6	1.	25	0.24
78	A	7	6	1.	25	0.24
79	A	11	7	1.	29	0.241

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	10	7	1.	29	0.241
81	A	8	7	1.	29	0.241
82	A	8	7	1.	27	0.259
83	A	16	11	1.	26	0.423
84	A	16	11	1.	29	0.379
85	A	24	12	1.	29	0.414
86	A	0	0	0.	0	0.
87	A	13	4	1.	26	0.154
88	A	10	4	1.	26	0.154
89	A	7	4	1.	24	0.167
90	A	6	3	1.	26	0.115
91	A	8	3	1.	26	0.115
92	A	10	3	1.	26	0.115
93	A	1	1	1.	19	0.053
94	A	1	1	1.	24	0.042
95	A	1	1	1.	26	0.038
96	A	2	2	1.	30	0.067
97	A	1	1	1.	21	0.048
98	A	1	1	1.	26	0.038
99	A	1	1	1.	28	0.036
100	A	2	2	1.	32	0.062
101	A	1	1	1.	18	0.056
102	A	1	1	1.	23	0.043
103	A	1	1	1.	25	0.04
104	A	3	3	1.	29	0.103
105	A	1	1	1.	19	0.053
106	A	1	1	1.	24	0.042
107	A	1	1	1.	26	0.038
108	A	2	2	1.	30	0.067
109	A	1	1	1.	19	0.053
110	A	1	1	1.	24	0.042
111	A	1	1	1.	26	0.038
112	A	2	2	1.	30	0.067
113	A	1	1	1.	21	0.048

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
114	A	1	1	1.	26	0.038
115	A	1	1	1.	28	0.036
116	A	2	2	1.	32	0.062
117	A	1	1	1.	21	0.048
118	A	1	1	1.	26	0.038
119	A	1	1	1.	28	0.036
120	A	2	2	1.	32	0.062
121	A	1	1	1.	18	0.056
122	A	1	1	1.	23	0.043
123	A	1	1	1.	25	0.04
124	A	4	3	1.	29	0.103
125	A	1	1	1.	18	0.056
126	A	1	1	1.	23	0.043
127	A	1	1	1.	25	0.04
128	A	3	3	1.	29	0.103
129	A	1	1	1.	19	0.053
130	A	1	1	1.	24	0.042
131	A	1	1	1.	26	0.038
132	A	2	2	1.	30	0.067
133	A	1	1	1.	21	0.048
134	A	1	1	1.	26	0.038
135	A	1	1	1.	28	0.036
136	A	2	2	1.	32	0.062
137	A	1	1	1.	18	0.056
138	A	1	1	1.	23	0.043
139	A	1	1	1.	25	0.04
140	A	2	2	1.	29	0.069
141	A	4	2	1.	29	0.069
142	A	5	3	1.	29	0.103
143	A	6	3	1.	29	0.103
144	A	3	3	1.	59	0.051
145	A	5	3	1.	31	0.097
146	A	5	3	1.	29	0.103
147	A	5	3	1.	27	0.111

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
148	A	5	3	1.	26	0.115
149	A	8	5	1.	29	0.172
150	A	5	3	1.	29	0.103
151	A	5	3	1.	29	0.103
152	A	10	4	1.	31	0.129
153	A	7	4	1.	29	0.138
154	A	2	2	1.	22	0.091
155	A	0	0	0.	0	0.
156	A	0	0	0.	0	0.

3 Listing of integrals

3.1 $\int \left(d+ex^3\right)^5 \left(a+bx^3+cx^6\right)\, dx$

Optimal. Leaf size=163

$$\frac{1}{16}e^3x^{16}\left(e(ae+5bd)+10cd^2\right)+\frac{5}{13}de^2x^{13}\left(e(ae+2bd)+2cd^2\right)+\frac{1}{2}d^2ex^{10}\left(2e(ae+bd)+cd^2\right)+\frac{1}{7}d^3x^7\left(5e(2ae+bd)+cd^2\right)+\frac{1}{4}d^4x^4(5ae+bd)+ad^5x+\frac{1}{19}e^4x^{19}(be+5cd)+\frac{1}{22}ce^5x^{22}$$

```
[Out] a*d^5*x + (d^4*(b*d + 5*a*e)*x^4)/4 + (d^3*(c*d^2 + 5*e*(b*d + 2*
a*e))*x^7)/7 + (d^2*e*(c*d^2 + 2*e*(b*d + a*e))*x^10)/2 + (5*d*e^
2*(2*c*d^2 + e*(2*b*d + a*e))*x^13)/13 + (e^3*(10*c*d^2 + e*(5*b*
d + a*e))*x^16)/16 + (e^4*(5*c*d + b*e)*x^19)/19 + (c*e^5*x^22)/2
2
```

Rubi [A] time = 0.383542, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{1}{16}e^3x^{16}\left(e(ae+5bd)+10cd^2\right)+\frac{5}{13}de^2x^{13}\left(e(ae+2bd)+2cd^2\right)+\frac{1}{2}d^2ex^{10}\left(2e(ae+bd)+cd^2\right)+\frac{1}{7}d^3x^7\left(5e(2ae+bd)+cd^2\right)+\frac{1}{4}d^4x^4(5ae+bd)+ad^5x+\frac{1}{19}e^4x^{19}(be+5cd)+\frac{1}{22}ce^5x^{22}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^3)^5*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^5*x + (d^4*(b*d + 5*a*e)*x^4)/4 + (d^3*(c*d^2 + 5*e*(b*d + 2*
a*e))*x^7)/7 + (d^2*e*(c*d^2 + 2*e*(b*d + a*e))*x^10)/2 + (5*d*e^
2*(2*c*d^2 + e*(2*b*d + a*e))*x^13)/13 + (e^3*(10*c*d^2 + e*(5*b*
d + a*e))*x^16)/16 + (e^4*(5*c*d + b*e)*x^19)/19 + (c*e^5*x^22)/2
2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ce^5x^{22}}{22}+d^5\int a\,dx+\frac{d^4x^4\left(5ae+bd\right)}{4}+\frac{d^3x^7\left(10ae^2+5bde+cd^2\right)}{7}+\frac{d^2ex^{10}\left(2ae^2+2bde+cd^2\right)}{2}+\frac{5de^2x^{13}\left(ae^2+2bde+2cd^2\right)}{13}+\frac{e^4x^{19}\left(be+5cd\right)}{19}+\frac{e^3x^{16}\left(ae^2+5bde+10cd^2\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**3+d)**5*(c*x**6+b*x**3+a),x)
```

```
[Out] c*e**5*x**22/22 + d**5*Integral(a, x) + d**4*x**4*(5*a*e + b*d)/4
 + d**3*x**7*(10*a*e**2 + 5*b*d*e + c*d**2)/7 + d**2*e*x**10*(2*a
*e**2 + 2*b*d*e + c*d**2)/2 + 5*d*e**2*x**13*(a*e**2 + 2*b*d*e +
2*c*d**2)/13 + e**4*x**19*(b*e + 5*c*d)/19 + e**3*x**16*(a*e**2 +
 5*b*d*e + 10*c*d**2)/16
```

Mathematica [A] time = 0.0799074, size = 164, normalized size = 1.01

$$\frac{5}{13}de^2x^{13}\left(ae^2+2bde+2cd^2\right)+\frac{1}{2}d^2ex^{10}\left(2ae^2+2bde+cd^2\right)+\frac{1}{16}e^3x^{16}\left(ae^2+5bde+10cd^2\right)+\frac{1}{7}d^3x^7\left(10ae^2+5bde+cd^2\right)+\frac{1}{4}d^4x^4(5ae+bd)+ad^5x+\frac{1}{19}e^4x^{19}(be+5cd)+\frac{1}{22}ce^5x^{22}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^3)^5*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^5*x + (d^4*(b*d + 5*a*e)*x^4)/4 + (d^3*(c*d^2 + 5*b*d*e + 10*
a*e^2)*x^7)/7 + (d^2*e*(c*d^2 + 2*b*d*e + 2*a*e^2)*x^10)/2 + (5*d
*e^2*(2*c*d^2 + 2*b*d*e + a*e^2)*x^13)/13 + (e^3*(10*c*d^2 + 5*b*
d*e + a*e^2)*x^16)/16 + (e^4*(5*c*d + b*e)*x^19)/19 + (c*e^5*x^22
)/22
```

Maple [A] time = 0.003, size = 169, normalized size = 1.

$$\frac{ce^5x^{22}}{22}+\frac{\left(e^5b+5\,de^4c\right)x^{19}}{19}+\frac{\left(e^5a+5\,de^4b+10\,d^2e^3c\right)x^{16}}{16}+\frac{\left(5\,de^4a+10\,d^2e^3b+10\,d^3e^2c\right)x^{13}}{13}+\frac{\left(10\,d^2e^3a+10\,d^3e^2b+5\,d^4ec\right)x^{10}}{10}+\frac{\left(10\,ad^3e^2+5\,d^4eb+cd^5\right)x^7}{7}+\frac{\left(5\,d^4ea+d^5b\right)x^4}{4}+ad^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^3+d)^5*(c*x^6+b*x^3+a),x)
```

```
[Out] 1/22*c*e^5*x^22+1/19*(b*e^5+5*c*d*e^4)*x^19+1/16*(a*e^5+5*b*d*e^4
+10*c*d^2*e^3)*x^16+1/13*(5*a*d*e^4+10*b*d^2*e^3+10*c*d^3*e^2)*x^
13+1/10*(10*a*d^2*e^3+10*b*d^3*e^2+5*c*d^4*e)*x^10+1/7*(10*a*d^3*
e^2+5*b*d^4*e+c*d^5)*x^7+1/4*(5*a*d^4*e+b*d^5)*x^4+a*d^5*x
```

Maxima [A] time = 0.740458, size = 224, normalized size = 1.37

$$\frac{1}{22}\,ce^{5}x^{22}+\frac{1}{19}\,\left(5\,cde^{4}+be^{5}\right)x^{19}+\frac{1}{16}\,\left(10\,cd^{2}e^{3}+5\,bde^{4}+ae^{5}\right)x^{16}+\frac{5}{13}\,\left(2\,cd^{3}e^{2}+2\,bd^{2}e^{3}+ade^{4}\right)x^{13}+\frac{1}{2}\,\left(cd^{4}e+2\,bd^{3}e^{2}+2\,ad^{2}e^{3}\right)x^{10}+\frac{1}{7}\,\left(cd^{5}+5\,bd^{4}e+10\,ad^{3}e^{2}\right)x^{7}+ad^{5}x+\frac{1}{4}\,\left(bd^{5}+5\,ad^{4}e\right)x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^5,x, algorithm="maxima")
```

```
[Out] 1/22*c*e^5*x^22 + 1/19*(5*c*d*e^4 + b*e^5)*x^19 + 1/16*(10*c*d^2*
e^3 + 5*b*d*e^4 + a*e^5)*x^16 + 5/13*(2*c*d^3*e^2 + 2*b*d^2*e^3 +
 a*d*e^4)*x^13 + 1/2*(c*d^4*e + 2*b*d^3*e^2 + 2*a*d^2*e^3)*x^10 +
 1/7*(c*d^5 + 5*b*d^4*e + 10*a*d^3*e^2)*x^7 + a*d^5*x + 1/4*(b*d^
5 + 5*a*d^4*e)*x^4
```

Fricas [A] time = 0.225141, size = 1, normalized size = 0.01

$$\frac{1}{22}x^{22}e^{5}c+\frac{5}{19}x^{19}e^{4}dc+\frac{1}{19}x^{19}e^{5}b+\frac{5}{8}x^{16}e^{3}d^{2}c+\frac{5}{16}x^{16}e^{4}db+\frac{1}{16}x^{16}e^{5}a+\frac{10}{13}x^{13}e^{2}d^{3}c+\frac{10}{13}x^{13}e^{3}d^{2}b+\frac{5}{13}x^{13}e^{4}da+\frac{1}{2}x^{10}ed^{4}c+x^{10}e^{2}d^{3}b+x^{10}e^{3}d^{2}a+\frac{1}{7}x^{7}d^{5}c+\frac{5}{7}x^{7}ed^{4}b+\frac{10}{7}x^{7}e^{2}d^{3}a+\frac{1}{4}x^{4}d^{5}b+\frac{5}{4}x^{4}ed^{4}a+xd^{5}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^5,x, algorithm="fricas")
```

```
[Out] 1/22*x^22*e^5*c + 5/19*x^19*e^4*d*c + 1/19*x^19*e^5*b + 5/8*x^16*
e^3*d^2*c + 5/16*x^16*e^4*d*b + 1/16*x^16*e^5*a + 10/13*x^13*e^2*
d^3*c + 10/13*x^13*e^3*d^2*b + 5/13*x^13*e^4*d*a + 1/2*x^10*e*d^4
*c + x^10*e^2*d^3*b + x^10*e^3*d^2*a + 1/7*x^7*d^5*c + 5/7*x^7*e*
d^4*b + 10/7*x^7*e^2*d^3*a + 1/4*x^4*d^5*b + 5/4*x^4*e*d^4*a + x*
d^5*a
```

Sympy [A] time = 0.187195, size = 187, normalized size = 1.15

$$ad^{5}x+\frac{ce^{5}x^{22}}{22}+x^{19}\left(\frac{be^{5}}{19}+\frac{5cde^{4}}{19}\right)+x^{16}\left(\frac{ae^{5}}{16}+\frac{5bde^{4}}{16}+\frac{5cd^{2}e^{3}}{8}\right)+x^{13}\left(\frac{5ade^{4}}{13}+\frac{10bd^{2}e^{3}}{13}+\frac{10cd^{3}e^{2}}{13}\right)+x^{10}\left(ad^{2}e^{3}+bd^{3}e^{2}+\frac{cd^{4}e}{2}\right)+x^{7}\left(\frac{10ad^{3}e^{2}}{7}+\frac{5bd^{4}e}{7}+\frac{cd^{5}}{7}\right)+x^{4}\left(\frac{5ad^{4}e}{4}+\frac{bd^{5}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**3+d)**5*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**5*x + c*e**5*x**22/22 + x**19*(b*e**5/19 + 5*c*d*e**4/19) +
x**16*(a*e**5/16 + 5*b*d*e**4/16 + 5*c*d**2*e**3/8) + x**13*(5*a*
d*e**4/13 + 10*b*d**2*e**3/13 + 10*c*d**3*e**2/13) + x**10*(a*d**
2*e**3 + b*d**3*e**2 + c*d**4*e/2) + x**7*(10*a*d**3*e**2/7 + 5*b
*d**4*e/7 + c*d**5/7) + x**4*(5*a*d**4*e/4 + b*d**5/4)
```

GIAC/XCAS [A] time = 0.272933, size = 234, normalized size = 1.44

$$\frac{1}{22}cx^{22}e^5 + \frac{5}{19}cdx^{19}e^4 + \frac{1}{19}bx^{19}e^5 + \frac{5}{8}cd^2x^{16}e^3 + \frac{5}{16}bdx^{16}e^4 + \frac{1}{16}ax^{16}e^5 + \frac{10}{13}cd^3x^{13}e^2 + \frac{10}{13}bd^2x^{13}e^3 + \frac{5}{13}adx^{13}e^4 + \frac{1}{2}cd^4x^{10}e + bd^3x^{10}e^2 + ad^2x^{10}e^3 + \frac{1}{7}cd^5x^7 + \frac{5}{7}bd^4x^7e + \frac{10}{7}ad^3x^7e^2 + \frac{1}{4}bd^5x^4 + \frac{5}{4}ad^4x^4e + ad^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^5,x, algorithm="giac")
```

```
[Out] 1/22*c*x^22*e^5 + 5/19*c*d*x^19*e^4 + 1/19*b*x^19*e^5 + 5/8*c*d^2
*x^16*e^3 + 5/16*b*d*x^16*e^4 + 1/16*a*x^16*e^5 + 10/13*c*d^3*x^1
3*e^2 + 10/13*b*d^2*x^13*e^3 + 5/13*a*d*x^13*e^4 + 1/2*c*d^4*x^10
*e + b*d^3*x^10*e^2 + a*d^2*x^10*e^3 + 1/7*c*d^5*x^7 + 5/7*b*d^4*
x^7*e + 10/7*a*d^3*x^7*e^2 + 1/4*b*d^5*x^4 + 5/4*a*d^4*x^4*e + a*
d^5*x
```

3.2 $\int \left(d+e x^3\right)^4 \left(a+b x^3+c x^6\right) \, dx$

Optimal. Leaf size=135

$$\frac{1}{13}e^2x^{13}\left(e(ae+4bd)+6cd^2\right)+\frac{1}{7}d^2x^7\left(6ae^2+4bde+cd^2\right)+\frac{1}{5}dex^{10}\left(e(2ae+3bd)+2cd^2\right)+\frac{1}{4}d^3x^4(4ae+bd)+ad^4x+\frac{1}{16}e^3x^{16}(be+4cd)+\frac{1}{19}ce^4x^{19}$$

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^4)/4 + (d^2*(c*d^2 + 4*b*d*e + 6*a
*e^2)*x^7)/7 + (d*e*(2*c*d^2 + e*(3*b*d + 2*a*e))*x^10)/5 + (e^2*
(6*c*d^2 + e*(4*b*d + a*e))*x^13)/13 + (e^3*(4*c*d + b*e)*x^16)/1
6 + (c*e^4*x^19)/19
```

Rubi [A] time = 0.255889, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{1}{13}e^2x^{13}\left(e(ae+4bd)+6cd^2\right)+\frac{1}{7}d^2x^7\left(6ae^2+4bde+cd^2\right)+\frac{1}{5}dex^{10}\left(e(2ae+3bd)+2cd^2\right)+\frac{1}{4}d^3x^4(4ae+bd)+ad^4x+\frac{1}{16}e^3x^{16}(be+4cd)+\frac{1}{19}ce^4x^{19}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^3)^4*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^4)/4 + (d^2*(c*d^2 + 4*b*d*e + 6*a
*e^2)*x^7)/7 + (d*e*(2*c*d^2 + e*(3*b*d + 2*a*e))*x^10)/5 + (e^2*
(6*c*d^2 + e*(4*b*d + a*e))*x^13)/13 + (e^3*(4*c*d + b*e)*x^16)/1
6 + (c*e^4*x^19)/19
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ce^4x^{19}}{19}+d^4\int a\,dx+\frac{d^3x^4\left(4ae+bd\right)}{4}+\frac{d^2x^7\left(6ae^2+4bde+cd^2\right)}{7}+\frac{dex^{10}\left(2ae^2+3bde+2cd^2\right)}{5}+\frac{e^3x^{16}\left(be+4cd\right)}{16}+\frac{e^2x^{13}\left(ae^2+4bde+6cd^2\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**3+d)**4*(c*x**6+b*x**3+a),x)
```

```
[Out] c*e**4*x**19/19 + d**4*Integral(a, x) + d**3*x**4*(4*a*e + b*d)/4
 + d**2*x**7*(6*a*e**2 + 4*b*d*e + c*d**2)/7 + d*e*x**10*(2*a*e**
2 + 3*b*d*e + 2*c*d**2)/5 + e**3*x**16*(b*e + 4*c*d)/16 + e**2*x*
*13*(a*e**2 + 4*b*d*e + 6*c*d**2)/13
```

Mathematica [A] time = 0.0706769, size = 135, normalized size = 1.

$$\frac{1}{13}e^2x^{13}\left(ae^2+4bde+6cd^2\right)+\frac{1}{5}dex^{10}\left(2ae^2+3bde+2cd^2\right)+\frac{1}{7}d^2x^7\left(6ae^2+4bde+cd^2\right)+\frac{1}{4}d^3x^4(4ae+bd)+ad^4x+\frac{1}{16}e^3x^{16}(be+4cd)+\frac{1}{19}ce^4x^{19}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^3)^4*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^4)/4 + (d^2*(c*d^2 + 4*b*d*e + 6*a
*e^2)*x^7)/7 + (d*e*(2*c*d^2 + 3*b*d*e + 2*a*e^2)*x^10)/5 + (e^2*
(6*c*d^2 + 4*b*d*e + a*e^2)*x^13)/13 + (e^3*(4*c*d + b*e)*x^16)/1
6 + (c*e^4*x^19)/19
```

Maple [A] time = 0.001, size = 136, normalized size = 1.

$$\frac{ce^4x^{19}}{19}+\frac{\left(e^4b+4\,de^3c\right)x^{16}}{16}+\frac{\left(ae^4+4\,de^3b+6\,d^2e^2c\right)x^{13}}{13}+\frac{\left(4\,de^3a+6\,d^2e^2b+4\,d^3ec\right)x^{10}}{10}+\frac{\left(6\,ad^2e^2+4\,bd^3e+cd^4\right)x^7}{7}+\frac{\left(4\,d^3ea+d^4b\right)x^4}{4}+ad^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^3+d)^4*(c*x^6+b*x^3+a),x)
```

```
[Out] 1/19*c*e^4*x^19+1/16*(b*e^4+4*c*d*e^3)*x^16+1/13*(a*e^4+4*b*d*e^3
+6*c*d^2*e^2)*x^13+1/10*(4*a*d*e^3+6*b*d^2*e^2+4*c*d^3*e)*x^10+1/
7*(6*a*d^2*e^2+4*b*d^3*e+c*d^4)*x^7+1/4*(4*a*d^3*e+b*d^4)*x^4+a*d
^4*x
```

Maxima [A] time = 0.738172, size = 182, normalized size = 1.35

$$\frac{1}{19}\,ce^4x^{19}+\frac{1}{16}\left(4\,cde^3+be^4\right)x^{16}+\frac{1}{13}\left(6\,cd^2e^2+4\,bde^3+ae^4\right)x^{13}+\frac{1}{5}\left(2\,cd^3e+3\,bd^2e^2+2\,ade^3\right)x^{10}+\frac{1}{7}\left(cd^4+4\,bd^3e+6\,ad^2e^2\right)x^7+ad^4x+\frac{1}{4}\left(bd^4+4\,ad^3e\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^4,x, algorithm="maxima")
```

```
[Out] 1/19*c*e^4*x^19 + 1/16*(4*c*d*e^3 + b*e^4)*x^16 + 1/13*(6*c*d^2*e
^2 + 4*b*d*e^3 + a*e^4)*x^13 + 1/5*(2*c*d^3*e + 3*b*d^2*e^2 + 2*a
*d*e^3)*x^10 + 1/7*(c*d^4 + 4*b*d^3*e + 6*a*d^2*e^2)*x^7 + a*d^4*
x + 1/4*(b*d^4 + 4*a*d^3*e)*x^4
```

Fricas [A] time = 0.237477, size = 1, normalized size = 0.01

$$\frac{1}{19}x^{19}e^4c + \frac{1}{4}x^{16}e^3dc + \frac{1}{16}x^{16}e^4b + \frac{6}{13}x^{13}e^2d^2c + \frac{4}{13}x^{13}e^3db + \frac{1}{13}x^{13}e^4a + \frac{2}{5}x^{10}ed^3c + \frac{3}{5}x^{10}e^2d^2b + \frac{2}{5}x^{10}e^3da + \frac{1}{7}x^7d^4c + \frac{4}{7}x^7ed^3b + \frac{6}{7}x^7e^2d^2a + \frac{1}{4}x^4d^4b + x^4ed^3a + xd^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^4,x, algorithm="fricas")
```

```
[Out] 1/19*x^19*e^4*c + 1/4*x^16*e^3*d*c + 1/16*x^16*e^4*b + 6/13*x^13*
e^2*d^2*c + 4/13*x^13*e^3*d*b + 1/13*x^13*e^4*a + 2/5*x^10*e*d^3*
c + 3/5*x^10*e^2*d^2*b + 2/5*x^10*e^3*d*a + 1/7*x^7*d^4*c + 4/7*x
^7*e*d^3*b + 6/7*x^7*e^2*d^2*a + 1/4*x^4*d^4*b + x^4*e*d^3*a + x*
d^4*a
```

Sympy [A] time = 0.164237, size = 151, normalized size = 1.12

$$ad^4x + \frac{ce^4x^{19}}{19} + x^{16}\left(\frac{be^4}{16} + \frac{cde^3}{4}\right) + x^{13}\left(\frac{ae^4}{13} + \frac{4bde^3}{13} + \frac{6cd^2e^2}{13}\right) + x^{10}\left(\frac{2ade^3}{5} + \frac{3bd^2e^2}{5} + \frac{2cd^3e}{5}\right) + x^7\left(\frac{6ad^2e^2}{7} + \frac{4bd^3e}{7} + \frac{cd^4}{7}\right) + x^4\left(ad^3e + \frac{bd^4}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**3+d)**4*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**4*x + c*e**4*x**19/19 + x**16*(b*e**4/16 + c*d*e**3/4) + x**
13*(a*e**4/13 + 4*b*d*e**3/13 + 6*c*d**2*e**2/13) + x**10*(2*a*d*
e**3/5 + 3*b*d**2*e**2/5 + 2*c*d**3*e/5) + x**7*(6*a*d**2*e**2/7
+ 4*b*d**3*e/7 + c*d**4/7) + x**4*(a*d**3*e + b*d**4/4)
```

GIAC/XCAS [A] time = 0.270159, size = 190, normalized size = 1.41

$$\frac{1}{19}\,cx^{19}e^{4}+\frac{1}{4}\,cdx^{16}e^{3}+\frac{1}{16}\,bx^{16}e^{4}+\frac{6}{13}\,cd^{2}x^{13}e^{2}+\frac{4}{13}\,bdx^{13}e^{3}+\frac{1}{13}\,ax^{13}e^{4}+\frac{2}{5}\,cd^{3}x^{10}e+\frac{3}{5}\,bd^{2}x^{10}e^{2}+\frac{2}{5}\,adx^{10}e^{3}+\frac{1}{7}\,cd^{4}x^{7}+\frac{4}{7}\,bd^{3}x^{7}e+\frac{6}{7}\,ad^{2}x^{7}e^{2}+\frac{1}{4}\,bd^{4}x^{4}+ad^{3}x^{4}e+ad^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^4,x, algorithm="giac")
```

```
[Out] 1/19*c*x^19*e^4 + 1/4*c*d*x^16*e^3 + 1/16*b*x^16*e^4 + 6/13*c*d^2
*x^13*e^2 + 4/13*b*d*x^13*e^3 + 1/13*a*x^13*e^4 + 2/5*c*d^3*x^10*
e + 3/5*b*d^2*x^10*e^2 + 2/5*a*d*x^10*e^3 + 1/7*c*d^4*x^7 + 4/7*b
*d^3*x^7*e + 6/7*a*d^2*x^7*e^2 + 1/4*b*d^4*x^4 + a*d^3*x^4*e + a*
d^4*x
```

3.3 $\int \left(d+ex^3\right)^3 \left(a+bx^3+cx^6\right)\, dx$

Optimal. Leaf size=103

$$\frac{1}{10} e x^{10} \left(e(ae+3bd)+3cd^2\right)+\frac{1}{7} d x^7 \left(3e(ae+bd)+cd^2\right)+\frac{1}{4} d^2 x^4 (3ae+bd)+ad^3 x+\frac{1}{13} e^2 x^{13} (be+3cd)+\frac{1}{16} c e^3 x^{16}$$

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^4)/4 + (d*(c*d^2 + 3*e*(b*d + a*e)
)*x^7)/7 + (e*(3*c*d^2 + e*(3*b*d + a*e))*x^10)/10 + (e^2*(3*c*d
+ b*e)*x^13)/13 + (c*e^3*x^16)/16
```

Rubi [A] time = 0.20299, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{1}{10} e x^{10} \left(e(ae+3bd)+3cd^2\right)+\frac{1}{7} d x^7 \left(3e(ae+bd)+cd^2\right)+\frac{1}{4} d^2 x^4 (3ae+bd)+ad^3 x+\frac{1}{13} e^2 x^{13} (be+3cd)+\frac{1}{16} c e^3 x^{16}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^3)^3*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^4)/4 + (d*(c*d^2 + 3*e*(b*d + a*e)
)*x^7)/7 + (e*(3*c*d^2 + e*(3*b*d + a*e))*x^10)/10 + (e^2*(3*c*d
+ b*e)*x^13)/13 + (c*e^3*x^16)/16
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ce^3x^{16}}{16}+d^3\int a\,dx+\frac{d^2x^4\,(3ae+bd)}{4}+\frac{dx^7\left(3ae^2+3bde+cd^2\right)}{7}+\frac{e^2x^{13}\,(be+3cd)}{13}+\frac{ex^{10}\left(ae^2+3bde+3cd^2\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**3+d)**3*(c*x**6+b*x**3+a),x)
```

```
[Out] c*e**3*x**16/16 + d**3*Integral(a, x) + d**2*x**4*(3*a*e + b*d)/4
 + d*x**7*(3*a*e**2 + 3*b*d*e + c*d**2)/7 + e**2*x**13*(b*e + 3*c
*d)/13 + e*x**10*(a*e**2 + 3*b*d*e + 3*c*d**2)/10
```

Mathematica [A] time = 0.0467761, size = 104, normalized size = 1.01

$$\frac{1}{10}ex^{10}\left(ae^2+3bde+3cd^2\right)+\frac{1}{7}dx^7\left(3ae^2+3bde+cd^2\right)+\frac{1}{4}d^2x^4(3ae+bd)+ad^3x+\frac{1}{13}e^2x^{13}(be+3cd)+\frac{1}{16}ce^3x^{16}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^3)^3*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^4)/4 + (d*(c*d^2 + 3*b*d*e + 3*a*e
^2)*x^7)/7 + (e*(3*c*d^2 + 3*b*d*e + a*e^2)*x^10)/10 + (e^2*(3*c*
d + b*e)*x^13)/13 + (c*e^3*x^16)/16
```

Maple [A] time = 0.001, size = 103, normalized size = 1.

$$\frac{ce^3x^{16}}{16}+\frac{\left(e^3b+3\,cde^2\right)x^{13}}{13}+\frac{\left(e^3a+3\,bde^2+3\,d^2ec\right)x^{10}}{10}+\frac{\left(3\,e^2da+3\,bd^2e+cd^3\right)x^7}{7}+\frac{\left(3\,ad^2e+bd^3\right)x^4}{4}+ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^3+d)^3*(c*x^6+b*x^3+a),x)
```

```
[Out] 1/16*c*e^3*x^16+1/13*(b*e^3+3*c*d*e^2)*x^13+1/10*(a*e^3+3*b*d*e^2
+3*c*d^2*e)*x^10+1/7*(3*a*d*e^2+3*b*d^2*e+c*d^3)*x^7+1/4*(3*a*d^2
*e+b*d^3)*x^4+a*d^3*x
```

Maxima [A] time = 0.741819, size = 138, normalized size = 1.34

$$\frac{1}{16}\,ce^3x^{16}+\frac{1}{13}\left(3\,cde^2+be^3\right)x^{13}+\frac{1}{10}\left(3\,cd^2e+3\,bde^2+ae^3\right)x^{10}+\frac{1}{7}\left(cd^3+3\,bd^2e+3\,ade^2\right)x^7+ad^3x+\frac{1}{4}\left(bd^3+3\,ad^2e\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*c*e^3*x^16 + 1/13*(3*c*d*e^2 + b*e^3)*x^13 + 1/10*(3*c*d^2*e
 + 3*b*d*e^2 + a*e^3)*x^10 + 1/7*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*
x^7 + a*d^3*x + 1/4*(b*d^3 + 3*a*d^2*e)*x^4
```

Fricas [A] time = 0.235499, size = 1, normalized size = 0.01

$$\frac{1}{16}x^{16}e^3c + \frac{3}{13}x^{13}e^2dc + \frac{1}{13}x^{13}e^3b + \frac{3}{10}x^{10}ed^2c + \frac{3}{10}x^{10}e^2db + \frac{1}{10}x^{10}e^3a + \frac{1}{7}x^7d^3c + \frac{3}{7}x^7ed^2b + \frac{3}{7}x^7e^2da + \frac{1}{4}x^4d^3b + \frac{3}{4}x^4ed^2a + xd^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*e^3*c + 3/13*x^13*e^2*d*c + 1/13*x^13*e^3*b + 3/10*x^10
*e*d^2*c + 3/10*x^10*e^2*d*b + 1/10*x^10*e^3*a + 1/7*x^7*d^3*c +
3/7*x^7*e*d^2*b + 3/7*x^7*e^2*d*a + 1/4*x^4*d^3*b + 3/4*x^4*e*d^2
*a + x*d^3*a
```

Sympy [A] time = 0.150497, size = 117, normalized size = 1.14

$$ad^3x + \frac{ce^3x^{16}}{16} + x^{13}\left(\frac{be^3}{13} + \frac{3cde^2}{13}\right) + x^{10}\left(\frac{ae^3}{10} + \frac{3bde^2}{10} + \frac{3cd^2e}{10}\right) + x^7\left(\frac{3ade^2}{7} + \frac{3bd^2e}{7} + \frac{cd^3}{7}\right) + x^4\left(\frac{3ad^2e}{4} + \frac{bd^3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**3+d)**3*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**3*x + c*e**3*x**16/16 + x**13*(b*e**3/13 + 3*c*d*e**2/13) +
x**10*(a*e**3/10 + 3*b*d*e**2/10 + 3*c*d**2*e/10) + x**7*(3*a*d*e
**2/7 + 3*b*d**2*e/7 + c*d**3/7) + x**4*(3*a*d**2*e/4 + b*d**3/4)
```

GIAC/XCAS [A] time = 0.269901, size = 147, normalized size = 1.43

$$\frac{1}{16}\,cx^{16}e^3 + \frac{3}{13}\,cdx^{13}e^2 + \frac{1}{13}\,bx^{13}e^3 + \frac{3}{10}\,cd^2x^{10}e + \frac{3}{10}\,bdx^{10}e^2 + \frac{1}{10}\,ax^{10}e^3 + \frac{1}{7}\,cd^3x^7 + \frac{3}{7}\,bd^2x^7e + \frac{3}{7}\,adx^7e^2 + \frac{1}{4}\,bd^3x^4 + \frac{3}{4}\,ad^2x^4e + ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^3,x, algorithm="giac")
```

```
[Out] 1/16*c*x^16*e^3 + 3/13*c*d*x^13*e^2 + 1/13*b*x^13*e^3 + 3/10*c*d^
2*x^10*e + 3/10*b*d*x^10*e^2 + 1/10*a*x^10*e^3 + 1/7*c*d^3*x^7 +
3/7*b*d^2*x^7*e + 3/7*a*d*x^7*e^2 + 1/4*b*d^3*x^4 + 3/4*a*d^2*x^4
*e + a*d^3*x
```

3.4 $\int \left(d+ex^3\right)^2 \left(a+bx^3+cx^6\right)\,dx$

Optimal. Leaf size=73

$$\frac{1}{7}x^7\left(e(ae+2bd)+cd^2\right)+\frac{1}{4}dx^4(2ae+bd)+ad^2x+\frac{1}{10}ex^{10}(be+2cd)+\frac{1}{13}ce^2x^{13}$$

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^4)/4 + ((c*d^2 + e*(2*b*d + a*e))*x^
7)/7 + (e*(2*c*d + b*e)*x^10)/10 + (c*e^2*x^13)/13
```

Rubi [A] time = 0.132012, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{1}{7}x^7\left(e(ae+2bd)+cd^2\right)+\frac{1}{4}dx^4(2ae+bd)+ad^2x+\frac{1}{10}ex^{10}(be+2cd)+\frac{1}{13}ce^2x^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^3)^2*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^4)/4 + ((c*d^2 + e*(2*b*d + a*e))*x^
7)/7 + (e*(2*c*d + b*e)*x^10)/10 + (c*e^2*x^13)/13
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ce^2x^{13}}{13}+d^2\int a\,dx+\frac{dx^4\,(2ae+bd)}{4}+\frac{ex^{10}\,(be+2cd)}{10}+x^7\left(\frac{ae^2}{7}+\frac{2bde}{7}+\frac{cd^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**3+d)**2*(c*x**6+b*x**3+a),x)
```

```
[Out] c*e**2*x**13/13 + d**2*Integral(a, x) + d*x**4*(2*a*e + b*d)/4 +
e*x**10*(b*e + 2*c*d)/10 + x**7*(a*e**2/7 + 2*b*d*e/7 + c*d**2/7)
```

Mathematica [A] time = 0.0368057, size = 73, normalized size = 1.

$$\frac{1}{7}x^7\left(ae^2+2bde+cd^2\right)+\frac{1}{4}dx^4(2ae+bd)+ad^2x+\frac{1}{10}ex^{10}(be+2cd)+\frac{1}{13}ce^2x^{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^3)^2*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^4)/4 + ((c*d^2 + 2*b*d*e + a*e^2)*x^
7)/7 + (e*(2*c*d + b*e)*x^10)/10 + (c*e^2*x^13)/13
```

Maple [A] time = 0.001, size = 70, normalized size = 1.

$$\frac{ce^2x^{13}}{13} + \frac{\left(be^2 + 2\,cde\right)x^{10}}{10} + \frac{\left(ae^2 + 2\,bde + cd^2\right)x^7}{7} + \frac{\left(2\,ade + bd^2\right)x^4}{4} + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^3+d)^2*(c*x^6+b*x^3+a),x)
```

```
[Out] 1/13*c*e^2*x^13+1/10*(b*e^2+2*c*d*e)*x^10+1/7*(a*e^2+2*b*d*e+c*d^
2)*x^7+1/4*(2*a*d*e+b*d^2)*x^4+a*d^2*x
```

Maxima [A] time = 0.743146, size = 93, normalized size = 1.27

$$\frac{1}{13}\,ce^2x^{13} + \frac{1}{10}\left(2\,cde + be^2\right)x^{10} + \frac{1}{7}\left(cd^2 + 2\,bde + ae^2\right)x^7 + \frac{1}{4}\left(bd^2 + 2\,ade\right)x^4 + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^2,x, algorithm="maxima")
```

```
[Out] 1/13*c*e^2*x^13 + 1/10*(2*c*d*e + b*e^2)*x^10 + 1/7*(c*d^2 + 2*b*
d*e + a*e^2)*x^7 + 1/4*(b*d^2 + 2*a*d*e)*x^4 + a*d^2*x
```

Fricas [A] time = 0.232728, size = 1, normalized size = 0.01

$$\frac{1}{13}x^{13}e^2c + \frac{1}{5}x^{10}edc + \frac{1}{10}x^{10}e^2b + \frac{1}{7}x^7d^2c + \frac{2}{7}x^7edb + \frac{1}{7}x^7e^2a + \frac{1}{4}x^4d^2b + \frac{1}{2}x^4eda + xd^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*e^2*c + 1/5*x^10*e*d*c + 1/10*x^10*e^2*b + 1/7*x^7*d^2*
c + 2/7*x^7*e*d*b + 1/7*x^7*e^2*a + 1/4*x^4*d^2*b + 1/2*x^4*e*d*a
 + x*d^2*a
```

Sympy [A] time = 0.121311, size = 75, normalized size = 1.03

$$ad^2x + \frac{ce^2x^{13}}{13} + x^{10}\left(\frac{be^2}{10} + \frac{cde}{5}\right) + x^7\left(\frac{ae^2}{7} + \frac{2bde}{7} + \frac{cd^2}{7}\right) + x^4\left(\frac{ade}{2} + \frac{bd^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**3+d)**2*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**2*x + c*e**2*x**13/13 + x**10*(b*e**2/10 + c*d*e/5) + x**7*(
a*e**2/7 + 2*b*d*e/7 + c*d**2/7) + x**4*(a*d*e/2 + b*d**2/4)
```

GIAC/XCAS [A] time = 0.268413, size = 103, normalized size = 1.41

$$\frac{1}{13}\,cx^{13}e^2 + \frac{1}{5}\,cdx^{10}e + \frac{1}{10}\,bx^{10}e^2 + \frac{1}{7}\,cd^2x^7 + \frac{2}{7}\,bdx^7e + \frac{1}{7}\,ax^7e^2 + \frac{1}{4}\,bd^2x^4 + \frac{1}{2}\,adx^4e + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^2,x, algorithm="giac")
```

```
[Out] 1/13*c*x^13*e^2 + 1/5*c*d*x^10*e + 1/10*b*x^10*e^2 + 1/7*c*d^2*x^
7 + 2/7*b*d*x^7*e + 1/7*a*x^7*e^2 + 1/4*b*d^2*x^4 + 1/2*a*d*x^4*e
 + a*d^2*x
```

3.5 $\int (d + ex^3) (a + bx^3 + cx^6) \, dx$

Optimal. Leaf size=42

$$\frac{1}{4}x^4(ae + bd) + adx + \frac{1}{7}x^7(be + cd) + \frac{1}{10}cex^{10}$$

```
[Out] a*d*x + ((b*d + a*e)*x^4)/4 + ((c*d + b*e)*x^7)/7 + (c*e*x^10)/10
```

Rubi [A] time = 0.0614345, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$\frac{1}{4}x^4(ae + bd) + adx + \frac{1}{7}x^7(be + cd) + \frac{1}{10}cex^{10}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^3)*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^4)/4 + ((c*d + b*e)*x^7)/7 + (c*e*x^10)/10
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{cex^{10}}{10} + d\int a\,dx + x^7\left(\frac{be}{7} + \frac{cd}{7}\right) + x^4\left(\frac{ae}{4} + \frac{bd}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**3+d)*(c*x**6+b*x**3+a),x)
```

```
[Out] c*e*x**10/10 + d*Integral(a, x) + x**7*(b*e/7 + c*d/7) + x**4*(a*
e/4 + b*d/4)
```

Mathematica [A] time = 0.0149326, size = 42, normalized size = 1.

$$\frac{1}{4}x^4(ae + bd) + adx + \frac{1}{7}x^7(be + cd) + \frac{1}{10}cex^{10}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^3)*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^4)/4 + ((c*d + b*e)*x^7)/7 + (c*e*x^10)/10
```

Maple [A] time = 0.001, size = 37, normalized size = 0.9

$$adx + \frac{(ae + bd)\,x^4}{4} + \frac{(be + cd)\,x^7}{7} + \frac{cex^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^3+d)*(c*x^6+b*x^3+a),x)
```

```
[Out] a*d*x+1/4*(a*e+b*d)*x^4+1/7*(b*e+c*d)*x^7+1/10*c*e*x^10
```

Maxima [A] time = 0.743519, size = 49, normalized size = 1.17

$$\frac{1}{10}\,cex^{10} + \frac{1}{7}\,(cd + be)x^7 + \frac{1}{4}\,(bd + ae)x^4 + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d),x, algorithm="maxima")
```

```
[Out] 1/10*c*e*x^10 + 1/7*(c*d + b*e)*x^7 + 1/4*(b*d + a*e)*x^4 + a*d*x
```

Fricas [A] time = 0.228144, size = 1, normalized size = 0.02

$$\frac{1}{10}x^{10}ec + \frac{1}{7}x^7dc + \frac{1}{7}x^7eb + \frac{1}{4}x^4db + \frac{1}{4}x^4ea + xda$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e*c + 1/7*x^7*d*c + 1/7*x^7*e*b + 1/4*x^4*d*b + 1/4*x^4
*e*a + x*d*a
```

Sympy [A] time = 0.087369, size = 39, normalized size = 0.93

$$adx + \frac{cex^{10}}{10} + x^7\left(\frac{be}{7} + \frac{cd}{7}\right) + x^4\left(\frac{ae}{4} + \frac{bd}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**3+d)*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d*x + c*e*x**10/10 + x**7*(b*e/7 + c*d/7) + x**4*(a*e/4 + b*d/4
)
```

GIAC/XCAS [A] time = 0.266413, size = 58, normalized size = 1.38

$$\frac{1}{10}\,cx^{10}e + \frac{1}{7}\,cdx^7 + \frac{1}{7}\,bx^7e + \frac{1}{4}\,bdx^4 + \frac{1}{4}\,ax^4e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d),x, algorithm="giac")
```

```
[Out] 1/10*c*x^10*e + 1/7*c*d*x^7 + 1/7*b*x^7*e + 1/4*b*d*x^4 + 1/4*a*x
^4*e + a*d*x
```

3.6 $\int \frac{a+bx^3+cx^6}{d+ex^3} dx$

Optimal. Leaf size=188

$$-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(ae^2-bde+cd^2\right)}{6d^{2/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(ae^2-bde+cd^2\right)}{3d^{2/3}e^{7/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(ae^2-bde+cd^2\right)}{\sqrt{3}d^{2/3}e^{7/3}}-\frac{x(cd-be)}{e^2}+\frac{cx^4}{4e}$$

```
[Out] -(((c*d - b*e)*x)/e^2) + (c*x^4)/(4*e) - ((c*d^2 - b*d*e + a*e^2)
*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2
/3)*e^(7/3)) + ((c*d^2 - b*d*e + a*e^2)*Log[d^(1/3) + e^(1/3)*x])
/(3*d^(2/3)*e^(7/3)) - ((c*d^2 - b*d*e + a*e^2)*Log[d^(2/3) - d^(
1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(7/3))
```

Rubi [A] time = 0.410791, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(ae^2-bde+cd^2\right)}{6d^{2/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(ae^2-bde+cd^2\right)}{3d^{2/3}e^{7/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(ae^2-bde+cd^2\right)}{\sqrt{3}d^{2/3}e^{7/3}}-\frac{x(cd-be)}{e^2}+\frac{cx^4}{4e}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)/(d + e*x^3),x]
```


```
[Out] -(((c*d - b*e)*x)/e^2) + (c*x^4)/(4*e) - ((c*d^2 - b*d*e + a*e^2)
*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2
/3)*e^(7/3)) + ((c*d^2 - b*d*e + a*e^2)*Log[d^(1/3) + e^(1/3)*x])
/(3*d^(2/3)*e^(7/3)) - ((c*d^2 - b*d*e + a*e^2)*Log[d^(2/3) - d^(
1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(7/3))
```

Rubi in Sympy [A] time = 49.891, size = 175, normalized size = 0.93

$$\frac{cx^4}{4e} + \frac{x(be-cd)}{e^2} + \frac{(ae^2-bde+cd^2)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{\frac{2}{3}}e^{\frac{7}{3}}} - \frac{(ae^2-bde+cd^2)\log\left(d^{\frac{2}{3}}-\sqrt[3]{d}\sqrt[3]{e}x+e^{\frac{2}{3}}x^2\right)}{6d^{\frac{2}{3}}e^{\frac{7}{3}}} - \frac{\sqrt{3}(ae^2-bde+cd^2)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{d}}{3}-\frac{2\sqrt[3]{e}x}{3}\right)}{\sqrt[3]{d}}\right)}{3d^{\frac{2}{3}}e^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)/(e*x**3+d),x)
```

```
[Out] c*x**4/(4*e) + x*(b*e - c*d)/e**2 + (a*e**2 - b*d*e + c*d**2)*log
(d**(1/3) + e**(1/3)*x)/(3*d**(2/3)*e**(7/3)) - (a*e**2 - b*d*e +
 c*d**2)*log(d**(2/3) - d**(1/3)*e**(1/3)*x + e**(2/3)*x**2)/(6*d
**(2/3)*e**(7/3)) - sqrt(3)*(a*e**2 - b*d*e + c*d**2)*atan(sqrt(3
)*(d**(1/3)/3 - 2*e**(1/3)*x/3)/d**(1/3))/(3*d**(2/3)*e**(7/3))
```

Mathematica [A] time = 0.297117, size = 176, normalized size = 0.94

$$\frac{-\frac{2\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)(e(ae-bd)+cd^2)}{d^{2/3}} + \frac{4\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)(e(ae-bd)+cd^2)}{d^{2/3}} - \frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)(e(ae-bd)+cd^2)}{d^{2/3}} + 12\sqrt[3]{e}x(be-cd) +}{12e^{7/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3),x]
```

```
[Out] (12*e^(1/3)*(-(c*d) + b*e)*x + 3*c*e^(4/3)*x^4 - (4*Sqrt[3]*(c*d^
2 + e*(-(b*d) + a*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]]
)/d^(2/3) + (4*(c*d^2 + e*(-(b*d) + a*e))*Log[d^(1/3) + e^(1/3)*x
])/d^(2/3) - (2*(c*d^2 + e*(-(b*d) + a*e))*Log[d^(2/3) - d^(1/3)*
e^(1/3)*x + e^(2/3)*x^2])/d^(2/3))/(12*e^(7/3))
```

Maple [B] time = 0.008, size = 313, normalized size = 1.7

$$\frac{cx^4}{4e}+\frac{bx}{e}-\frac{cdx}{e^2}+\frac{a}{3e}\ln\left(x+\sqrt[3]{\frac{d}{e}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}-\frac{bd}{3e^2}\ln\left(x+\sqrt[3]{\frac{d}{e}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}+\frac{cd^2}{3e^3}\ln\left(x+\sqrt[3]{\frac{d}{e}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}-\frac{a}{6e}\ln\left(x^2-x\sqrt[3]{\frac{d}{e}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}+\frac{bd}{6e^2}\ln\left(x^2-x\sqrt[3]{\frac{d}{e}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}-\frac{cd^2}{6e^3}\ln\left(x^2-x\sqrt[3]{\frac{d}{e}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}+\frac{\sqrt{3}a}{3e}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{d}{e}}}-1\right)\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}-\frac{\sqrt{3}bd}{3e^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{d}{e}}}-1\right)\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}+\frac{\sqrt{3}cd^2}{3e^3}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{d}{e}}}-1\right)\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)/(e*x^3+d),x)
```

```
[Out] 1/4*c*x^4/e+1/e*b*x-c*d*x/e^2+1/3/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))
*a-1/3/e^2/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*b*d+1/3/e^3/(d/e)^(2/3)*
ln(x+(d/e)^(1/3))*c*d^2-1/6/e/(d/e)^(2/3)*ln(x^2-x*(d/e)^(1/3)+(d
/e)^(2/3))*a+1/6/e^2/(d/e)^(2/3)*ln(x^2-x*(d/e)^(1/3)+(d/e)^(2/3)
)*b*d-1/6/e^3/(d/e)^(2/3)*ln(x^2-x*(d/e)^(1/3)+(d/e)^(2/3))*c*d^2
+1/3/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1)
)*a-1/3/e^2/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)
*x-1))*b*d+1/3/e^3/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e
)^(1/3)*x-1))*c*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.28968, size = 231, normalized size = 1.23

$$-\frac{\sqrt{3}\left(2\sqrt{3}\left(cd^2-bde+ae^2\right)\log\left(\left(d^2e\right)^{\frac{2}{3}}x^2-\left(d^2e\right)^{\frac{1}{3}}dx+d^2\right)-4\sqrt{3}\left(cd^2-bde+ae^2\right)\log\left(\left(d^2e\right)^{\frac{1}{3}}x+d\right)-12\left(cd^2-bde\right.\right.}{36\left(d^2e\right)^{\frac{1}{3}}e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(3)*(2*sqrt(3)*(c*d^2 - b*d*e + a*e^2)*log((d^2*e)^(2/3
)*x^2 - (d^2*e)^(1/3)*d*x + d^2) - 4*sqrt(3)*(c*d^2 - b*d*e + a*e
^2)*log((d^2*e)^(1/3)*x + d) - 12*(c*d^2 - b*d*e + a*e^2)*arctan(
1/3*(2*sqrt(3)*(d^2*e)^(1/3)*x - sqrt(3)*d)/d) - 3*sqrt(3)*(c*e*x
^4 - 4*(c*d - b*e)*x)*(d^2*e)^(1/3))/((d^2*e)^(1/3)*e^2)
```

Sympy [A] time = 3.72324, size = 175, normalized size = 0.93

$$\frac{cx^4}{4e}$$
$$+\mathrm{RootSum}\left(27t^3d^2e^7-a^3e^6+3a^2bde^5-3a^2cd^2e^4-3ab^2d^2e^4+6abcd^3e^3-3ac^2d^4e^2+b^3d^3e^3-3b^2cd^4e^2+3bc^2d^5e-c^3d\right.$$
$$+\frac{x(be-cd)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)/(e*x**3+d),x)
```

```
[Out] c*x**4/(4*e) + RootSum(27*_t**3*d**2*e**7 - a**3*e**6 + 3*a**2*b*
d*e**5 - 3*a**2*c*d**2*e**4 - 3*a*b**2*d**2*e**4 + 6*a*b*c*d**3*e
**3 - 3*a*c**2*d**4*e**2 + b**3*d**3*e**3 - 3*b**2*c*d**4*e**2 +
3*b*c**2*d**5*e - c**3*d**6, Lambda(_t, _t*log(3*_t*d*e**2/(a*e**
2 - b*d*e + c*d**2) + x))) + x*(b*e - c*d)/e**2
```

GIAC/XCAS [A] time = 0.284519, size = 279, normalized size = 1.48

$$\frac{\sqrt{3}\left(\left(-de^{2}\right)^{\frac{1}{3}}cd^{2}-\left(-de^{2}\right)^{\frac{1}{3}}bde+\left(-de^{2}\right)^{\frac{1}{3}}ae^{2}\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-3)}}{3d}-\frac{\left(cd^{2}e^{2}-bde^{3}+ae^{4}\right)\left(-de^{(-1)}\right)^{\frac{1}{3}}e^{(-4)}\ln\left(\left|x-\left(-de^{(-1)}\right)^{\frac{1}{3}}\right|\right)}{3d}+\frac{1}{4}\left(cx^{4}e^{3}-4cdxe^{2}+4bxe^{3}\right)e^{(-4)}+\frac{\left(\left(-de^{2}\right)^{\frac{1}{3}}cd^{2}-\left(-de^{2}\right)^{\frac{1}{3}}bde+\left(-de^{2}\right)^{\frac{1}{3}}ae^{2}\right)e^{(-3)}\ln\left(x^{2}+\left(-de^{(-1)}\right)^{\frac{1}{3}}x+\left(-de^{(-1)}\right)^{\frac{2}{3}}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-d*e^2)^(1/3)*c*d^2 - (-d*e^2)^(1/3)*b*d*e + (-d*e^
2)^(1/3)*a*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d*e^(-1))^(1/3))/(-d*
e^(-1))^(1/3))*e^(-3)/d - 1/3*(c*d^2*e^2 - b*d*e^3 + a*e^4)*(-d*e
^(-1))^(1/3)*e^(-4)*ln(abs(x - (-d*e^(-1))^(1/3)))/d + 1/4*(c*x^4
*e^3 - 4*c*d*x*e^2 + 4*b*x*e^3)*e^(-4) + 1/6*((-d*e^2)^(1/3)*c*d^
2 - (-d*e^2)^(1/3)*b*d*e + (-d*e^2)^(1/3)*a*e^2)*e^(-3)*ln(x^2 +
(-d*e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3))/d
```

3.7 $\int \frac{a+bx^3+cx^6}{(d+ex^3)^2}\,dx$

Optimal. Leaf size=213

$$\frac{x\left(ae^2-bde+cd^2\right)}{3de^2\left(d+ex^3\right)}+\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(4cd^2-e(2ae+bd)\right)}{18d^{5/3}e^{7/3}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(4cd^2-e(2ae+bd)\right)}{9d^{5/3}e^{7/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(4cd^2-e(2ae+bd)\right)}{3\sqrt{3}d^{5/3}e^{7/3}}+\frac{cx}{e^2}$$

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(3*d*e^2*(d + e*x^3)) + (
(4*c*d^2 - e*(b*d + 2*a*e))*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[
3]*d^(1/3))])/(3*Sqrt[3]*d^(5/3)*e^(7/3)) - ((4*c*d^2 - e*(b*d +
2*a*e))*Log[d^(1/3) + e^(1/3)*x])/(9*d^(5/3)*e^(7/3)) + ((4*c*d^2
 - e*(b*d + 2*a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2
])/(18*d^(5/3)*e^(7/3))
```

Rubi [A] time = 0.448877, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$\frac{x\left(ae^2-bde+cd^2\right)}{3de^2\left(d+ex^3\right)}+\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(4cd^2-e(2ae+bd)\right)}{18d^{5/3}e^{7/3}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(4cd^2-e(2ae+bd)\right)}{9d^{5/3}e^{7/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(4cd^2-e(2ae+bd)\right)}{3\sqrt{3}d^{5/3}e^{7/3}}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(3*d*e^2*(d + e*x^3)) + (
(4*c*d^2 - e*(b*d + 2*a*e))*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[
3]*d^(1/3))])/(3*Sqrt[3]*d^(5/3)*e^(7/3)) - ((4*c*d^2 - e*(b*d +
2*a*e))*Log[d^(1/3) + e^(1/3)*x])/(9*d^(5/3)*e^(7/3)) + ((4*c*d^2
 - e*(b*d + 2*a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2
])/(18*d^(5/3)*e^(7/3))
```

Rubi in Sympy [A] time = 64.4256, size = 202, normalized size = 0.95

$$\frac{cx}{e^2} + \frac{x\left(ae^2 - bde + cd^2\right)}{3de^2\left(d + ex^3\right)} + \frac{\left(2ae^2 + bde - 4cd^2\right)\log\left(\sqrt[3]{d} + \sqrt[3]{e}x\right)}{9d^{\frac{5}{3}}e^{\frac{7}{3}}} - \frac{\left(2ae^2 + bde - 4cd^2\right)\log\left(d^{\frac{2}{3}} - \sqrt[3]{d}\sqrt[3]{e}x + e^{\frac{2}{3}}x^2\right)}{18d^{\frac{5}{3}}e^{\frac{7}{3}}} - \frac{\sqrt{3}\left(2ae^2 + bde - 4cd^2\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{d}}{3} - \frac{2\sqrt[3]{e}x}{3}\right)}{\sqrt[3]{d}}\right)}{9d^{\frac{5}{3}}e^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)/(e*x**3+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(3*d*e**2*(d + e*x**3)) +
(2*a*e**2 + b*d*e - 4*c*d**2)*log(d**(1/3) + e**(1/3)*x)/(9*d**(5
/3)*e**(7/3)) - (2*a*e**2 + b*d*e - 4*c*d**2)*log(d**(2/3) - d**(
1/3)*e**(1/3)*x + e**(2/3)*x**2)/(18*d**(5/3)*e**(7/3)) - sqrt(3)
*(2*a*e**2 + b*d*e - 4*c*d**2)*atan(sqrt(3)*(d**(1/3)/3 - 2*e**(1
/3)*x/3)/d**(1/3))/(9*d**(5/3)*e**(7/3))
```

Mathematica [A] time = 0.492418, size = 199, normalized size = 0.93

$$\frac{\frac{6\sqrt[3]{e}x\left(e(ae-bd)+cd^2\right)}{d\left(d+ex^3\right)} + \frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(4cd^2-e(2ae+bd)\right)}{d^{5/3}} - \frac{2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(4cd^2-e(2ae+bd)\right)}{d^{5/3}} + \frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)\left(4cd^2-e(2ae+l\right.}{d^{5/3}}}{18e^{7/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^2,x]
```

```
[Out] (18*c*e^(1/3)*x + (6*e^(1/3)*(c*d^2 + e*(-(b*d) + a*e))*x)/(d*(d
+ e*x^3)) + (2*Sqrt[3]*(4*c*d^2 - e*(b*d + 2*a*e))*ArcTan[(1 - (2
*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/d^(5/3) - (2*(4*c*d^2 - e*(b*d + 2
*a*e))*Log[d^(1/3) + e^(1/3)*x])/d^(5/3) + ((4*c*d^2 - e*(b*d + 2
*a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/d^(5/3))/(
18*e^(7/3))
```

Maple [A] time = 0.014, size = 345, normalized size = 1.6

$$\frac{cx}{e^2}+\frac{ax}{3\,d\,(ex^3+d)}-\frac{bx}{3\,e\,(ex^3+d)}+\frac{cdx}{3\,e^2\,(ex^3+d)}+\frac{2\,a}{9\,de}\ln\left(x+\sqrt[3]{\frac{d}{e}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}+\frac{b}{9\,e^2}\ln\left(x+\sqrt[3]{\frac{d}{e}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}-\frac{4\,cd}{9\,e^3}\ln\left(x+\sqrt[3]{\frac{d}{e}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}-\frac{a}{9\,de}\ln\left(x^2-x\sqrt[3]{\frac{d}{e}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}-\frac{b}{18\,e^2}\ln\left(x^2-x\sqrt[3]{\frac{d}{e}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}+\frac{2\,cd}{9\,e^3}\ln\left(x^2-x\sqrt[3]{\frac{d}{e}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}+\frac{2\,\sqrt{3}a}{9\,de}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{d}{e}}}-1\right)\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}+\frac{\sqrt{3}b}{9\,e^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{d}{e}}}-1\right)\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}-\frac{4\,d\sqrt{3}c}{9\,e^3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{d}{e}}}-1\right)\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)/(e*x^3+d)^2,x)
```

```
[Out] c*x/e^2+1/3/d*x/(e*x^3+d)*a-1/3/e*x/(e*x^3+d)*b+1/3/e^2*d*x/(e*x^
3+d)*c+2/9/e/d/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a+1/9/e^2/(d/e)^(2/3
)*ln(x+(d/e)^(1/3))*b-4/9/e^3*d/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*c-1
/9/e/d/(d/e)^(2/3)*ln(x^2-x*(d/e)^(1/3)+(d/e)^(2/3))*a-1/18/e^2/(
d/e)^(2/3)*ln(x^2-x*(d/e)^(1/3)+(d/e)^(2/3))*b+2/9/e^3*d/(d/e)^(2
/3)*ln(x^2-x*(d/e)^(1/3)+(d/e)^(2/3))*c+2/9/e/d/(d/e)^(2/3)*3^(1/
2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a+1/9/e^2/(d/e)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*b-4/9/e^3*d/(d/e)
^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.291672, size = 385, normalized size = 1.81

$$\frac{\sqrt{3}\left(\sqrt{3}\left(4\,cd^3 - bd^2e - 2\,ade^2 + \left(4\,cd^2e - bde^2 - 2\,ae^3\right)x^3\right)\log\left(\left(d^2e\right)^{\frac{2}{3}}x^2 - \left(d^2e\right)^{\frac{1}{3}}dx + d^2\right) - 2\,\sqrt{3}\left(4\,cd^3 - bd^2e - 2\,ade^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^2,x, algorithm="fricas")
```

```
[Out] 1/54*sqrt(3)*(sqrt(3)*(4*c*d^3 - b*d^2*e - 2*a*d*e^2 + (4*c*d^2*e
 - b*d*e^2 - 2*a*e^3)*x^3)*log((d^2*e)^(2/3)*x^2 - (d^2*e)^(1/3)*
d*x + d^2) - 2*sqrt(3)*(4*c*d^3 - b*d^2*e - 2*a*d*e^2 + (4*c*d^2*
e - b*d*e^2 - 2*a*e^3)*x^3)*log((d^2*e)^(1/3)*x + d) - 6*(4*c*d^3
 - b*d^2*e - 2*a*d*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*arc
tan(1/3*(2*sqrt(3)*(d^2*e)^(1/3)*x - sqrt(3)*d)/d) + 6*sqrt(3)*(3
*c*d*e*x^4 + (4*c*d^2 - b*d*e + a*e^2)*x)*(d^2*e)^(1/3))/((d*e^3*
x^3 + d^2*e^2)*(d^2*e)^(1/3))
```

Sympy [A] time = 6.96547, size = 206, normalized size = 0.97

$$\frac{cx}{e^2} + \frac{x\left(ae^2 - bde + cd^2\right)}{3d^2e^2 + 3de^3x^3}$$
$$+\mathrm{RootSum}\left(729t^3d^5e^7 - 8a^3e^6 - 12a^2bde^5 + 48a^2cd^2e^4 - 6ab^2d^2e^4 + 48abcd^3e^3 - 96ac^2d^4e^2 - b^3d^3e^3 + 12b^2cd^4e^2 - 48bc^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)/(e*x**3+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(3*d**2*e**2 + 3*d*e**3*x*
*3) + RootSum(729*_t**3*d**5*e**7 - 8*a**3*e**6 - 12*a**2*b*d*e**
5 + 48*a**2*c*d**2*e**4 - 6*a*b**2*d**2*e**4 + 48*a*b*c*d**3*e**3
 - 96*a*c**2*d**4*e**2 - b**3*d**3*e**3 + 12*b**2*c*d**4*e**2 - 4
8*b*c**2*d**5*e + 64*c**3*d**6, Lambda(_t, _t*log(9*_t*d**2*e**2/
(2*a*e**2 + b*d*e - 4*c*d**2) + x)))
```

GIAC/XCAS [A] time = 0.279325, size = 306, normalized size = 1.44

$$cxe^{(-2)} - \frac{\sqrt{3}\left(4\,\left(-de^2\right)^{\frac{1}{3}}cd^2 - \left(-de^2\right)^{\frac{1}{3}}bde - 2\,\left(-de^2\right)^{\frac{1}{3}}ae^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-3)}}{9\,d^2} + \frac{\left(4\,cd^2 - bde - 2\,ae^2\right)\left(-de^{(-1)}\right)^{\frac{1}{3}}e^{(-2)}\ln\left(\left|x - \left(-de^{(-1)}\right)^{\frac{1}{3}}\right|\right)}{9\,d^2} - \frac{\left(4\,\left(-de^2\right)^{\frac{1}{3}}cd^2 - \left(-de^2\right)^{\frac{1}{3}}bde - 2\,\left(-de^2\right)^{\frac{1}{3}}ae^2\right)e^{(-3)}\ln\left(x^2 + \left(-de^{(-1)}\right)^{\frac{1}{3}}x + \left(-de^{(-1)}\right)^{\frac{2}{3}}\right)}{18\,d^2} + \frac{\left(cd^2x - bdxe + axe^2\right)e^{(-2)}}{3\,(x^3e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) - 1/9*sqrt(3)*(4*(-d*e^2)^(1/3)*c*d^2 - (-d*e^2)^(1/3)
*b*d*e - 2*(-d*e^2)^(1/3)*a*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d*e^
(-1))^(1/3))/(-d*e^(-1))^(1/3))*e^(-3)/d^2 + 1/9*(4*c*d^2 - b*d*e
 - 2*a*e^2)*(-d*e^(-1))^(1/3)*e^(-2)*ln(abs(x - (-d*e^(-1))^(1/3)
))/d^2 - 1/18*(4*(-d*e^2)^(1/3)*c*d^2 - (-d*e^2)^(1/3)*b*d*e - 2*
(-d*e^2)^(1/3)*a*e^2)*e^(-3)*ln(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e
^(-1))^(2/3))/d^2 + 1/3*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-2)/((x^
3*e + d)*d)
```

3.8 $\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx$

Optimal. Leaf size=242

$$-\frac{x\left(7cd^2-e(5ae+bd)\right)}{18d^2e^2\left(d+ex^3\right)}+\frac{x\left(ae^2-bde+cd^2\right)}{6de^2\left(d+ex^3\right)^2}-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(e(5ae+bd)+2cd^2\right)}{54d^{8/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(e(5ae+bd)+2cd^2\right)}{27d^{8/3}e^{7/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(e(5ae+bd)+2cd^2\right)}{9\sqrt{3}d^{8/3}e^{7/3}}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)*x)/(6*d*e^2*(d + e*x^3)^2) - ((7*c*d^2 -
 e*(b*d + 5*a*e))*x)/(18*d^2*e^2*(d + e*x^3)) - ((2*c*d^2 + e*(b*
d + 5*a*e))*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(9
*Sqrt[3]*d^(8/3)*e^(7/3)) + ((2*c*d^2 + e*(b*d + 5*a*e))*Log[d^(1
/3) + e^(1/3)*x])/(27*d^(8/3)*e^(7/3)) - ((2*c*d^2 + e*(b*d + 5*a
*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(54*d^(8/3)*
e^(7/3))
```

Rubi [A] time = 0.524643, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$-\frac{x\left(7cd^2-e(5ae+bd)\right)}{18d^2e^2\left(d+ex^3\right)}+\frac{x\left(ae^2-bde+cd^2\right)}{6de^2\left(d+ex^3\right)^2}-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(e(5ae+bd)+2cd^2\right)}{54d^{8/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(e(5ae+bd)+2cd^2\right)}{27d^{8/3}e^{7/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(e(5ae+bd)+2cd^2\right)}{9\sqrt{3}d^{8/3}e^{7/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^3,x]
```


```
[Out] ((c*d^2 - b*d*e + a*e^2)*x)/(6*d*e^2*(d + e*x^3)^2) - ((7*c*d^2 -
 e*(b*d + 5*a*e))*x)/(18*d^2*e^2*(d + e*x^3)) - ((2*c*d^2 + e*(b*
d + 5*a*e))*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(9
*Sqrt[3]*d^(8/3)*e^(7/3)) + ((2*c*d^2 + e*(b*d + 5*a*e))*Log[d^(1
/3) + e^(1/3)*x])/(27*d^(8/3)*e^(7/3)) - ((2*c*d^2 + e*(b*d + 5*a
*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(54*d^(8/3)*
e^(7/3))
```

Rubi in Sympy [A] time = 66.6659, size = 233, normalized size = 0.96

$$\frac{x\left(ae^2-bde+cd^2\right)}{6de^2\left(d+ex^3\right)^2}+\frac{x\left(5ae^2+bde-7cd^2\right)}{18d^2e^2\left(d+ex^3\right)}+\frac{\left(5ae^2+bde+2cd^2\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{27d^{\frac{8}{3}}e^{\frac{7}{3}}}-\frac{\left(5ae^2+bde+2cd^2\right)\log\left(d^{\frac{2}{3}}-\sqrt[3]{d}\sqrt[3]{e}x+e^{\frac{2}{3}}x^2\right)}{54d^{\frac{8}{3}}e^{\frac{7}{3}}}-\frac{\sqrt{3}\left(5ae^2+bde+2cd^2\right)\operatorname{atan}\left(\frac{\sqrt{3}\left(\frac{\sqrt[3]{d}}{3}-\frac{2\sqrt[3]{e}x}{3}\right)}{\sqrt[3]{d}}\right)}{27d^{\frac{8}{3}}e^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)/(e*x**3+d)**3,x)
```

```
[Out] x*(a*e**2 - b*d*e + c*d**2)/(6*d*e**2*(d + e*x**3)**2) + x*(5*a*e
**2 + b*d*e - 7*c*d**2)/(18*d**2*e**2*(d + e*x**3)) + (5*a*e**2 +
 b*d*e + 2*c*d**2)*log(d**(1/3) + e**(1/3)*x)/(27*d**(8/3)*e**(7/
3)) - (5*a*e**2 + b*d*e + 2*c*d**2)*log(d**(2/3) - d**(1/3)*e**(1
/3)*x + e**(2/3)*x**2)/(54*d**(8/3)*e**(7/3)) - sqrt(3)*(5*a*e**2
 + b*d*e + 2*c*d**2)*atan(sqrt(3)*(d**(1/3)/3 - 2*e**(1/3)*x/3)/d
**(1/3))/(27*d**(8/3)*e**(7/3))
```

Mathematica [A] time = 0.490439, size = 209, normalized size = 0.86

$$\frac{2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(e(5ae+bd)+2cd^2\right)-2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)\left(e(5ae+bd)+2cd^2\right)-\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(e(5ae\right.}{54d^{8/3}e^{7/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^3,x]
```

```
[Out] ((-3*d^(2/3)*e^(1/3)*x*(c*d^2*(4*d + 7*e*x^3) - e*(b*d*(-2*d + e*
x^3) + a*e*(8*d + 5*e*x^3))))/(d + e*x^3)^2 - 2*Sqrt[3]*(2*c*d^2
+ e*(b*d + 5*a*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] +
2*(2*c*d^2 + e*(b*d + 5*a*e))*Log[d^(1/3) + e^(1/3)*x] - (2*c*d^2
 + e*(b*d + 5*a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2
])/(54*d^(8/3)*e^(7/3))
```

Maple [A] time = 0.015, size = 362, normalized size = 1.5

$$\frac{1}{(ex^3+d)^2}\left(\frac{(5\,ae^2+bde-7\,cd^2)\,x^4}{18\,d^2e}+\frac{(4\,ae^2-bde-2\,cd^2)\,x}{9\,e^2d}\right)+\frac{5\,a}{27\,d^2e}\ln\left(x+\sqrt[3]{\frac{d}{e}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}+\frac{b}{27\,e^2d}\ln\left(x+\sqrt[3]{\frac{d}{e}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}+\frac{2\,c}{27\,e^3}\ln\left(x+\sqrt[3]{\frac{d}{e}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}-\frac{5\,a}{54\,d^2e}\ln\left(x^2-x\sqrt[3]{\frac{d}{e}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}-\frac{b}{54\,e^2d}\ln\left(x^2-x\sqrt[3]{\frac{d}{e}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}-\frac{c}{27\,e^3}\ln\left(x^2-x\sqrt[3]{\frac{d}{e}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}+\frac{5\sqrt{3}a}{27\,d^2e}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{d}{e}}}-1\right)\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}+\frac{\sqrt{3}b}{27\,e^2d}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{d}{e}}}-1\right)\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}+\frac{2\sqrt{3}c}{27\,e^3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{d}{e}}}-1\right)\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)/(e*x^3+d)^3,x)
```

```
[Out] (1/18*(5*a*e^2+b*d*e-7*c*d^2)/d^2/e*x^4+1/9*(4*a*e^2-b*d*e-2*c*d^
2)/e^2/d*x)/(e*x^3+d)^2+5/27/d^2/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*
a+1/27/d/e^2/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*b+2/27/e^3/(d/e)^(2/3)
*ln(x+(d/e)^(1/3))*c-5/54/d^2/e/(d/e)^(2/3)*ln(x^2-x*(d/e)^(1/3)+
(d/e)^(2/3))*a-1/54/d/e^2/(d/e)^(2/3)*ln(x^2-x*(d/e)^(1/3)+(d/e)^
(2/3))*b-1/27/e^3/(d/e)^(2/3)*ln(x^2-x*(d/e)^(1/3)+(d/e)^(2/3))*c
+5/27/d^2/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)
*x-1))*a+1/27/d/e^2/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/
e)^(1/3)*x-1))*b+2/27/e^3/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*
(2/(d/e)^(1/3)*x-1))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.26324, size = 552, normalized size = 2.28

$$-\frac{\sqrt{3}\left(\sqrt{3}\left(\left(2\,cd^2e^2 + bde^3 + 5\,ae^4\right)x^6 + 2\,cd^4 + bd^3e + 5\,ad^2e^2 + 2\left(2\,cd^3e + bd^2e^2 + 5\,ade^3\right)x^3\right)\log\left(\left(d^2e\right)^{\frac{2}{3}}x^2 - \left(d^2e\right)^{\frac{1}{3}}dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^3,x, algorithm="fricas")
```

```
[Out] -1/162*sqrt(3)*(sqrt(3)*((2*c*d^2*e^2 + b*d*e^3 + 5*a*e^4)*x^6 +
2*c*d^4 + b*d^3*e + 5*a*d^2*e^2 + 2*(2*c*d^3*e + b*d^2*e^2 + 5*a*
d*e^3)*x^3)*log((d^2*e)^(2/3)*x^2 - (d^2*e)^(1/3)*d*x + d^2) - 2*
sqrt(3)*((2*c*d^2*e^2 + b*d*e^3 + 5*a*e^4)*x^6 + 2*c*d^4 + b*d^3*
e + 5*a*d^2*e^2 + 2*(2*c*d^3*e + b*d^2*e^2 + 5*a*d*e^3)*x^3)*log(
(d^2*e)^(1/3)*x + d) - 6*((2*c*d^2*e^2 + b*d*e^3 + 5*a*e^4)*x^6 +
 2*c*d^4 + b*d^3*e + 5*a*d^2*e^2 + 2*(2*c*d^3*e + b*d^2*e^2 + 5*a
*d*e^3)*x^3)*arctan(1/3*(2*sqrt(3)*(d^2*e)^(1/3)*x - sqrt(3)*d)/d
) + 3*sqrt(3)*((7*c*d^2*e - b*d*e^2 - 5*a*e^3)*x^4 + 2*(2*c*d^3 +
 b*d^2*e - 4*a*d*e^2)*x)*(d^2*e)^(1/3))/((d^2*e^4*x^6 + 2*d^3*e^3
*x^3 + d^4*e^2)*(d^2*e)^(1/3))
```

Sympy [A] time = 22.2383, size = 246, normalized size = 1.02

$$\frac{x^4\left(5ae^3 + bde^2 - 7cd^2e\right) + x\left(8ade^2 - 2bd^2e - 4cd^3\right)}{18d^4e^2 + 36d^3e^3x^3 + 18d^2e^4x^6}$$
$$+\mathrm{RootSum}\left(19683t^3d^8e^7 - 125a^3e^6 - 75a^2bde^5 - 150a^2cd^2e^4 - 15ab^2d^2e^4 - 60abcd^3e^3 - 60ac^2d^4e^2 - b^3d^3e^3 - 6b^2cd^4e^2 -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)/(e*x**3+d)**3,x)
```

```
[Out] (x**4*(5*a*e**3 + b*d*e**2 - 7*c*d**2*e) + x*(8*a*d*e**2 - 2*b*d*
*2*e - 4*c*d**3))/(18*d**4*e**2 + 36*d**3*e**3*x**3 + 18*d**2*e**
4*x**6) + RootSum(19683*_t**3*d**8*e**7 - 125*a**3*e**6 - 75*a**2
*b*d*e**5 - 150*a**2*c*d**2*e**4 - 15*a*b**2*d**2*e**4 - 60*a*b*c
*d**3*e**3 - 60*a*c**2*d**4*e**2 - b**3*d**3*e**3 - 6*b**2*c*d**4
*e**2 - 12*b*c**2*d**5*e - 8*c**3*d**6, Lambda(_t, _t*log(27*_t*d
**3*e**2/(5*a*e**2 + b*d*e + 2*c*d**2) + x)))
```

GIAC/XCAS [A] time = 0.280327, size = 340, normalized size = 1.4

$$\frac{\sqrt{3}\left(2\left(-de^{2}\right)^{\frac{1}{3}}cd^{2}+\left(-de^{2}\right)^{\frac{1}{3}}bde+5\left(-de^{2}\right)^{\frac{1}{3}}ae^{2}\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-3)}}{27d^{3}} - \frac{\left(2cd^{2}+bde+5ae^{2}\right)\left(-de^{(-1)}\right)^{\frac{1}{3}}e^{(-2)}\ln\left(\left|x-\left(-de^{(-1)}\right)^{\frac{1}{3}}\right|\right)}{27d^{3}} + \frac{\left(2\left(-de^{2}\right)^{\frac{1}{3}}cd^{2}+\left(-de^{2}\right)^{\frac{1}{3}}bde+5\left(-de^{2}\right)^{\frac{1}{3}}ae^{2}\right)e^{(-3)}\ln\left(x^{2}+\left(-de^{(-1)}\right)^{\frac{1}{3}}x+\left(-de^{(-1)}\right)^{\frac{2}{3}}\right)}{54d^{3}} - \frac{\left(7cd^{2}x^{4}e-bdx^{4}e^{2}-5ax^{4}e^{3}+4cd^{3}x+2bd^{2}xe-8adxe^{2}\right)e^{(-2)}}{18\left(x^{3}e+d\right)^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(2*(-d*e^2)^(1/3)*c*d^2 + (-d*e^2)^(1/3)*b*d*e + 5*(
-d*e^2)^(1/3)*a*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d*e^(-1))^(1/3))
/(-d*e^(-1))^(1/3))*e^(-3)/d^3 - 1/27*(2*c*d^2 + b*d*e + 5*a*e^2)
*(-d*e^(-1))^(1/3)*e^(-2)*ln(abs(x - (-d*e^(-1))^(1/3)))/d^3 + 1/
54*(2*(-d*e^2)^(1/3)*c*d^2 + (-d*e^2)^(1/3)*b*d*e + 5*(-d*e^2)^(1
/3)*a*e^2)*e^(-3)*ln(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3
))/d^3 - 1/18*(7*c*d^2*x^4*e - b*d*x^4*e^2 - 5*a*x^4*e^3 + 4*c*d^
3*x + 2*b*d^2*x*e - 8*a*d*x*e^2)*e^(-2)/((x^3*e + d)^2*d^2)
```

3.9 $\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=132

$$-\frac{(ace+b^2(-e)+bcd)\log(a+bx^3+cx^6)}{6c^3}-\frac{(3abce-2ac^2d+b^3(-e)+b^2cd)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^3\sqrt{b^2-4ac}}+\frac{x^3(cd-be)}{3c^2}+\frac{ex^6}{6c}$$

```
[Out] ((c*d - b*e)*x^3)/(3*c^2) + (e*x^6)/(6*c) - ((b^2*c*d - 2*a*c^2*d
 - b^3*e + 3*a*b*c*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(
3*c^3*Sqrt[b^2 - 4*a*c]) - ((b*c*d - b^2*e + a*c*e)*Log[a + b*x^3
 + c*x^6])/(6*c^3)
```

Rubi [A] time = 0.441345, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24

$$-\frac{(ace+b^2(-e)+bcd)\log(a+bx^3+cx^6)}{6c^3}-\frac{(3abce-2ac^2d+b^3(-e)+b^2cd)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^3\sqrt{b^2-4ac}}+\frac{x^3(cd-be)}{3c^2}+\frac{ex^6}{6c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((c*d - b*e)*x^3)/(3*c^2) + (e*x^6)/(6*c) - ((b^2*c*d - 2*a*c^2*d
 - b^3*e + 3*a*b*c*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(
3*c^3*Sqrt[b^2 - 4*a*c]) - ((b*c*d - b^2*e + a*c*e)*Log[a + b*x^3
 + c*x^6])/(6*c^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\left(\frac{be}{3}-\frac{cd}{3}\right)\int^{x^3}\frac{1}{c^2}\,dx+\frac{e\int^{x^3}x\,dx}{3c}+\frac{(-ace+b^2e-bcd)\log(a+bx^3+cx^6)}{6c^3}+\frac{(-3abce+2ac^2d+b^3e-b^2cd)\,\mathrm{atanh}\left(\frac{b+2cx^3}{\sqrt{-4ac+b^2}}\right)}{3c^3\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] -(b*e/3 - c*d/3)*Integral(c**(-2), (x, x**3)) + e*Integral(x, (x,
 x**3))/(3*c) + (-a*c*e + b**2*e - b*c*d)*log(a + b*x**3 + c*x**6
)/(6*c**3) + (-3*a*b*c*e + 2*a*c**2*d + b**3*e - b**2*c*d)*atanh(
(b + 2*c*x**3)/sqrt(-4*a*c + b**2))/(3*c**3*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.108351, size = 126, normalized size = 0.95

$$\frac{\left(-ace+b^2e-bcd\right)\log\left(a+bx^3+cx^6\right)+\frac{2\left(3abce-2ac^2d+b^3(-e)+b^2cd\right)\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+2cx^3(cd-be)+c^2ex^6}{6c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*c*(c*d - b*e)*x^3 + c^2*e*x^6 + (2*(b^2*c*d - 2*a*c^2*d - b^3*
e + 3*a*b*c*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^
2 + 4*a*c] + (-(b*c*d) + b^2*e - a*c*e)*Log[a + b*x^3 + c*x^6])/(
6*c^3)
```

Maple [B] time = 0.008, size = 260, normalized size = 2.

$$\frac{ex^6}{6\,c}-\frac{bex^3}{3\,c^2}+\frac{dx^3}{3\,c}-\frac{\ln\left(cx^6+bx^3+a\right)ae}{6\,c^2}+\frac{\ln\left(cx^6+bx^3+a\right)b^2e}{6\,c^3}-\frac{\ln\left(cx^6+bx^3+a\right)bd}{6\,c^2}$$
$$+\frac{abe}{c^2}\arctan\left((2\,cx^3+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}-\frac{2\,ad}{3\,c}\arctan\left((2\,cx^3+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$
$$-\frac{b^3e}{3\,c^3}\arctan\left((2\,cx^3+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{b^2d}{3\,c^2}\arctan\left((2\,cx^3+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/6*e*x^6/c-1/3/c^2*x^3*b*e+1/3/c*d*x^3-1/6/c^2*ln(c*x^6+b*x^3+a)
*a*e+1/6/c^3*ln(c*x^6+b*x^3+a)*b^2*e-1/6/c^2*ln(c*x^6+b*x^3+a)*b*
d+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*a
*b*e-2/3/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2)
)*a*d-1/3/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1
/2))*b^3*e+1/3/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^
2)^(1/2))*b^2*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)*x^8/(c*x^6 + b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.612669, size = 1, normalized size = 0.01

$$\left[\frac{\left(\left(b^2c-2\,ac^2\right)d-\left(b^3-3\,abc\right)e\right)\log\left(-\frac{2\,\left(b^2c-4\,ac^2\right)x^3+b^3-4\,abc-\left(2\,c^2x^6+2\,bcx^3+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)+\left(c^2ex^6+2\,\left(c^2d-bce\right)x^3-\right.}{6\,\sqrt{b^2-4\,ac}c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)*x^8/(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] [1/6*(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e)*log(-(2*(b^2*c -
4*a*c^2)*x^3 + b^3 - 4*a*b*c - (2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a
*c)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) + (c^2*e*x^6 + 2*(c^2
*d - b*c*e)*x^3 - (b*c*d - (b^2 - a*c)*e)*log(c*x^6 + b*x^3 + a))
*sqrt(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c)*c^3), 1/6*(2*((b^2*c - 2*a
*c^2)*d - (b^3 - 3*a*b*c)*e)*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*
a*c)/(b^2 - 4*a*c)) + (c^2*e*x^6 + 2*(c^2*d - b*c*e)*x^3 - (b*c*d
 - (b^2 - a*c)*e)*log(c*x^6 + b*x^3 + a))*sqrt(-b^2 + 4*a*c))/(sq
rt(-b^2 + 4*a*c)*c^3)]
```

Sympy [A] time = 46.1755, size = 619, normalized size = 4.69

$$\left(-\frac{\sqrt{-4ac+b^2}\left(3abce-2ac^2d-b^3e+b^2cd\right)}{6c^3\left(4ac-b^2\right)}-\frac{ace-b^2e+bcd}{6c^3}\right)\log\left(x^3+\frac{2a^2ce-ab^2e+abcd+12ac^3\left(-\frac{\sqrt{-4ac+b^2}(3abce-2ac^2d-b^3e+b^2cd)}{6c^3(4ac-b^2)}-\frac{ace-b^2e+bcd}{6c^3}\right)-3b^2c^2\left(-\frac{\sqrt{-4}}{}\right.}{3abce-2ac^2d-b^3e+b^2cd}\right.$$

$$+\left(\frac{\sqrt{-4ac+b^2}\left(3abce-2ac^2d-b^3e+b^2cd\right)}{6c^3\left(4ac-b^2\right)}-\frac{ace-b^2e+bcd}{6c^3}\right)\log\left(x^3+\frac{2a^2ce-ab^2e+abcd+12ac^3\left(\frac{\sqrt{-4ac+b^2}(3abce-2ac^2d-b^3e+b^2cd)}{6c^3(4ac-b^2)}-\frac{ace-b^2e+bcd}{6c^3}\right)-3b^2c^2\left(\frac{\sqrt{-4ac+}}{}\right.}{3abce-2ac^2d-b^3e+b^2cd}\right.$$

$$+\frac{ex^6}{6c}-\frac{x^3\left(be-cd\right)}{3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d
)/(6*c**3*(4*a*c - b**2)) - (a*c*e - b**2*e + b*c*d)/(6*c**3))*lo
g(x**3 + (2*a**2*c*e - a*b**2*e + a*b*c*d + 12*a*c**3*(-sqrt(-4*a
*c + b**2)*(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)/(6*c**3*(
4*a*c - b**2)) - (a*c*e - b**2*e + b*c*d)/(6*c**3)) - 3*b**2*c**2
*(-sqrt(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*
d)/(6*c**3*(4*a*c - b**2)) - (a*c*e - b**2*e + b*c*d)/(6*c**3)))/
(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)) + (sqrt(-4*a*c + b*
*2)*(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c -
 b**2)) - (a*c*e - b**2*e + b*c*d)/(6*c**3))*log(x**3 + (2*a**2*c
*e - a*b**2*e + a*b*c*d + 12*a*c**3*(sqrt(-4*a*c + b**2)*(3*a*b*c
*e - 2*a*c**2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c - b**2)) - (a
*c*e - b**2*e + b*c*d)/(6*c**3)) - 3*b**2*c**2*(sqrt(-4*a*c + b**
2)*(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c -
b**2)) - (a*c*e - b**2*e + b*c*d)/(6*c**3)))/(3*a*b*c*e - 2*a*c**
2*d - b**3*e + b**2*c*d)) + e*x**6/(6*c) - x**3*(b*e - c*d)/(3*c*
*2)
```

GIAC/XCAS [A] time = 0.269936, size = 177, normalized size = 1.34

$$\frac{cx^6e+2\,cdx^3-2\,bx^3e}{6\,c^2}-\frac{\left(bcd-b^2e+ace\right)\ln\left(cx^6+bx^3+a\right)}{6\,c^3}+\frac{\left(b^2cd-2\,ac^2d-b^3e+3\,abce\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)*x^8/(c*x^6 + b*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/6*(c*x^6*e + 2*c*d*x^3 - 2*b*x^3*e)/c^2 - 1/6*(b*c*d - b^2*e +
a*c*e)*ln(c*x^6 + b*x^3 + a)/c^3 + 1/3*(b^2*c*d - 2*a*c^2*d - b^3
*e + 3*a*b*c*e)*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b
^2 + 4*a*c)*c^3)
```

3.10 $\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=97

$$\frac{\left(2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}+\frac{(cd-be)\log\left(a+bx^3+cx^6\right)}{6c^2}+\frac{ex^3}{3c}$$

```
[Out] (e*x^3)/(3*c) + ((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^3)/
Sqrt[b^2 - 4*a*c]])/(3*c^2*Sqrt[b^2 - 4*a*c]) + ((c*d - b*e)*Log[
a + b*x^3 + c*x^6])/(6*c^2)
```

Rubi [A] time = 0.255665, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24

$$\frac{\left(2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}+\frac{(cd-be)\log\left(a+bx^3+cx^6\right)}{6c^2}+\frac{ex^3}{3c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x^3)/(3*c) + ((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^3)/
Sqrt[b^2 - 4*a*c]])/(3*c^2*Sqrt[b^2 - 4*a*c]) + ((c*d - b*e)*Log[
a + b*x^3 + c*x^6])/(6*c^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int^{x^3} e\,dx}{3c}-\frac{(be-cd)\log\left(a+bx^3+cx^6\right)}{6c^2}-\frac{\left(-2ace+b^2e-bcd\right)\operatorname{atanh}\left(\frac{b+2cx^3}{\sqrt{-4ac+b^2}}\right)}{3c^2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] Integral(e, (x, x**3))/(3*c) - (b*e - c*d)*log(a + b*x**3 + c*x**
6)/(6*c**2) - (-2*a*c*e + b**2*e - b*c*d)*atanh((b + 2*c*x**3)/sq
rt(-4*a*c + b**2))/(3*c**2*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.11327, size = 93, normalized size = 0.96

$$\frac{\frac{2\left(-2ace+b^2e-bcd\right)\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+(cd-be)\log\left(a+bx^3+cx^6\right)+2cex^3}{6c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*c*e*x^3 + (2*(-(b*c*d) + b^2*e - 2*a*c*e)*ArcTan[(b + 2*c*x^3)
/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c*d - b*e)*Log[a + b*
x^3 + c*x^6])/(6*c^2)
```

Maple [A] time = 0.005, size = 175, normalized size = 1.8

$$\frac{ex^3}{3c}-\frac{\ln\left(cx^6+bx^3+a\right)be}{6c^2}+\frac{\ln\left(cx^6+bx^3+a\right)d}{6c}-\frac{2ae}{3c}\arctan\left((2cx^3+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}+\frac{b^2e}{3c^2}\arctan\left((2cx^3+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}-\frac{bd}{3c}\arctan\left((2cx^3+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3/c*x^3*e-1/6/c^2*ln(c*x^6+b*x^3+a)*b*e+1/6/c*ln(c*x^6+b*x^3+a)
*d-2/3/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*
a*e+1/3/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2
))*b^2*e-1/3/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(
1/2))*b*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)*x^5/(c*x^6 + b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.353454, size = 1, normalized size = 0.01

$$\left[\frac{\left(bcd-\left(b^2-2\,ac\right)e\right)\log\left(\frac{2\left(b^2c-4\,ac^2\right)x^3+b^3-4\,abc+\left(2\,c^2x^6+2\,bcx^3+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)+\left(2\,cex^3+\left(cd-be\right)\log\left(cx^6+bx^3+a\right)\right)\sqrt{b^2-4\,ac}}{6\,\sqrt{b^2-4\,ac}c^2}, -\frac{2\left(bcd-\left(b^2-2\,ac\right)e\right)\arctan\left(-\frac{\left(2\,cx^3+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-\left(2\,cex^3+\left(cd-be\right)\log\left(cx^6+bx^3+a\right)\right)\sqrt{-b^2+4\,ac}}{6\,\sqrt{-b^2+4\,ac}c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)*x^5/(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] [1/6*((b*c*d - (b^2 - 2*a*c)*e)*log((2*(b^2*c - 4*a*c^2)*x^3 + b^
3 - 4*a*b*c + (2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c)*sqrt(b^2 - 4*
a*c))/(c*x^6 + b*x^3 + a)) + (2*c*e*x^3 + (c*d - b*e)*log(c*x^6 +
 b*x^3 + a))*sqrt(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c)*c^2), -1/6*(2*
(b*c*d - (b^2 - 2*a*c)*e)*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c
)/(b^2 - 4*a*c)) - (2*c*e*x^3 + (c*d - b*e)*log(c*x^6 + b*x^3 + a
))*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)]
```

Sympy [A] time = 29.424, size = 434, normalized size = 4.47

$$\left(-\frac{\sqrt{-4ac+b^2}\left(2ace-b^2e+bcd\right)}{6c^2\left(4ac-b^2\right)}-\frac{be-cd}{6c^2}\right)\log\left(x^3+\frac{-abe-12ac^2\left(-\frac{\sqrt{-4ac+b^2}\left(2ace-b^2e+bcd\right)}{6c^2\left(4ac-b^2\right)}-\frac{be-cd}{6c^2}\right)+2acd+3b^2c\left(-\frac{\sqrt{-4ac+b^2}\left(2ace-b^2e+bcd\right)}{6c^2\left(4ac-b^2\right)}-\frac{be-cd}{6c^2}\right)}{2ace-b^2e+bcd}\right)+\left(\frac{\sqrt{-4ac+b^2}\left(2ace-b^2e+bcd\right)}{6c^2\left(4ac-b^2\right)}-\frac{be-cd}{6c^2}\right)\log\left(x^3+\frac{-abe-12ac^2\left(\frac{\sqrt{-4ac+b^2}\left(2ace-b^2e+bcd\right)}{6c^2\left(4ac-b^2\right)}-\frac{be-cd}{6c^2}\right)+2acd+3b^2c\left(\frac{\sqrt{-4ac+b^2}\left(2ace-b^2e+bcd\right)}{6c^2\left(4ac-b^2\right)}-\frac{be-cd}{6c^2}\right)}{2ace-b^2e+bcd}\right)+\frac{ex^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c -
 b**2)) - (b*e - c*d)/(6*c**2))*log(x**3 + (-a*b*e - 12*a*c**2*(-
sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b
**2)) - (b*e - c*d)/(6*c**2)) + 2*a*c*d + 3*b**2*c*(-sqrt(-4*a*c
+ b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b*e
 - c*d)/(6*c**2)))/(2*a*c*e - b**2*e + b*c*d)) + (sqrt(-4*a*c + b
**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b*e -
c*d)/(6*c**2))*log(x**3 + (-a*b*e - 12*a*c**2*(sqrt(-4*a*c + b**2
)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b*e - c*d
)/(6*c**2)) + 2*a*c*d + 3*b**2*c*(sqrt(-4*a*c + b**2)*(2*a*c*e -
b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2)))/
(2*a*c*e - b**2*e + b*c*d)) + e*x**3/(3*c)
```

GIAC/XCAS [A] time = 0.271663, size = 128, normalized size = 1.32

$$\frac{x^3e}{3\,c} + \frac{(cd - be)\ln\left(cx^6 + bx^3 + a\right)}{6\,c^2} - \frac{\left(bcd - b^2e + 2\,ace\right)\,\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2 + 4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)*x^5/(c*x^6 + b*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/3*x^3*e/c + 1/6*(c*d - b*e)*ln(c*x^6 + b*x^3 + a)/c^2 - 1/3*(b*
c*d - b^2*e + 2*a*c*e)*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(
sqrt(-b^2 + 4*a*c)*c^2)
```

3.11 $\int \frac{x^2\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=72

$$\frac{e\log\left(a+bx^3+cx^6\right)}{6c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}$$

```
[Out] -((2*c*d - b*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c*Sq
rt[b^2 - 4*a*c]) + (e*Log[a + b*x^3 + c*x^6])/(6*c)
```

Rubi [A] time = 0.167914, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{e\log\left(a+bx^3+cx^6\right)}{6c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] -((2*c*d - b*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c*Sq
rt[b^2 - 4*a*c]) + (e*Log[a + b*x^3 + c*x^6])/(6*c)
```

Rubi in Sympy [A] time = 23.8993, size = 63, normalized size = 0.88

$$\frac{e\log\left(a+bx^3+cx^6\right)}{6c}+\frac{(be-2cd)\operatorname{atanh}\left(\frac{b+2cx^3}{\sqrt{-4ac+b^2}}\right)}{3c\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] e*log(a + b*x**3 + c*x**6)/(6*c) + (b*e - 2*c*d)*atanh((b + 2*c*x
**3)/sqrt(-4*a*c + b**2))/(3*c*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.0856646, size = 71, normalized size = 0.99

$$\frac{e\log\left(a+bx^3+cx^6\right)-\frac{2(be-2cd)\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{6c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((-2*(-2*c*d + b*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqr
t[-b^2 + 4*a*c] + e*Log[a + b*x^3 + c*x^6])/(6*c)
```

Maple [A] time = 0.004, size = 99, normalized size = 1.4

$$\frac{e\ln\left(cx^6+bx^3+a\right)}{6\,c}+\frac{2\,d}{3}\arctan\left((2\,cx^3+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}-\frac{be}{3\,c}\arctan\left((2\,cx^3+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/6*e*ln(c*x^6+b*x^3+a)/c+2/3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b
)/(4*a*c-b^2)^(1/2))*d-1/3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(
4*a*c-b^2)^(1/2))/c*b*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)*x^2/(c*x^6 + b*x^3 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.295373, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{b^2-4\,ac}e\log\left(cx^6+bx^3+a\right)-\left(2\,cd-be\right)\log\left(\frac{2\left(b^2c-4\,ac^2\right)x^3+b^3-4\,abc+\left(2\,c^2x^6+2\,bcx^3+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)}{6\sqrt{b^2-4\,ac}c}, \frac{\sqrt{-b^2+4\,ac}e\log\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)*x^2/(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(b^2 - 4*a*c)*e*log(c*x^6 + b*x^3 + a) - (2*c*d - b*e)*
log((2*(b^2*c - 4*a*c^2)*x^3 + b^3 - 4*a*b*c + (2*c^2*x^6 + 2*b*c
*x^3 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)))/(sqr
t(b^2 - 4*a*c)*c), 1/6*(sqrt(-b^2 + 4*a*c)*e*log(c*x^6 + b*x^3 +
a) + 2*(2*c*d - b*e)*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^
2 - 4*a*c)))/(sqrt(-b^2 + 4*a*c)*c)]
```

Sympy [A] time = 13.7803, size = 287, normalized size = 3.99

$$\left(\frac{e}{6c} - \frac{\sqrt{-4ac+b^2}\,(be-2cd)}{6c\,(4ac-b^2)}\right)\log\left(x^3+\frac{-12ac\left(\frac{e}{6c}-\frac{\sqrt{-4ac+b^2}(be-2cd)}{6c(4ac-b^2)}\right)+2ae+3b^2\left(\frac{e}{6c}-\frac{\sqrt{-4ac+b^2}(be-2cd)}{6c(4ac-b^2)}\right)-bd}{be-2cd}\right) + \left(\frac{e}{6c} + \frac{\sqrt{-4ac+b^2}\,(be-2cd)}{6c\,(4ac-b^2)}\right)\log\left(x^3+\frac{-12ac\left(\frac{e}{6c}+\frac{\sqrt{-4ac+b^2}(be-2cd)}{6c(4ac-b^2)}\right)+2ae+3b^2\left(\frac{e}{6c}+\frac{\sqrt{-4ac+b^2}(be-2cd)}{6c(4ac-b^2)}\right)-bd}{be-2cd}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] (e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2))
)*log(x**3 + (-12*a*c*(e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d
)/(6*c*(4*a*c - b**2))) + 2*a*e + 3*b**2*(e/(6*c) - sqrt(-4*a*c +
 b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*d))
+ (e/(6*c) + sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2
)))*log(x**3 + (-12*a*c*(e/(6*c) + sqrt(-4*a*c + b**2)*(b*e - 2*c
*d)/(6*c*(4*a*c - b**2))) + 2*a*e + 3*b**2*(e/(6*c) + sqrt(-4*a*c
 + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*d)
```

)

GIAC/XCAS [A] time = 0.27378, size = 95, normalized size = 1.32

$$\frac{e\ln\left(cx^6+bx^3+a\right)}{6\,c}+\frac{(2\,cd-be)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)*x^2/(c*x^6 + b*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/6*e*ln(c*x^6 + b*x^3 + a)/c + 1/3*(2*c*d - b*e)*arctan((2*c*x^3
 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

3.12 $\int \frac{d+ex^3}{x(a+bx^3+cx^6)}\,dx$

Optimal. Leaf size=78

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bx^3+cx^6\right)}{6a}+\frac{d\log(x)}{a}$$

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*a*Sqr
t[b^2 - 4*a*c]) + (d*Log[x])/a - (d*Log[a + b*x^3 + c*x^6])/(6*a)
```

Rubi [A] time = 0.26991, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bx^3+cx^6\right)}{6a}+\frac{d\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*a*Sqr
t[b^2 - 4*a*c]) + (d*Log[x])/a - (d*Log[a + b*x^3 + c*x^6])/(6*a)
```

Rubi in Sympy [A] time = 36.0732, size = 73, normalized size = 0.94

$$\frac{d\log\left(x^3\right)}{3a}-\frac{d\log\left(a+bx^3+cx^6\right)}{6a}-\frac{(2ae-bd)\,\mathrm{atanh}\left(\frac{b+2cx^3}{\sqrt{-4ac+b^2}}\right)}{3a\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**3+d)/x/(c*x**6+b*x**3+a),x)
```

```
[Out] d*log(x**3)/(3*a) - d*log(a + b*x**3 + c*x**6)/(6*a) - (2*a*e - b
*d)*atanh((b + 2*c*x**3)/sqrt(-4*a*c + b**2))/(3*a*sqrt(-4*a*c +
b**2))
```

Mathematica [C] time = 0.0555417, size = 80, normalized size = 1.03

$$\frac{d\log(x)}{a}-\frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{2\#1^3c+b}\&\right]}{3a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] (d*Log[x])/a - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] -
 a*e*Log[x - #1] + c*d*Log[x - #1]*#1^3)/(b + 2*c*#1^3) & ]/(3*a)
```

Maple [A] time = 0.01, size = 106, normalized size = 1.4

$$\frac{d\ln(x)}{a}-\frac{d\ln\left(cx^6+bx^3+a\right)}{6a}+\frac{2e}{3}\arctan\left((2cx^3+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$
$$-\frac{bd}{3a}\arctan\left((2cx^3+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^3+d)/x/(c*x^6+b*x^3+a),x)
```

```
[Out] d*ln(x)/a-1/6*d*ln(c*x^6+b*x^3+a)/a+2/3/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x^3+b)/(4*a*c-b^2)^(1/2))*e-1/3/a/(4*a*c-b^2)^(1/2)*arctan((
2*c*x^3+b)/(4*a*c-b^2)^(1/2))*b*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.423968, size = 1, normalized size = 0.01

$$\left[-\frac{(bd-2\,ae)\log\left(-\frac{2\,(b^2c-4\,ac^2)x^3+b^3-4\,abc-(2\,c^2x^6+2\,bcx^3+b^2-2\,ac)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)+\sqrt{b^2-4\,ac}\left(d\log\left(cx^6+bx^3+a\right)-6\,d\log\left(x\right)\right)}{6\,\sqrt{b^2-4\,ac}a},\right.$$
$$\left.-\frac{2\,(bd-2\,ae)\arctan\left(-\frac{(2\,cx^3+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+\sqrt{-b^2+4\,ac}\left(d\log\left(cx^6+bx^3+a\right)-6\,d\log\left(x\right)\right)}{6\,\sqrt{-b^2+4\,ac}a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x),x, algorithm="fricas")
```

```
[Out] [-1/6*((b*d - 2*a*e)*log(-(2*(b^2*c - 4*a*c^2)*x^3 + b^3 - 4*a*b*
c - (2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x
^6 + b*x^3 + a)) + sqrt(b^2 - 4*a*c)*(d*log(c*x^6 + b*x^3 + a) -
6*d*log(x)))/(sqrt(b^2 - 4*a*c)*a), -1/6*(2*(b*d - 2*a*e)*arctan(
-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + sqrt(-b^2 + 4*
a*c)*(d*log(c*x^6 + b*x^3 + a) - 6*d*log(x)))/(sqrt(-b^2 + 4*a*c)
*a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**3+d)/x/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.275199, size = 103, normalized size = 1.32

$$-\frac{d\ln\left(cx^6+bx^3+a\right)}{6\,a}+\frac{d\ln\left(|x|\right)}{a}-\frac{(bd-2\,ae)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x),x, algorithm="giac")
```

```
[Out] -1/6*d*ln(c*x^6 + b*x^3 + a)/a + d*ln(abs(x))/a - 1/3*(b*d - 2*a*
e)*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a
)
```

3.13 $\int \frac{d+ex^3}{x^4(a+bx^3+cx^6)}\,dx$

Optimal. Leaf size=112

$$-\frac{\left(-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}+\frac{(bd-ae)\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{\log(x)(bd-ae)}{a^2}-\frac{d}{3ax^3}$$

```
[Out] -d/(3*a*x^3) - ((b^2*d - 2*a*c*d - a*b*e)*ArcTanh[(b + 2*c*x^3)/S
qrt[b^2 - 4*a*c]])/(3*a^2*Sqrt[b^2 - 4*a*c]) - ((b*d - a*e)*Log[x
])/a^2 + ((b*d - a*e)*Log[a + b*x^3 + c*x^6])/(6*a^2)
```

Rubi [A] time = 0.402598, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$-\frac{\left(-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}+\frac{(bd-ae)\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{\log(x)(bd-ae)}{a^2}-\frac{d}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -d/(3*a*x^3) - ((b^2*d - 2*a*c*d - a*b*e)*ArcTanh[(b + 2*c*x^3)/S
qrt[b^2 - 4*a*c]])/(3*a^2*Sqrt[b^2 - 4*a*c]) - ((b*d - a*e)*Log[x
])/a^2 + ((b*d - a*e)*Log[a + b*x^3 + c*x^6])/(6*a^2)
```

Rubi in Sympy [A] time = 54.575, size = 105, normalized size = 0.94

$$-\frac{d}{3ax^3}+\frac{(ae-bd)\log\left(x^3\right)}{3a^2}-\frac{(ae-bd)\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{\left(-abe-2acd+b^2d\right)\operatorname{atanh}\left(\frac{b+2cx^3}{\sqrt{-4ac+b^2}}\right)}{3a^2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**3+d)/x**4/(c*x**6+b*x**3+a),x)
```

```
[Out] -d/(3*a*x**3) + (a*e - b*d)*log(x**3)/(3*a**2) - (a*e - b*d)*log(
a + b*x**3 + c*x**6)/(6*a**2) - (-a*b*e - 2*a*c*d + b**2*d)*atanh
((b + 2*c*x**3)/sqrt(-4*a*c + b**2))/(3*a**2*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0810235, size = 130, normalized size = 1.16

$$\frac{\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{-\#1^3ace\log(x-\#1)+\#1^3bcd\log(x-\#1)-abe\log(x-\#1)-acd\log(x-\#1)+b^2d\log(x-\#1)}{2\#1^3c+b}\&\right]}{3a^2}+\frac{\log(x)(ae-bd)}{a^2}-\frac{d}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -d/(3*a*x^3) + ((-(b*d) + a*e)*Log[x])/a^2 + RootSum[a + b*#1^3 +
 c*#1^6 & , (b^2*d*Log[x - #1] - a*c*d*Log[x - #1] - a*b*e*Log[x
- #1] + b*c*d*Log[x - #1]*#1^3 - a*c*e*Log[x - #1]*#1^3)/(b + 2*c
*#1^3) & ]/(3*a^2)
```

Maple [A] time = 0.013, size = 191, normalized size = 1.7

$$-\frac{d}{3\,ax^3}+\frac{\ln(x)\,e}{a}-\frac{\ln(x)\,bd}{a^2}-\frac{\ln\left(cx^6+bx^3+a\right)\,e}{6\,a}+\frac{\ln\left(cx^6+bx^3+a\right)\,bd}{6\,a^2}-\frac{be}{3\,a}\arctan\left((2\,cx^3+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}-\frac{2\,cd}{3\,a}\arctan\left((2\,cx^3+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{b^2d}{3\,a^2}\arctan\left((2\,cx^3+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x)
```

```
[Out] -1/3*d/a/x^3+1/a*ln(x)*e-1/a^2*ln(x)*b*d-1/6/a*ln(c*x^6+b*x^3+a)*
e+1/6/a^2*ln(c*x^6+b*x^3+a)*b*d-1/3/a/(4*a*c-b^2)^(1/2)*arctan((2
*c*x^3+b)/(4*a*c-b^2)^(1/2))*b*e-2/3/a/(4*a*c-b^2)^(1/2)*arctan((
2*c*x^3+b)/(4*a*c-b^2)^(1/2))*c*d+1/3/a^2/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*b^2*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.826442, size = 1, normalized size = 0.01

$$\left[\frac{\left(abe-\left(b^{2}-2\,ac\right)d\right)x^{3}\log\left(\frac{2\left(b^{2}c-4\,ac^{2}\right)x^{3}+b^{3}-4\,abc+\left(2\,c^{2}x^{6}+2\,bcx^{3}+b^{2}-2\,ac\right)\sqrt{b^{2}-4\,ac}}{cx^{6}+bx^{3}+a}\right)+\left((bd-ae)x^{3}\log\left(cx^{6}+bx^{3}+a\right)-6\,(b\cdots}{6\,\sqrt{b^{2}-4\,ac}a^{2}x^{3}}\right.$$

$$-\frac{2\left(abe-\left(b^{2}-2\,ac\right)d\right)x^{3}\arctan\left(-\frac{\left(2\,cx^{3}+b\right)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)-\left((bd-ae)x^{3}\log\left(cx^{6}+bx^{3}+a\right)-6\,(bd-ae)x^{3}\log\left(x\right)-2\,ad\right)\cdots}{6\,\sqrt{-b^{2}+4\,ac}a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x^4),x, algorithm="fricas")
```

```
[Out] [1/6*((a*b*e - (b^2 - 2*a*c)*d)*x^3*log((2*(b^2*c - 4*a*c^2)*x^3
+ b^3 - 4*a*b*c + (2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c)*sqrt(b^2
- 4*a*c))/(c*x^6 + b*x^3 + a)) + ((b*d - a*e)*x^3*log(c*x^6 + b*x
^3 + a) - 6*(b*d - a*e)*x^3*log(x) - 2*a*d)*sqrt(b^2 - 4*a*c))/(s
qrt(b^2 - 4*a*c)*a^2*x^3), -1/6*(2*(a*b*e - (b^2 - 2*a*c)*d)*x^3*
arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b*d -
 a*e)*x^3*log(c*x^6 + b*x^3 + a) - 6*(b*d - a*e)*x^3*log(x) - 2*a
*d)*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**3+d)/x**4/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.28305, size = 173, normalized size = 1.54

$$\frac{(bd-ae)\ln\left(cx^6+bx^3+a\right)}{6\,a^2}-\frac{(bd-ae)\ln\left(|x|\right)}{a^2}+\frac{\left(b^2d-2\,acd-abe\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}a^2}+\frac{bdx^3-ax^3e-ad}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x^4),x, algorithm="giac")
```

```
[Out] 1/6*(b*d - a*e)*ln(c*x^6 + b*x^3 + a)/a^2 - (b*d - a*e)*ln(abs(x)
)/a^2 + 1/3*(b^2*d - 2*a*c*d - a*b*e)*arctan((2*c*x^3 + b)/sqrt(-
b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/3*(b*d*x^3 - a*x^3*e -
 a*d)/(a^2*x^3)
```

3.14 $\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=723

$$\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$+\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}$$
$$-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}$$
$$-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$
$$-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{ex^2}{2c}$$

```
[Out] (e*x^2)/(2*c) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2
- 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c
])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(5/3)*(b - Sqrt[b^2 - 4*a*
c])^(1/3)) - ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4
*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^
(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])
^(1/3)) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*
c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^
(2/3)*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - ((c*d - b*e + (b*c
*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*
c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*c^(5/3)*(b + Sqrt[b^2
- 4*a*c])^(1/3)) + ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b
```

```
^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*
(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3
)*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((c*d - b*e + (b*c*d -
 b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^
(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)
*c^(2/3)*x^2])/(6*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))
```

Rubi [A] time = 3.97019, antiderivative size = 723, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32

$$\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{ex^2}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x^2)/(2*c) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2
- 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c
])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(5/3)*(b - Sqrt[b^2 - 4*a*
c])^(1/3)) - ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4
*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^
(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])
^(1/3)) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*
c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^
(2/3)*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - ((c*d - b*e + (b*c
*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*
c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*c^(5/3)*(b + Sqrt[b^2
- 4*a*c])^(1/3)) + ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b
^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*
(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3
)*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((c*d - b*e + (b*c*d -
 b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^
(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)
*c^(2/3)*x^2])/(6*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.078496, size = 88, normalized size = 0.12

$$\frac{3ex^2 - 2\text{RootSum}\left[\#1^6 c + \#1^3 b + a\&, \frac{\#1^3 be\log(x-\#1)+\#1^3(-c)d\log(x-\#1)+ae\log(x-\#1)}{2\#1^4 c+\#1 b}\&\right]}{6c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (3*e*x^2 - 2*RootSum[a + b*#1^3 + c*#1^6 & , (a*e*Log[x - #1] - c
*d*Log[x - #1]*#1^3 + b*e*Log[x - #1]*#1^3)/(b*#1 + 2*c*#1^4) & ]
)/(6*c)
```

Maple [C] time = 0.031, size = 70, normalized size = 0.1

$$\frac{ex^2}{2c} - \frac{1}{3c} \sum_{\_R = RootOf\left(\_Z^6c + \_Z^3b + a\right)} \frac{\left((be - cd)\,\_R^4 + ae\_R\right)\ln\left(x - \_R\right)}{2\,\_R^5c + \_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/2*e*x^2/c-1/3/c*sum(((b*e-c*d)*_R^4+a*e*_R)/(2*_R^5*c+_R^2*b)*l
n(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{ex^2}{2c} - \frac{-\int \frac{(cd-be)x^4 - aex}{cx^6+bx^3+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)*x^4/(c*x^6 + b*x^3 + a),x, algorithm="maxima")
```

```
[Out] 1/2*e*x^2/c - integrate(-((c*d - b*e)*x^4 - a*e*x)/(c*x^6 + b*x^3
 + a), x)/c
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)*x^4/(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^3 + d)\,x^4}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)*x^4/(c*x^6 + b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^3 + d)*x^4/(c*x^6 + b*x^3 + a), x)
```

3.15 $\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=718

$$-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$
$$+\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$+\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$
$$-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$
$$-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{ex}{c}$$

```
[Out] (e*x)/c - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*
c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/
3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2
/3)) - ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])
*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))
/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)
) + ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Lo
g[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*
c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((c*d - b*e + (b*c*d - b
^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1
/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*
```

```
c])^(2/3)) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4
*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - S
qrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(4
/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((c*d - b*e + (b*c*d - b^2*e
 + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3)
- 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/
3)*x^2])/(6*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rubi [A] time = 3.44774, antiderivative size = 718, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$

$$-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{ex}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x)/c - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*
c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/
3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2
/3)) - ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])
*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))
/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)
) + ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Lo
g[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*
c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((c*d - b*e + (b*c*d - b
^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1
/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*
c])^(2/3)) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4
*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - S
qrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(4
/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((c*d - b*e + (b*c*d - b^2*e
 + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3)
- 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/
3)*x^2])/(6*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0818609, size = 88, normalized size = 0.12

$$\frac{ex}{c} - \frac{\text{RootSum}\left[\#1^6 c + \#1^3 b + a\&, \frac{\#1^3 be\log(x-\#1)+\#1^3(-c)d\log(x-\#1)+ae\log(x-\#1)}{2\#1^5 c+\#1^2 b}\&\right]}{3c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x)/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*e*Log[x - #1] - c*d*
Log[x - #1]*#1^3 + b*e*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/
(3*c)
```

Maple [C] time = 0.007, size = 67, normalized size = 0.1

$$\frac{ex}{c}+\frac{1}{3\,c}\sum_{\_R=RootOf\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\left(-be+cd\right)\_R^{3}-ae\right)\ln\left(x-\_R\right)}{2\,\_R^{5}c+\_R^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] e*x/c+1/3/c*sum(((-b*e+c*d)*_R^3-a*e)/(2*_R^5*c+_R^2*b)*ln(x-_R),
_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{ex}{c}-\frac{-\int\frac{(cd-be)x^{3}-ae}{cx^{6}+bx^{3}+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)*x^3/(c*x^6 + b*x^3 + a),x, algorithm="maxima")
```

```
[Out] e*x/c - integrate(-((c*d - b*e)*x^3 - a*e)/(c*x^6 + b*x^3 + a), x
)/c
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)*x^3/(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^3+d\right)x^3}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)*x^3/(c*x^6 + b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^3 + d)*x^3/(c*x^6 + b*x^3 + a), x)
```

3.16 $\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=634

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{2/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{2/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}$$

```
[Out] -(((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c
^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[
3]*c^(2/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3))) - ((e - (2*c*d - b*e)/
Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^
2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)*(b + Sqrt[b
^2 - 4*a*c])^(1/3)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[
(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*c^
(2/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - ((e - (2*c*d - b*e)/Sqrt[b
^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*
x])/(3*2^(2/3)*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + ((e + (2*
c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) -
 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3
)*x^2])/(6*2^(2/3)*c^(2/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((e -
 (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/
3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^
(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))
```

Rubi [A] time = 1.88656, antiderivative size = 634, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] -(((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c
^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[
3]*c^(2/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3))) - ((e - (2*c*d - b*e)/
Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^
2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)*(b + Sqrt[b
^2 - 4*a*c])^(1/3)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[
(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*c^
(2/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - ((e - (2*c*d - b*e)/Sqrt[b
^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*
x])/(3*2^(2/3)*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + ((e + (2*
c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) -
 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3
)*x^2])/(6*2^(2/3)*c^(2/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((e -
 (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/
3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^
(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0470714, size = 59, normalized size = 0.09

$$\frac{1}{3}\text{RootSum}\left[\#1^6c+\#1^3b+a\&,\frac{\#1^3e\log(x-\#1)+d\log(x-\#1)}{2\#1^4c+\#1b}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (d*Log[x - #1] + e*Log[x - #1]*#1
^3)/(b*#1 + 2*c*#1^4) & ]/3
```

Maple [C] time = 0.005, size = 49, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=RootOf\left(\_Z^6c+\_Z^3b+a\right)}\frac{\left(\_R^4e+\_R\,d\right)\ln\left(x-\_R\right)}{2\,\_R^5c+\_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*sum((_R^4*e+_R*d)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c
+_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ex^3+d\right)x}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)*x/(c*x^6 + b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^3 + d)*x/(c*x^6 + b*x^3 + a), x)
```

Fricas [A] time = 4.37104, size = 19340, normalized size = 30.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)*x/(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*(1/2)^(1/3)*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 + (a*
b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*
b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e
^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3
*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6
*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2
 - 4*a^2*c^3))^(1/3)*arctan(-(1/2)^(2/3)*(sqrt(3)*((a*b^6*c^3 - 1
2*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*d^2 - (a^2*b^6*c^2 -
 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)*e^2)*sqrt((b^2*c^4
*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(
a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)
*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b
^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2
*c^6 - 64*a^5*c^7)) - sqrt(3)*((b^4*c^3 - 4*a*b^2*c^4)*d^5 - 10*(
a*b^3*c^3 - 4*a^2*b*c^4)*d^4*e + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 2
4*a^3*c^4)*d^3*e^2 - (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^
2*e^3 + (7*a^2*b^4*c - 36*a^3*b^2*c^2 + 32*a^4*c^3)*d*e^4 - (a^2*
b^5 - 6*a^3*b^3*c + 8*a^4*b*c^2)*e^5))*(-(c^2*d^3 - 3*a*c*d*e^2 +
 a*b*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4
*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^
2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2
*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)
*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7
)))/(a*b^2*c^2 - 4*a^2*c^3))^(2/3)/((1/2)^(2/3)*((b^4*c^3 - 4*a*b
^2*c^4)*d^5 - 10*(a*b^3*c^3 - 4*a^2*b*c^4)*d^4*e + 4*(a*b^4*c^2 +
 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^2 - (a*b^5*c + 12*a^2*b^3*c^2
- 64*a^3*b*c^3)*d^2*e^3 + (7*a^2*b^4*c - 36*a^3*b^2*c^2 + 32*a^4*
c^3)*d*e^4 - (a^2*b^5 - 6*a^3*b^3*c + 8*a^4*b*c^2)*e^5 - ((a*b^6*
c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*d^2 - (a^2*b^
6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)*e^2)*sqrt((
b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^
2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a
^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 -
4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*
a^4*b^2*c^6 - 64*a^5*c^7)))*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 +
```

```
(a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*
(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^
3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*
a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*
b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*
c^2 - 4*a^2*c^3))^(2/3) - 4*(b*c^4*d^7 - 2*(b^2*c^3 + 3*a*c^4)*d^
6*e + (b^3*c^2 + 17*a*b*c^3)*d^5*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3
)*d^4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^4 - (a*b^4 + 6*a^2*b^
2*c + 2*a^3*c^2)*d^2*e^5 + (2*a^2*b^3 - a^3*b*c)*d*e^6 - (a^3*b^2
 - 2*a^4*c)*e^7)*x - 4*sqrt(1/2)*(b*c^4*d^7 - 2*(b^2*c^3 + 3*a*c^
4)*d^6*e + (b^3*c^2 + 17*a*b*c^3)*d^5*e^2 - 5*(3*a*b^2*c^2 + 2*a^
2*c^3)*d^4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^4 - (a*b^4 + 6*a
^2*b^2*c + 2*a^3*c^2)*d^2*e^5 + (2*a^2*b^3 - a^3*b*c)*d*e^6 - (a^
3*b^2 - 2*a^4*c)*e^7)*sqrt((2*(b*c^4*d^7 - 2*(b^2*c^3 + 3*a*c^4)*
d^6*e + (b^3*c^2 + 17*a*b*c^3)*d^5*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c
^3)*d^4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^4 - (a*b^4 + 6*a^2*
b^2*c + 2*a^3*c^2)*d^2*e^5 + (2*a^2*b^3 - a^3*b*c)*d*e^6 - (a^3*b
^2 - 2*a^4*c)*e^7)*x^2 + (1/2)^(2/3)*(((a*b^6*c^3 - 12*a^2*b^4*c^
4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*d^2 - (a^2*b^6*c^2 - 12*a^3*b^4*
c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)*e^2)*x*sqrt((b^2*c^4*d^6 - 12*
a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2
+ 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 -
 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a
^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*
a^5*c^7)) - ((b^4*c^3 - 4*a*b^2*c^4)*d^5 - 10*(a*b^3*c^3 - 4*a^2*
b*c^4)*d^4*e + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^2
 - (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^2*e^3 + (7*a^2*b^4
*c - 36*a^3*b^2*c^2 + 32*a^4*c^3)*d*e^4 - (a^2*b^5 - 6*a^3*b^3*c
+ 8*a^4*b*c^2)*e^5)*x)*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 + (a*b^
2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^
2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3
 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b
*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c
^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 -
 4*a^2*c^3))^(2/3) + (1/2)^(1/3)*((b^3*c^3 - 4*a*b*c^4)*d^6 - (b^
4*c^2 + 2*a*b^2*c^3 - 24*a^2*c^4)*d^5*e + 10*(a*b^3*c^2 - 4*a^2*b
*c^3)*d^4*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^3 +
 (a*b^5 - 3*a^2*b^3*c - 4*a^3*b*c^2)*d^2*e^4 - (a^2*b^4 - 6*a^3*b
^2*c + 8*a^4*c^2)*d*e^5 - ((a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*
c^5)*d^3 - (a*b^6*c^2 - 6*a^2*b^4*c^3 + 32*a^4*c^5)*d^2*e + 3*(a^
2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d*e^2 - 2*(a^3*b^4*c^2
- 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4
*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^
2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2
*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)
*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7
)))*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*
sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*
d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2
 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*
b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5
 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3))
/(b*c^4*d^7 - 2*(b^2*c^3 + 3*a*c^4)*d^6*e + (b^3*c^2 + 17*a*b*c^3
)*d^5*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^3 + 5*(a*b^3*c + 3*
```

a^2*b*c^2)*d^3*e^4 - (a*b^4 + 6*a^2*b^2*c + 2*a^3*c^2)*d^2*e^5 +
(2*a^2*b^3 - a^3*b*c)*d*e^6 - (a^3*b^2 - 2*a^4*c)*e^7)))) + 2/3*s
qrt(3)*(1/2)^(1/3)*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b^2*c^
2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^
3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3
*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2
)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 -
 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a
^2*c^3))^(1/3)*arctan(-(1/2)^(2/3)*(sqrt(3)*((a*b^6*c^3 - 12*a^2*
b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*d^2 - (a^2*b^6*c^2 - 12*a^
3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)*e^2)*sqrt((b^2*c^4*d^6 -
 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*
c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e
^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c +
 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 -
 64*a^5*c^7)) + sqrt(3)*((b^4*c^3 - 4*a*b^2*c^4)*d^5 - 10*(a*b^3*
c^3 - 4*a^2*b*c^4)*d^4*e + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*a^3*
c^4)*d^3*e^2 - (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^2*e^3
+ (7*a^2*b^4*c - 36*a^3*b^2*c^2 + 32*a^4*c^3)*d*e^4 - (a^2*b^5 -
6*a^3*b^3*c + 8*a^4*b*c^2)*e^5))*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e
^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e
 + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^
3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c
 - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/
(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a
*b^2*c^2 - 4*a^2*c^3))^(2/3)/((1/2)^(2/3)*((b^4*c^3 - 4*a*b^2*c^4
)*d^5 - 10*(a*b^3*c^3 - 4*a^2*b*c^4)*d^4*e + 4*(a*b^4*c^2 + 2*a^2
*b^2*c^3 - 24*a^3*c^4)*d^3*e^2 - (a*b^5*c + 12*a^2*b^3*c^2 - 64*a
^3*b*c^3)*d^2*e^3 + (7*a^2*b^4*c - 36*a^3*b^2*c^2 + 32*a^4*c^3)*d
*e^4 - (a^2*b^5 - 6*a^3*b^3*c + 8*a^4*b*c^2)*e^5 + ((a*b^6*c^3 -
12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*d^2 - (a^2*b^6*c^2
- 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)*e^2)*sqrt((b^2*c^
4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*
(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3
)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*
b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^
2*c^6 - 64*a^5*c^7)))*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b^2
*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2
*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3
+ 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*
c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^
4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 -
4*a^2*c^3))^(2/3) - 4*(b*c^4*d^7 - 2*(b^2*c^3 + 3*a*c^4)*d^6*e +
(b^3*c^2 + 17*a*b*c^3)*d^5*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*
e^3 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^4 - (a*b^4 + 6*a^2*b^2*c +
2*a^3*c^2)*d^2*e^5 + (2*a^2*b^3 - a^3*b*c)*d*e^6 - (a^3*b^2 - 2*a
^4*c)*e^7)*x - 4*sqrt(1/2)*(b*c^4*d^7 - 2*(b^2*c^3 + 3*a*c^4)*d^6
*e + (b^3*c^2 + 17*a*b*c^3)*d^5*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)
*d^4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^4 - (a*b^4 + 6*a^2*b^2
*c + 2*a^3*c^2)*d^2*e^5 + (2*a^2*b^3 - a^3*b*c)*d*e^6 - (a^3*b^2
- 2*a^4*c)*e^7)*sqrt((2*(b*c^4*d^7 - 2*(b^2*c^3 + 3*a*c^4)*d^6*e
+ (b^3*c^2 + 17*a*b*c^3)*d^5*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^
4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^4 - (a*b^4 + 6*a^2*b^2*c
+ 2*a^3*c^2)*d^2*e^5 + (2*a^2*b^3 - a^3*b*c)*d*e^6 - (a^3*b^2 - 2

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*a^4*c)*e^7)*x^2 - (1/2)^(2/3)*(((a*b^6*c^3 - 12*a^2*b^4*c^4 + 48
*a^3*b^2*c^5 - 64*a^4*c^6)*d^2 - (a^2*b^6*c^2 - 12*a^3*b^4*c^3 +
48*a^4*b^2*c^4 - 64*a^5*c^5)*e^2)*x*sqrt((b^2*c^4*d^6 - 12*a*b*c^
4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a
^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^
2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2
)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^
7)) + ((b^4*c^3 - 4*a*b^2*c^4)*d^5 - 10*(a*b^3*c^3 - 4*a^2*b*c^4)
*d^4*e + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^2 - (a*
b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^2*e^3 + (7*a^2*b^4*c - 3
6*a^3*b^2*c^2 + 32*a^4*c^3)*d*e^4 - (a^2*b^5 - 6*a^3*b^3*c + 8*a^
4*b*c^2)*e^5)*x)*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b^2*c^2
- 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3
+ 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(
7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*
d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 1
2*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2
*c^3))^(2/3) + (1/2)^(1/3)*((b^3*c^3 - 4*a*b*c^4)*d^6 - (b^4*c^2
+ 2*a*b^2*c^3 - 24*a^2*c^4)*d^5*e + 10*(a*b^3*c^2 - 4*a^2*b*c^3)*
d^4*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^3 + (a*b^
5 - 3*a^2*b^3*c - 4*a^3*b*c^2)*d^2*e^4 - (a^2*b^4 - 6*a^3*b^2*c +
 8*a^4*c^2)*d*e^5 + ((a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d
^3 - (a*b^6*c^2 - 6*a^2*b^4*c^3 + 32*a^4*c^5)*d^2*e + 3*(a^2*b^5*
c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d*e^2 - 2*(a^3*b^4*c^2 - 8*a^
4*b^2*c^3 + 16*a^5*c^4)*e^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e
 + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^
3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c
 - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/
(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))*(-
(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((
b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^
2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a
^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 -
4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*
a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3))/(b*c^
4*d^7 - 2*(b^2*c^3 + 3*a*c^4)*d^6*e + (b^3*c^2 + 17*a*b*c^3)*d^5*
e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^3 + 5*(a*b^3*c + 3*a^2*b*
c^2)*d^3*e^4 - (a*b^4 + 6*a^2*b^2*c + 2*a^3*c^2)*d^2*e^5 + (2*a^2
*b^3 - a^3*b*c)*d*e^6 - (a^3*b^2 - 2*a^4*c)*e^7)))) - 1/6*(1/2)^(
1/3)*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 + (a*b^2*c^2 - 4*a^2*c^3)
*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)
*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^
2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2
*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^
5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3)
*log(2*(b*c^4*d^7 - 2*(b^2*c^3 + 3*a*c^4)*d^6*e + (b^3*c^2 + 17*a
*b*c^3)*d^5*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^3 + 5*(a*b^3*
c + 3*a^2*b*c^2)*d^3*e^4 - (a*b^4 + 6*a^2*b^2*c + 2*a^3*c^2)*d^2*
e^5 + (2*a^2*b^3 - a^3*b*c)*d*e^6 - (a^3*b^2 - 2*a^4*c)*e^7)*x^2
+ (1/2)^(2/3)*(((a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64
*a^4*c^6)*d^2 - (a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 -
64*a^5*c^5)*e^2)*x*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^
2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3
 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b
```

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*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c
^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)) - ((b^4*c^3 -
 4*a*b^2*c^4)*d^5 - 10*(a*b^3*c^3 - 4*a^2*b*c^4)*d^4*e + 4*(a*b^4
*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^2 - (a*b^5*c + 12*a^2*b^
3*c^2 - 64*a^3*b*c^3)*d^2*e^3 + (7*a^2*b^4*c - 36*a^3*b^2*c^2 + 3
2*a^4*c^3)*d*e^4 - (a^2*b^5 - 6*a^3*b^3*c + 8*a^4*b*c^2)*e^5)*x)*
(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt
((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*
e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8
*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4
- 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 4
8*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(2/3) + (1
/2)^(1/3)*((b^3*c^3 - 4*a*b*c^4)*d^6 - (b^4*c^2 + 2*a*b^2*c^3 - 2
4*a^2*c^4)*d^5*e + 10*(a*b^3*c^2 - 4*a^2*b*c^3)*d^4*e^2 - 4*(a*b^
4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^3 + (a*b^5 - 3*a^2*b^3*c -
 4*a^3*b*c^2)*d^2*e^4 - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*e^5
 - ((a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^3 - (a*b^6*c^2 -
 6*a^2*b^4*c^3 + 32*a^4*c^5)*d^2*e + 3*(a^2*b^5*c^2 - 8*a^3*b^3*c
^3 + 16*a^4*b*c^4)*d*e^2 - 2*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^
5*c^4)*e^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 +
 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7
*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d
*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12
*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))*(-(c^2*d^3 - 3*a*c*
d*e^2 + a*b*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*
a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2
+ 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 -
 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a
^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*
a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3)) - 1/6*(1/2)^(1/3)*(-(c
^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^
2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2
- 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3
*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*
a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^
4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3)*log(2*(b
*c^4*d^7 - 2*(b^2*c^3 + 3*a*c^4)*d^6*e + (b^3*c^2 + 17*a*b*c^3)*d
^5*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^3 + 5*(a*b^3*c + 3*a^2
*b*c^2)*d^3*e^4 - (a*b^4 + 6*a^2*b^2*c + 2*a^3*c^2)*d^2*e^5 + (2*
a^2*b^3 - a^3*b*c)*d*e^6 - (a^3*b^2 - 2*a^4*c)*e^7)*x^2 - (1/2)^(
2/3)*(((a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)
*d^2 - (a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^
5)*e^2)*x*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6
*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a
^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e
^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a
^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)) + ((b^4*c^3 - 4*a*b^2*
c^4)*d^5 - 10*(a*b^3*c^3 - 4*a^2*b*c^4)*d^4*e + 4*(a*b^4*c^2 + 2*
a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^2 - (a*b^5*c + 12*a^2*b^3*c^2 - 6
4*a^3*b*c^3)*d^2*e^3 + (7*a^2*b^4*c - 36*a^3*b^2*c^2 + 32*a^4*c^3
)*d*e^4 - (a^2*b^5 - 6*a^3*b^3*c + 8*a^4*b*c^2)*e^5)*x)*(-(c^2*d^
3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4
*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(
a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)
```

```
*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b
^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2
*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(2/3) + (1/2)^(1/3)
*((b^3*c^3 - 4*a*b*c^4)*d^6 - (b^4*c^2 + 2*a*b^2*c^3 - 24*a^2*c^4
)*d^5*e + 10*(a*b^3*c^2 - 4*a^2*b*c^3)*d^4*e^2 - 4*(a*b^4*c - 3*a
^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^3 + (a*b^5 - 3*a^2*b^3*c - 4*a^3*b*
c^2)*d^2*e^4 - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*e^5 + ((a*b^
5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^3 - (a*b^6*c^2 - 6*a^2*b^
4*c^3 + 32*a^4*c^5)*d^2*e + 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a
^4*b*c^4)*d*e^2 - 2*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^
3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^
4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*
c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a
^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*
c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))*(-(c^2*d^3 - 3*a*c*d*e^2 + a
*b*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d
^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*
b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b
^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e
^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7))
)/(a*b^2*c^2 - 4*a^2*c^3))^(1/3)) + 1/3*(1/2)^(1/3)*(-(c^2*d^3 -
3*a*c*d*e^2 + a*b*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6
 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^
3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2
*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c
 + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6
 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3)*log((1/2)^(2/3)*(
(b^4*c^3 - 4*a*b^2*c^4)*d^5 - 10*(a*b^3*c^3 - 4*a^2*b*c^4)*d^4*e
+ 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^2 - (a*b^5*c +
 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^2*e^3 + (7*a^2*b^4*c - 36*a^3*b
^2*c^2 + 32*a^4*c^3)*d*e^4 - (a^2*b^5 - 6*a^3*b^3*c + 8*a^4*b*c^2
)*e^5 - ((a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^
6)*d^2 - (a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*
c^5)*e^2)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6
*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a
^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e
^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a
^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))*(-(c^2*d^3 - 3*a*c*d*
e^2 + a*b*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*
b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 +
16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6
*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4
*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^
5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(2/3) + 2*(b*c^4*d^7 - 2*(b^2*c
^3 + 3*a*c^4)*d^6*e + (b^3*c^2 + 17*a*b*c^3)*d^5*e^2 - 5*(3*a*b^2
*c^2 + 2*a^2*c^3)*d^4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^4 - (
a*b^4 + 6*a^2*b^2*c + 2*a^3*c^2)*d^2*e^5 + (2*a^2*b^3 - a^3*b*c)*
d*e^6 - (a^3*b^2 - 2*a^4*c)*e^7)*x) + 1/3*(1/2)^(1/3)*(-(c^2*d^3
- 3*a*c*d*e^2 + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d
^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*
b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d
^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2
*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c
^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3)*log((1/2)^(2/3)
```

```
*((b^4*c^3 - 4*a*b^2*c^4)*d^5 - 10*(a*b^3*c^3 - 4*a^2*b*c^4)*d^4*
e + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^2 - (a*b^5*c
 + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^2*e^3 + (7*a^2*b^4*c - 36*a^3
*b^2*c^2 + 32*a^4*c^3)*d*e^4 - (a^2*b^5 - 6*a^3*b^3*c + 8*a^4*b*c
^2)*e^5 + ((a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*
c^6)*d^2 - (a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^
5*c^5)*e^2)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 +
 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7
*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d
*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12
*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))*(-(c^2*d^3 - 3*a*c*
d*e^2 + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*
a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2
+ 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 -
 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a
^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*
a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(2/3) + 2*(b*c^4*d^7 - 2*(b^2
*c^3 + 3*a*c^4)*d^6*e + (b^3*c^2 + 17*a*b*c^3)*d^5*e^2 - 5*(3*a*b
^2*c^2 + 2*a^2*c^3)*d^4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^4 -
 (a*b^4 + 6*a^2*b^2*c + 2*a^3*c^2)*d^2*e^5 + (2*a^2*b^3 - a^3*b*c
)*d*e^6 - (a^3*b^2 - 2*a^4*c)*e^7)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^3 + d)x}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)*x/(c*x^6 + b*x^3 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^3 + d)*x/(c*x^6 + b*x^3 + a), x)
```

3.17 $\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=634

$$-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$

```
[Out] -(((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c
^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[
3]*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3))) - ((e - (2*c*d - b*e)/
Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^
2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)*(b + Sqrt[b
^2 - 4*a*c])^(2/3)) + ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[
(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*c^
(1/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((e - (2*c*d - b*e)/Sqrt[b
^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*
x])/(3*2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((e + (2*
c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) -
 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3
)*x^2])/(6*2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((e -
 (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/
3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^
(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rubi [A] time = 1.65044, antiderivative size = 634, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318

$$-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^3)/(a + b*x^3 + c*x^6),x]
```

```
[Out] -(((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c
^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[
3]*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3))) - ((e - (2*c*d - b*e)/
Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^
2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)*(b + Sqrt[b
^2 - 4*a*c])^(2/3)) + ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[
(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*c^
(1/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((e - (2*c*d - b*e)/Sqrt[b
^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*
x])/(3*2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((e + (2*
c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) -
 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3
)*x^2])/(6*2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((e -
 (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/
3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^
(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rubi in Sympy [A] time = 174.044, size = 661, normalized size = 1.04

$$\frac{2^{\frac{2}{3}}\left(be-2cd+e\sqrt{-4ac+b^2}\right)\log\left(\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b+\sqrt{-4ac+b^2}}\right)}{6\sqrt[3]{c}\left(b+\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\sqrt{-4ac+b^2}} - \frac{2^{\frac{2}{3}}\left(be-2cd+e\sqrt{-4ac+b^2}\right)\log\left(c^{\frac{2}{3}}x^2-\frac{2^{\frac{2}{3}}\sqrt[3]{c}x\sqrt[3]{b+\sqrt{-4ac+b^2}}}{2}+\frac{\sqrt[3]{2}\left(b+\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}}{2}\right)}{12\sqrt[3]{c}\left(b+\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\sqrt{-4ac+b^2}} - \frac{2^{\frac{2}{3}}\sqrt{3}\left(be-2cd+e\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\sqrt{3}\left(-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{3\sqrt[3]{b+\sqrt{-4ac+b^2}}}+\frac{1}{3}\right)\right)}{6\sqrt[3]{c}\left(b+\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\sqrt{-4ac+b^2}} - \frac{2^{\frac{2}{3}}\left(be-2cd-e\sqrt{-4ac+b^2}\right)\log\left(\sqrt[3]{2}\sqrt[3]{c}x+\sqrt[3]{b-\sqrt{-4ac+b^2}}\right)}{6\sqrt[3]{c}\left(b-\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\sqrt{-4ac+b^2}} + \frac{2^{\frac{2}{3}}\left(be-2cd-e\sqrt{-4ac+b^2}\right)\log\left(c^{\frac{2}{3}}x^2-\frac{2^{\frac{2}{3}}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{-4ac+b^2}}}{2}+\frac{\sqrt[3]{2}\left(b-\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}}{2}\right)}{12\sqrt[3]{c}\left(b-\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\sqrt{-4ac+b^2}} + \frac{2^{\frac{2}{3}}\sqrt{3}\left(be-2cd-e\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\sqrt{3}\left(-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{3\sqrt[3]{b-\sqrt{-4ac+b^2}}}+\frac{1}{3}\right)\right)}{6\sqrt[3]{c}\left(b-\sqrt{-4ac+b^2}\right)^{\frac{2}{3}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] 2**(2/3)*(b*e - 2*c*d + e*sqrt(-4*a*c + b**2))*log(2**(1/3)*c**(1
/3)*x + (b + sqrt(-4*a*c + b**2))**(1/3))/(6*c**(1/3)*(b + sqrt(-
4*a*c + b**2))**(2/3)*sqrt(-4*a*c + b**2)) - 2**(2/3)*(b*e - 2*c*
d + e*sqrt(-4*a*c + b**2))*log(c**(2/3)*x**2 - 2**(2/3)*c**(1/3)*
x*(b + sqrt(-4*a*c + b**2))**(1/3)/2 + 2**(1/3)*(b + sqrt(-4*a*c
+ b**2))**(2/3)/2)/(12*c**(1/3)*(b + sqrt(-4*a*c + b**2))**(2/3)*
sqrt(-4*a*c + b**2)) - 2**(2/3)*sqrt(3)*(b*e - 2*c*d + e*sqrt(-4*
a*c + b**2))*atan(sqrt(3)*(-2*2**(1/3)*c**(1/3)*x/(3*(b + sqrt(-4
```

```
*a*c + b**2))**(1/3)) + 1/3))/(6*c**(1/3)*(b + sqrt(-4*a*c + b**2
))**(2/3)*sqrt(-4*a*c + b**2)) - 2**(2/3)*(b*e - 2*c*d - e*sqrt(-
4*a*c + b**2))*log(2**(1/3)*c**(1/3)*x + (b - sqrt(-4*a*c + b**2)
)**(1/3))/(6*c**(1/3)*(b - sqrt(-4*a*c + b**2))**(2/3)*sqrt(-4*a*
c + b**2)) + 2**(2/3)*(b*e - 2*c*d - e*sqrt(-4*a*c + b**2))*log(c
**(2/3)*x**2 - 2**(2/3)*c**(1/3)*x*(b - sqrt(-4*a*c + b**2))**(1/
3)/2 + 2**(1/3)*(b - sqrt(-4*a*c + b**2))**(2/3)/2)/(12*c**(1/3)*
(b - sqrt(-4*a*c + b**2))**(2/3)*sqrt(-4*a*c + b**2)) + 2**(2/3)*
sqrt(3)*(b*e - 2*c*d - e*sqrt(-4*a*c + b**2))*atan(sqrt(3)*(-2*2*
*(1/3)*c**(1/3)*x/(3*(b - sqrt(-4*a*c + b**2))**(1/3)) + 1/3))/(6
*c**(1/3)*(b - sqrt(-4*a*c + b**2))**(2/3)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0479341, size = 61, normalized size = 0.1

$$\frac{1}{3}\text{RootSum}\left[\#1^6c + \#1^3b + a\&, \frac{\#1^3e\log(x-\#1) + d\log(x-\#1)}{2\#1^5c + \#1^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^3)/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (d*Log[x - #1] + e*Log[x - #1]*#1
^3)/(b*#1^2 + 2*c*#1^5) & ]/3
```

Maple [C] time = 0.004, size = 47, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=RootOf\left(\_Z^6c+\_Z^3b+a\right)}\frac{\left(\_R^3e+d\right)\ln\left(x-\_R\right)}{2\_R^5c+\_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*sum((_R^3*e+d)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z
^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^3+d}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)/(c*x^6 + b*x^3 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^3 + d)/(c*x^6 + b*x^3 + a), x)
```

Fricas [A] time = 1.87965, size = 18004, normalized size = 28.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)/(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*(1/2)^(1/3)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2
*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*
c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*
d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16
*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6
*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c
 - 4*a^3*c^2))^(1/3)*arctan(-(1/2)^(1/3)*(sqrt(3)*((a^2*b^5*c - 8
*a^3*b^3*c^2 + 16*a^4*b*c^3)*d - 2*(a^3*b^4*c - 8*a^4*b^2*c^2 + 1
6*a^5*c^3)*e)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 -
4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e
- 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b
*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 -
 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) - sqrt(3)*((b^4*c
 - 6*a*b^2*c^2 + 8*a^2*c^3)*d^4 - 3*(a*b^3*c - 4*a^2*b*c^2)*d^3*e
 + 6*(a^2*b^2*c - 4*a^3*c^2)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^
3))*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*s
qrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4
*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*
c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 -
 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3
 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3)/
(4*(10*a^2*b*c*d^2*e^3 + a^3*b*e^5 - (b^2*c^2 - 2*a*c^3)*d^5 + (b
^3*c + a*b*c^2)*d^4*e - 4*(a*b^2*c + a^2*c^2)*d^3*e^2 - (a^2*b^2
+ 6*a^3*c)*d*e^4)*x + 4*sqrt(1/2)*(10*a^2*b*c*d^2*e^3 + a^3*b*e^5
 - (b^2*c^2 - 2*a*c^3)*d^5 + (b^3*c + a*b*c^2)*d^4*e - 4*(a*b^2*c
 + a^2*c^2)*d^3*e^2 - (a^2*b^2 + 6*a^3*c)*d*e^4)*sqrt((2*(a^4*b*e
^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4
*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c^2)
*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^
3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6)*x^2 - (1/2)^(2/3)*(
(b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*d^5 - 5*(a*b^
5*c - 6*a^2*b^3*c^2 + 8*a^3*b*c^3)*d^4*e + 2*(7*a^2*b^4*c - 36*a^
3*b^2*c^2 + 32*a^4*c^3)*d^3*e^2 - (a^2*b^5 + 12*a^3*b^3*c - 64*a^
4*b*c^2)*d^2*e^3 + 2*(a^3*b^4 + 2*a^4*b^2*c - 24*a^5*c^2)*d*e^4 -
 2*(a^4*b^3 - 4*a^5*b*c)*e^5 - ((a^2*b^7*c - 12*a^3*b^5*c^2 + 48*
```

a^4*b^3*c^3 - 64*a^5*b*c^4)*d^2 - 2*(a^3*b^6*c - 12*a^4*b^4*c^2 +
48*a^5*b^2*c^3 - 64*a^6*c^4)*d*e)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*
b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2
- 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*
(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^
2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^
5)))*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*
sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 +
4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2
*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3
- 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^
3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(2/3)
+ (1/2)^(1/3)*(((a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^3
- (a^2*b^6*c - 6*a^3*b^4*c^2 + 32*a^5*c^4)*d^2*e + 3*(a^3*b^5*c
- 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d*e^2 - 2*(a^4*b^4*c - 8*a^5*b^2*
c^2 + 16*a^6*c^3)*e^3)*x*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 -
(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*
c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c
+ 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^
4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) - ((b^
4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^6 - (b^5*c - 3*a*b^3*c^2 - 4*a
^2*b*c^3)*d^5*e + 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^4*e^2
- 10*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e^3 + (a^2*b^4 + 2*a^3*b^2*c
- 24*a^4*c^2)*d^2*e^4 - (a^3*b^3 - 4*a^4*b*c)*d*e^5)*x)*((b*c*d^3
- 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*
b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6
+ 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c
^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c
+ 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2
*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3))/(a^4*b*e^7 -
(b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c +
6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4
e^3 - 5(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*
c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6)) - (1/2)^(1/3)*((b^4*c
- 6*a*b^2*c^2 + 8*a^2*c^3)*d^4 - 3*(a*b^3*c - 4*a^2*b*c^2)*d^3*e
+ 6*(a^2*b^2*c - 4*a^3*c^2)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3
- ((a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d - 2*(a^3*b^4*c -
8*a^4*b^2*c^2 + 16*a^5*c^3)*e)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2
*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2
*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^
2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e
^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5))
)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqr
t(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a
^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^
2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6
*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 +
48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3)))
+ 2/3*sqrt(3)*(1/2)^(1/3)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 - (a^
2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4
*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)
*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 1
6*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^
6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*

c - 4*a^3*c^2))^(1/3)*arctan(-(1/2)^(1/3)*(sqrt(3)*((a^2*b^5*c -
8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d - 2*(a^3*b^4*c - 8*a^4*b^2*c^2 +
16*a^5*c^3)*e)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 -
4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e
- 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*
b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2
- 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) + sqrt(3)*((b^4*
c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^4 - 3*(a*b^3*c - 4*a^2*b*c^2)*d^3*
e + 6*(a^2*b^2*c - 4*a^3*c^2)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e
^3))*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*
sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 +
4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2
*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3
- 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^
3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3)
/(4*(10*a^2*b*c*d^2*e^3 + a^3*b*e^5 - (b^2*c^2 - 2*a*c^3)*d^5 + (
b^3*c + a*b*c^2)*d^4*e - 4*(a*b^2*c + a^2*c^2)*d^3*e^2 - (a^2*b^2
+ 6*a^3*c)*d*e^4)*x + 4*sqrt(1/2)*(10*a^2*b*c*d^2*e^3 + a^3*b*e^
5 - (b^2*c^2 - 2*a*c^3)*d^5 + (b^3*c + a*b*c^2)*d^4*e - 4*(a*b^2*
c + a^2*c^2)*d^3*e^2 - (a^2*b^2 + 6*a^3*c)*d*e^4)*sqrt((2*(a^4*b*
e^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^
4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c^2
)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a
^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6)*x^2 - (1/2)^(2/3)*
((b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*d^5 - 5*(a*b
^5*c - 6*a^2*b^3*c^2 + 8*a^3*b*c^3)*d^4*e + 2*(7*a^2*b^4*c - 36*a
^3*b^2*c^2 + 32*a^4*c^3)*d^3*e^2 - (a^2*b^5 + 12*a^3*b^3*c - 64*a
^4*b*c^2)*d^2*e^3 + 2*(a^3*b^4 + 2*a^4*b^2*c - 24*a^5*c^2)*d*e^4
- 2*(a^4*b^3 - 4*a^5*b*c)*e^5 + ((a^2*b^7*c - 12*a^3*b^5*c^2 + 48
*a^4*b^3*c^3 - 64*a^5*b*c^4)*d^2 - 2*(a^3*b^6*c - 12*a^4*b^4*c^2
+ 48*a^5*b^2*c^3 - 64*a^6*c^4)*d*e)*sqrt(-(12*a^4*b*c*d*e^5 - a^4
*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2
- 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2
*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d
^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c
^5)))*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)
*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 +
4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^
2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3
- 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c
^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(2/3
) - (1/2)^(1/3)*(((a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^
3 - (a^2*b^6*c - 6*a^3*b^4*c^2 + 32*a^5*c^4)*d^2*e + 3*(a^3*b^5*c
- 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d*e^2 - 2*(a^4*b^4*c - 8*a^5*b^2
*c^2 + 16*a^6*c^3)*e^3)*x*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 -
(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b
*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*
c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a
^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) + ((b
^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^6 - (b^5*c - 3*a*b^3*c^2 - 4*
a^2*b*c^3)*d^5*e + 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^4*e^
2 - 10*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e^3 + (a^2*b^4 + 2*a^3*b^2*c
- 24*a^4*c^2)*d^2*e^4 - (a^3*b^3 - 4*a^4*b*c)*d*e^5)*x)*((b*c*d^
3 - 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4

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*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^
6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*
c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*
c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^
2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3))/(a^4*b*e^7
- (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c
+ 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^
4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b
*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6)) - (1/2)^(1/3)*((b^4*c
 - 6*a*b^2*c^2 + 8*a^2*c^3)*d^4 - 3*(a*b^3*c - 4*a^2*b*c^2)*d^3*e
 + 6*(a^2*b^2*c - 4*a^3*c^2)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^
3 + ((a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d - 2*(a^3*b^4*c
- 8*a^4*b^2*c^2 + 16*a^5*c^3)*e)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^
2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 -
2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a
^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*
e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)
))*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sq
rt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*
a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c
^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 -
6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3
+ 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3)))
 - 1/6*(1/2)^(1/3)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c
 - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 -
4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e
- 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b
*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 -
 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a
^3*c^2))^(1/3)*log(2*(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^
3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^
2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^
2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*
c)*d*e^6)*x^2 - (1/2)^(2/3)*((b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^
3 - 16*a^3*c^4)*d^5 - 5*(a*b^5*c - 6*a^2*b^3*c^2 + 8*a^3*b*c^3)*d
^4*e + 2*(7*a^2*b^4*c - 36*a^3*b^2*c^2 + 32*a^4*c^3)*d^3*e^2 - (a
^2*b^5 + 12*a^3*b^3*c - 64*a^4*b*c^2)*d^2*e^3 + 2*(a^3*b^4 + 2*a^
4*b^2*c - 24*a^5*c^2)*d*e^4 - 2*(a^4*b^3 - 4*a^5*b*c)*e^5 - ((a^2
*b^7*c - 12*a^3*b^5*c^2 + 48*a^4*b^3*c^3 - 64*a^5*b*c^4)*d^2 - 2*
(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*d*e)*s
qrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4
*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*
c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 -
 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3
 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))*((b*c*d^3 - 3*a*c*d^2*e + a^2*e
^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^
6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^
2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b
^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)
/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(
a^2*b^2*c - 4*a^3*c^2))^(2/3) + (1/2)^(1/3)*(((a^2*b^5*c^2 - 8*a^
3*b^3*c^3 + 16*a^4*b*c^4)*d^3 - (a^2*b^6*c - 6*a^3*b^4*c^2 + 32*a
^5*c^4)*d^2*e + 3*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d*e^
2 - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e^3)*x*sqrt(-(12*a
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^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*
d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^
3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^
2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*
b^2*c^4 - 64*a^7*c^5)) - ((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^6
 - (b^5*c - 3*a*b^3*c^2 - 4*a^2*b*c^3)*d^5*e + 4*(a*b^4*c - 3*a^2
*b^2*c^2 - 4*a^3*c^3)*d^4*e^2 - 10*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*
e^3 + (a^2*b^4 + 2*a^3*b^2*c - 24*a^4*c^2)*d^2*e^4 - (a^3*b^3 - 4
*a^4*b*c)*d*e^5)*x)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*
c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 -
 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e
 - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*
b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2
- 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*
a^3*c^2))^(1/3)) - 1/6*(1/2)^(1/3)*((b*c*d^3 - 3*a*c*d^2*e + a^2*
e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e
^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a
^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*
b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4
)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/
(a^2*b^2*c - 4*a^3*c^2))^(1/3)*log(2*(a^4*b*e^7 - (b^2*c^3 - 2*a*
c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2
*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*
b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(
a^3*b^2 + 3*a^4*c)*d*e^6)*x^2 - (1/2)^(2/3)*((b^6*c - 8*a*b^4*c^2
 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*d^5 - 5*(a*b^5*c - 6*a^2*b^3*c^2
+ 8*a^3*b*c^3)*d^4*e + 2*(7*a^2*b^4*c - 36*a^3*b^2*c^2 + 32*a^4*c
^3)*d^3*e^2 - (a^2*b^5 + 12*a^3*b^3*c - 64*a^4*b*c^2)*d^2*e^3 + 2
*(a^3*b^4 + 2*a^4*b^2*c - 24*a^5*c^2)*d*e^4 - 2*(a^4*b^3 - 4*a^5*
b*c)*e^5 + ((a^2*b^7*c - 12*a^3*b^5*c^2 + 48*a^4*b^3*c^3 - 64*a^5
*b*c^4)*d^2 - 2*(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64
*a^6*c^4)*d*e)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 -
 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e
 - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*
b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2
- 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))*((b*c*d^3 - 3*a
*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*
e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(
a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^
4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a
^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 -
 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(2/3) - (1/2)^(1/3)*(((a^
2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^3 - (a^2*b^6*c - 6*a^
3*b^4*c^2 + 32*a^5*c^4)*d^2*e + 3*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16
*a^5*b*c^3)*d*e^2 - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e^
3)*x*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c
^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^
2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3
*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b
^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) + ((b^4*c^2 - 6*a*b^2*c^3
+ 8*a^2*c^4)*d^6 - (b^5*c - 3*a*b^3*c^2 - 4*a^2*b*c^3)*d^5*e + 4*
(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^4*e^2 - 10*(a^2*b^3*c - 4
*a^3*b*c^2)*d^3*e^3 + (a^2*b^4 + 2*a^3*b^2*c - 24*a^4*c^2)*d^2*e^
4 - (a^3*b^3 - 4*a^4*b*c)*d*e^5)*x)*((b*c*d^3 - 3*a*c*d^2*e + a^2
```

```
*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*
e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*
a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2
*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^
4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))
/(a^2*b^2*c - 4*a^3*c^2))^(1/3)) + 1/3*(1/2)^(1/3)*((b*c*d^3 - 3*
a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d
*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*
(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d
^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*
a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4
- 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3)*log(2*(10*a^2*b*c*
d^2*e^3 + a^3*b*e^5 - (b^2*c^2 - 2*a*c^3)*d^5 + (b^3*c + a*b*c^2)
*d^4*e - 4*(a*b^2*c + a^2*c^2)*d^3*e^2 - (a^2*b^2 + 6*a^3*c)*d*e^
4)*x + (1/2)^(1/3)*((b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^4 - 3*(a*
b^3*c - 4*a^2*b*c^2)*d^3*e + 6*(a^2*b^2*c - 4*a^3*c^2)*d^2*e^2 -
(a^2*b^3 - 4*a^3*b*c)*d*e^3 - ((a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^
4*b*c^3)*d - 2*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*e)*sqrt(-
(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*
c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 -
 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a
^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48
*a^6*b^2*c^4 - 64*a^7*c^5)))*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 +
(a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (
b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c
^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c
+ 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4
*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b
^2*c - 4*a^3*c^2))^(1/3)) + 1/3*(1/2)^(1/3)*((b*c*d^3 - 3*a*c*d^2
*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 -
a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*
c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2
+ 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2
)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^
7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3)*log(2*(10*a^2*b*c*d^2*e^3
 + a^3*b*e^5 - (b^2*c^2 - 2*a*c^3)*d^5 + (b^3*c + a*b*c^2)*d^4*e
- 4*(a*b^2*c + a^2*c^2)*d^3*e^2 - (a^2*b^2 + 6*a^3*c)*d*e^4)*x +
(1/2)^(1/3)*((b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^4 - 3*(a*b^3*c -
 4*a^2*b*c^2)*d^3*e + 6*(a^2*b^2*c - 4*a^3*c^2)*d^2*e^2 - (a^2*b^
3 - 4*a^3*b*c)*d*e^3 + ((a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3
)*d - 2*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*e)*sqrt(-(12*a^4
*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^
6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*
c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*
c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^
2*c^4 - 64*a^7*c^5)))*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 - (a^2*b^
2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2
 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5
*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^
3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^
2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c -
4*a^3*c^2))^(1/3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**3+d)/(c*x**6+b*x**3+a),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^3 + d}{cx^6 + bx^3 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)/(c*x^6 + b*x^3 + a),x, algorithm="giac")

[Out] integrate((e*x^3 + d)/(c*x^6 + b*x^3 + a), x)
```

3.18 $\int \frac{d+ex^3}{x^2(a+bx^3+cx^6)}\,dx$

Optimal. Leaf size=653

$$-\frac{\sqrt[3]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\sqrt[3]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\sqrt[3]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{d}{ax}$$

```
[Out] -(d/(a*x)) + (c^(1/3)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[
3]])/(2^(2/3)*Sqrt[3]*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)
*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(
1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]
*a*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(d + (b*d - 2*a*e)/S
qrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(
1/3)*x])/(3*2^(2/3)*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(
d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^
(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*a*(b + Sqrt[b^2 - 4*a*c])^
(1/3)) - (c^(1/3)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b -
Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c]
```

```
)^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*a*(b - Sqrt[b^2 - 4*
a*c])^(1/3)) - (c^(1/3)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log
[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 -
4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*a*(b + Sqrt[b^
2 - 4*a*c])^(1/3))
```

Rubi [A] time = 2.60559, antiderivative size = 653, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$

$$-\frac{\sqrt[3]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\sqrt[3]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\sqrt[3]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{d}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -(d/(a*x)) + (c^(1/3)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[
3]])/(2^(2/3)*Sqrt[3]*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)
*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(
1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]
*a*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(d + (b*d - 2*a*e)/S
qrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(
1/3)*x])/(3*2^(2/3)*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(
d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^
(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*a*(b + Sqrt[b^2 - 4*a*c])^
(1/3)) - (c^(1/3)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b -
Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c]
)^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*a*(b - Sqrt[b^2 - 4*
a*c])^(1/3)) - (c^(1/3)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log
[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 -
4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*a*(b + Sqrt[b^
2 - 4*a*c])^(1/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((e*x**3+d)/x**2/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0747336, size = 85, normalized size = 0.13

$$-\frac{\text{RootSum}\left[\#1^6 c+\#1^3 b+a\&,\frac{\#1^3 c d \log (x-\#1)-a e \log (x-\#1)+b d \log (x-\#1)}{2 \#1^4 c+\#1 b}\&\right]}{3 a}-\frac{d}{a x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -(d/(a*x)) - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] - a
*e*Log[x - #1] + c*d*Log[x - #1]*#1^3)/(b*#1 + 2*c*#1^4) & ]/(3*a
)
```

Maple [C] time = 0.01, size = 70, normalized size = 0.1

$$-\frac{d}{ax}-\frac{1}{3\,a}\sum_{\_R=RootOf\left(\_Z^6c+\_Z^3b+a\right)}\frac{\left(cd\_R^4+(-ae+bd)\,\_R\right)\ln\left(x-\_R\right)}{2\,\_R^5c+\_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x)
```

```
[Out] -d/a/x-1/3/a*sum((c*d*_R^4+(-a*e+b*d)*_R)/(2*_R^5*c+_R^2*b)*ln(x-
_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{cdx^4+(bd-ae)x}{cx^6+bx^3+a}\,dx}{a}-\frac{d}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((c*d*x^4 + (b*d - a*e)*x)/(c*x^6 + b*x^3 + a), x)/a -
d/(a*x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**3+d)/x**2/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^3 + d}{(cx^6 + bx^3 + a)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x^2),x, algorithm="giac")
```

```
[Out] integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x^2), x)
```

3.19 $\int \frac{d+ex^3}{x^3(a+bx^3+cx^6)}\,dx$

Optimal. Leaf size=655

$$\frac{c^{2/3}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{c^{2/3}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{c^{2/3}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{d}{2ax^2}$$

```
[Out] -d/(2*a*x^2) + (c^(2/3)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqr
t[3]])/(2^(1/3)*Sqrt[3]*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/
3)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c
^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[
3]*a*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(d + (b*d - 2*a*e)
/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c
^(1/3)*x])/(3*2^(1/3)*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)
*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c]
)^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b + Sqrt[b^2 - 4*a*c]
)^(2/3)) + (c^(2/3)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b
```

```
- Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*
c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*a*(b - Sqrt[b^2 -
4*a*c])^(2/3)) + (c^(2/3)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*L
og[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2
- 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*a*(b + Sqrt[
b^2 - 4*a*c])^(2/3))
```

<u>Rubi</u> [A] time = 2.514, antiderivative size = 655, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$

$$\frac{c^{2/3}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{c^{2/3}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{c^{2/3}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{d}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -d/(2*a*x^2) + (c^(2/3)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqr
t[3]])/(2^(1/3)*Sqrt[3]*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/
3)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c
^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[
3]*a*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(d + (b*d - 2*a*e)
/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c
^(1/3)*x])/(3*2^(1/3)*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)
*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c]
)^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b + Sqrt[b^2 - 4*a*c]
)^(2/3)) + (c^(2/3)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b
- Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*
c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*a*(b - Sqrt[b^2 -
4*a*c])^(2/3)) + (c^(2/3)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*L
og[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2
- 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*a*(b + Sqrt[
b^2 - 4*a*c])^(2/3))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**3+d)/x**3/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.0778205, size = 89, normalized size = 0.14

$$-\frac{\text{RootSum}\left[\#1^6 c+\#1^3 b+a\&,\frac{\#1^3 c d \log (x-\#1)-a e \log (x-\#1)+b d \log (x-\#1)}{2 \#1^5 c+\#1^2 b}\&\right]}{3 a}-\frac{d}{2 a x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -d/(2*a*x^2) - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] -
 a*e*Log[x - #1] + c*d*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/
(3*a)
```

Maple [C] time = 0.01, size = 68, normalized size = 0.1

$$\frac{1}{3a}\sum_{\_R=RootOf\left(\_Z^6c+\_Z^3b+a\right)}\frac{\left(-\_R^3cd+ae-bd\right)\ln\left(x-\_R\right)}{2\_R^5c+\_R^2b}-\frac{d}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3/a*sum((-_R^3*c*d+a*e-b*d)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootO
f(_Z^6*c+_Z^3*b+a))-1/2*d/a/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{cdx^3+bd-ae}{cx^6+bx^3+a}\,dx}{a}-\frac{d}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x^3),x, algorithm="maxima")
```

```
[Out] -integrate((c*d*x^3 + b*d - a*e)/(c*x^6 + b*x^3 + a), x)/a - 1/2*
d/(a*x^2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**3+d)/x**3/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^3 + d}{(cx^6 + bx^3 + a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x^3),x, algorithm="giac")
```

```
[Out] integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x^3), x)
```

3.20 $\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=46

$$-\frac{x^6}{6}-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

```
[Out] -x^6/6 - ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 +
x^6]/6
```

Rubi [A] time = 0.12434, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261

$$-\frac{x^6}{6}-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^8*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x^6/6 - ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 +
x^6]/6
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^3}{3}-\frac{1}{3}\right)\right)}{9}-\frac{\int^{x^3}x\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(sqrt(3)*(2*x**3/3 - 1/3))/9
 - Integral(x, (x, x**3))/3
```

Mathematica [A] time = 0.0245158, size = 46, normalized size = 1.

$$-\frac{x^6}{6}+\frac{\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^8*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x^6/6 + ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 +
 x^6]/6
```

Maple [A] time = 0.004, size = 38, normalized size = 0.8

$$-\frac{x^{6}}{6}+\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}+\frac{\sqrt{3}}{9}\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -1/6*x^6+1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/
2))
```

Maxima [A] time = 0.817217, size = 50, normalized size = 1.09

$$-\frac{1}{6}x^{6}+\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{3}-1\right)\right)+\frac{1}{6}\log\left(x^{6}-x^{3}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)*x^8/(x^6 - x^3 + 1),x, algorithm="maxima")
```

```
[Out] -1/6*x^6 + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(
x^6 - x^3 + 1)
```

Fricas [A] time = 0.256763, size = 59, normalized size = 1.28

$$-\frac{1}{18}\sqrt{3}\left(\sqrt{3}x^{6}-\sqrt{3}\log\left(x^{6}-x^{3}+1\right)-2\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{3}-1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)*x^8/(x^6 - x^3 + 1),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*x^6 - sqrt(3)*log(x^6 - x^3 + 1) - 2*arcta
n(1/3*sqrt(3)*(2*x^3 - 1)))
```

Sympy [A] time = 0.286037, size = 42, normalized size = 0.91

$$-\frac{x^6}{6} + \frac{\log\left(x^6 - x^3 + 1\right)}{6} + \frac{\sqrt{3}\,\text{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x**6/6 + log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3
- sqrt(3)/3)/9
```

GIAC/XCAS [A] time = 0.269074, size = 50, normalized size = 1.09

$$-\frac{1}{6}\,x^6 + \frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^3 - 1\right)\right) + \frac{1}{6}\ln\left(x^6 - x^3 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)*x^8/(x^6 - x^3 + 1),x, algorithm="giac")
```

```
[Out] -1/6*x^6 + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*ln(x
^6 - x^3 + 1)
```

3.21 $\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=31

$$-\frac{x^3}{3}-\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -x^3/3 - (2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rubi [A] time = 0.0831293, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174

$$-\frac{x^3}{3}-\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^5*(1 - x^3))/(1 - x^3 + x^6),x]

[Out] -x^3/3 - (2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rubi in Sympy [A] time = 14.2477, size = 29, normalized size = 0.94

$$-\frac{x^3}{3}+\frac{2\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x^3}{3}-\frac{1}{3}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(-x**3+1)/(x**6-x**3+1),x)

[Out] -x**3/3 + 2*sqrt(3)*atan(sqrt(3)*(2*x**3/3 - 1/3))/9
```

Mathematica [A] time = 0.0123437, size = 31, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^5*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x^3/3 + (2*ArcTan[(-1 + 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Maple [A] time = 0.004, size = 25, normalized size = 0.8

$$-\frac{x^3}{3}+\frac{2\sqrt{3}}{9}\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -1/3*x^3+2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A] time = 0.816813, size = 32, normalized size = 1.03

$$-\frac{1}{3}x^3+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)*x^5/(x^6 - x^3 + 1),x, algorithm="maxima")
```

```
[Out] -1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Fricas [A] time = 0.272018, size = 38, normalized size = 1.23

$$-\frac{1}{9}\sqrt{3}\left(\sqrt{3}x^3-2\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)*x^5/(x^6 - x^3 + 1),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*(sqrt(3)*x^3 - 2*arctan(1/3*sqrt(3)*(2*x^3 - 1)))
```

Sympy [A] time = 0.269313, size = 32, normalized size = 1.03

$$-\frac{x^3}{3}+\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x**3/3 + 2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

GIAC/XCAS [A] time = 0.271258, size = 32, normalized size = 1.03

$$-\frac{1}{3}x^3+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)*x^5/(x^6 - x^3 + 1),x, algorithm="giac")
```

```
[Out] -1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

3.22 $\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)$$

```
[Out] -ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) - Log[1 - x^3 + x^6]/6
```

Rubi [A] time = 0.0866248, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) - Log[1 - x^3 + x^6]/6
```

Rubi in Sympy [A] time = 13.6594, size = 34, normalized size = 0.87

$$-\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^3}{3}-\frac{1}{3}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(sqrt(3)*(2*x**3/3 - 1/3))/
9
```

Mathematica [A] time = 0.0134294, size = 39, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) - Log[1 - x^3 + x^6]/6
```

Maple [A] time = 0.004, size = 33, normalized size = 0.9

$$-\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}+\frac{\sqrt{3}}{9}\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A] time = 0.816035, size = 43, normalized size = 1.1

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{3}-1\right)\right)-\frac{1}{6}\log\left(x^{6}-x^{3}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)*x^2/(x^6 - x^3 + 1),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 +
 1)
```

Fricas [A] time = 0.260337, size = 49, normalized size = 1.26

$$-\frac{1}{18}\sqrt{3}\left(\sqrt{3}\log\left(x^{6}-x^{3}+1\right)-2\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{3}-1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)*x^2/(x^6 - x^3 + 1),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*log(x^6 - x^3 + 1) - 2*arctan(1/3*sqrt(3)*
(2*x^3 - 1)))
```

Sympy [A] time = 0.297888, size = 37, normalized size = 0.95

$$-\frac{\log\left(x^6 - x^3 + 1\right)}{6} + \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)
/3)/9
```

GIAC/XCAS [A] time = 0.273731, size = 43, normalized size = 1.1

$$\frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^3 - 1\right)\right) - \frac{1}{6}\ln\left(x^6 - x^3 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)*x^2/(x^6 - x^3 + 1),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*ln(x^6 - x^3 +
1)
```

3.23 $\int \frac{1-x^3}{x(1-x^3+x^6)}\,dx$

Optimal. Leaf size=41

$$\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

```
[Out] ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - Log[1 - x^3 +
x^6]/6
```

Rubi [A] time = 0.115386, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261

$$\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^3)/(x*(1 - x^3 + x^6)),x]
```

```
[Out] ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - Log[1 - x^3 +
x^6]/6
```

Rubi in Sympy [A] time = 15.4631, size = 41, normalized size = 1.

$$\frac{\log\left(x^3\right)}{3}-\frac{\log\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^3}{3}-\frac{1}{3}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**3+1)/x/(x**6-x**3+1),x)
```

```
[Out] log(x**3)/3 - log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(sqrt(3)*(2*x*
*3/3 - 1/3))/9
```

Mathematica [C] time = 0.0204808, size = 44, normalized size = 1.07

$$\log(x)-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^3)/(x*(1 - x^3 + x^6)),x]
```

```
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^3)/(-1 + 2*#
1^3) & ]/3
```

Maple [A] time = 0.008, size = 35, normalized size = 0.9

$$\ln(x) - \frac{\ln\left(x^6 - x^3 + 1\right)}{6} - \frac{\sqrt{3}}{9}\arctan\left(\frac{\left(2x^3 - 1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^3+1)/x/(x^6-x^3+1),x)
```

```
[Out] ln(x)-1/6*ln(x^6-x^3+1)-1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A] time = 0.818721, size = 51, normalized size = 1.24

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3 - 1\right)\right) - \frac{1}{6}\log\left(x^6 - x^3 + 1\right) + \frac{1}{3}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)/((x^6 - x^3 + 1)*x),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3
+ 1) + 1/3*log(x^3)
```

Fricas [A] time = 0.269618, size = 58, normalized size = 1.41

$$-\frac{1}{18}\sqrt{3}\left(\sqrt{3}\log\left(x^6 - x^3 + 1\right) - 6\sqrt{3}\log(x) + 2\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3 - 1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)/((x^6 - x^3 + 1)*x),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*log(x^6 - x^3 + 1) - 6*sqrt(3)*log(x) + 2*
arctan(1/3*sqrt(3)*(2*x^3 - 1)))
```

Sympy [A] time = 0.339523, size = 41, normalized size = 1.

$$\log(x) - \frac{\log(x^6 - x^3 + 1)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**3+1)/x/(x**6-x**3+1),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 -
 sqrt(3)/3)/9
```

GIAC/XCAS [A] time = 0.288027, size = 47, normalized size = 1.15

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3 - 1)\right) - \frac{1}{6}\ln\left(x^6 - x^3 + 1\right) + \ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)/((x^6 - x^3 + 1)*x),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*ln(x^6 - x^3 +
 1) + ln(abs(x))
```

3.24 $\int \frac{1-x^3}{x^4(1-x^3+x^6)}\,dx$

Optimal. Leaf size=31

$$\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3x^3}$$

```
[Out] -1/(3*x^3) + (2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rubi [A] time = 0.102452, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174

$$\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^3)/(x^4*(1 - x^3 + x^6)),x]

[Out] -1/(3*x^3) + (2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rubi in Sympy [A] time = 14.5165, size = 32, normalized size = 1.03

$$-\frac{2\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^3}{3}-\frac{1}{3}\right)\right)}{9}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**3+1)/x**4/(x**6-x**3+1),x)

[Out] -2*sqrt(3)*atan(sqrt(3)*(2*x**3/3 - 1/3))/9 - 1/(3*x**3)
```

Mathematica [C] time = 0.0202159, size = 45, normalized size = 1.45

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^3)/(x^4*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(3*x^3) - RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-1 + 2*#1^3
) & ]/3
```

Maple [A] time = 0.006, size = 25, normalized size = 0.8

$$-\frac{1}{3\,x^3}-\frac{2\,\sqrt{3}}{9}\arctan\left(\frac{\left(2\,x^3-1\right)\,\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^3+1)/x^4/(x^6-x^3+1),x)
```

```
[Out] -1/3/x^3-2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A] time = 0.816764, size = 32, normalized size = 1.03

$$-\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^3-1\right)\right)-\frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)/((x^6 - x^3 + 1)*x^4),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3
```

Fricas [A] time = 0.253693, size = 41, normalized size = 1.32

$$-\frac{\sqrt{3}\left(2\,x^3\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^3-1\right)\right)+\sqrt{3}\right)}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)/((x^6 - x^3 + 1)*x^4),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*(2*x^3*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + sqrt(3))/x^
3
```

Sympy [A] time = 0.375926, size = 36, normalized size = 1.16

$$-\frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**3+1)/x**4/(x**6-x**3+1),x)
```

```
[Out] -2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9 - 1/(3*x**3)
```

GIAC/XCAS [A] time = 0.27478, size = 32, normalized size = 1.03

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)/((x^6 - x^3 + 1)*x^4),x, algorithm="giac")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3
```

3.25 $\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=418

$$-\frac{x^4}{4}-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

```
[Out] -x^4/4 - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/
3))/Sqrt[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I - Sqrt[3])*
ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)
*(1 + I*Sqrt[3])^(2/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1
/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((3 - I*Sqr
t[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 + I*S
qrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*
(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[
3])^(2/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 +
 I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^
(2/3))
```

Rubi [A] time = 1.12425, antiderivative size = 418, normalized size of antiderivative = 1., number of

steps used = 15, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$

$$-\frac{x^4}{4}-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^6*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x^4/4 - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/
3))/Sqrt[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I - Sqrt[3])*
ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)
*(1 + I*Sqrt[3])^(2/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1
/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((3 - I*Sqr
t[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 + I*S
qrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*
(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[
3])^(2/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 +
 I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^
(2/3))
```

Rubi in Sympy [A] time = 131.602, size = 340, normalized size = 0.81

$$-\frac{x^4}{4}+\frac{2^{\frac{2}{3}}\sqrt{3}i\sqrt[3]{1-\sqrt{3}i}\log\left(\sqrt[3]{2}x-\sqrt[3]{1-\sqrt{3}i}\right)}{18}-\frac{2^{\frac{2}{3}}\sqrt{3}i\sqrt[3]{1+\sqrt{3}i}\log\left(\sqrt[3]{2}x-\sqrt[3]{1+\sqrt{3}i}\right)}{18}$$
$$-\frac{2^{\frac{2}{3}}\sqrt{3}i\sqrt[3]{1-\sqrt{3}i}\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1-\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{36}$$
$$+\frac{2^{\frac{2}{3}}\sqrt{3}i\sqrt[3]{1+\sqrt{3}i}\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1+\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{36}$$
$$-\frac{2^{\frac{2}{3}}i\sqrt[3]{1-\sqrt{3}i}\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1-\sqrt{3}i}}+\frac{1}{3}\right)\right)}{6}+\frac{2^{\frac{2}{3}}i\sqrt[3]{1+\sqrt{3}i}\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1+\sqrt{3}i}}+\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x**4/4 + 2**(2/3)*sqrt(3)*I*(1 - sqrt(3)*I)**(1/3)*log(2**(1/3)*
x - (1 - sqrt(3)*I)**(1/3))/18 - 2**(2/3)*sqrt(3)*I*(1 + sqrt(3)*
I)**(1/3)*log(2**(1/3)*x - (1 + sqrt(3)*I)**(1/3))/18 - 2**(2/3)*
sqrt(3)*I*(1 - sqrt(3)*I)**(1/3)*log(x**2 + 2**(2/3)*x*(1 - sqrt(
3)*I)**(1/3)/2 + 2**(1/3)*(1 - sqrt(3)*I)**(2/3)/2)/36 + 2**(2/3)
*sqrt(3)*I*(1 + sqrt(3)*I)**(1/3)*log(x**2 + 2**(2/3)*x*(1 + sqrt
(3)*I)**(1/3)/2 + 2**(1/3)*(1 + sqrt(3)*I)**(2/3)/2)/36 - 2**(2/3
)*I*(1 - sqrt(3)*I)**(1/3)*atan(sqrt(3)*(2*2**(1/3)*x/(3*(1 - sqr
t(3)*I)**(1/3)) + 1/3))/6 + 2**(2/3)*I*(1 + sqrt(3)*I)**(1/3)*ata
n(sqrt(3)*(2*2**(1/3)*x/(3*(1 + sqrt(3)*I)**(1/3)) + 1/3))/6
```

Mathematica [C] time = 0.0194534, size = 47, normalized size = 0.11

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^6*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x^4/4 + RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^
3) & ]/3
```

Maple [C] time = 0.009, size = 46, normalized size = 0.1

$$-\frac{x^4}{4}+\frac{1}{3}\sum_{\_R=RootOf(\_Z^6-\_Z^3+1)}\frac{\_R^3\ln(x-\_R)}{2\_R^5-\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -1/4*x^4+1/3*sum(_R^3/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+
1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}x^4+\int\frac{x^3}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)*x^6/(x^6 - x^3 + 1),x, algorithm="maxima")
```

```
[Out] -1/4*x^4 + integrate(x^3/(x^6 - x^3 + 1), x)
```

Fricas [A] time = 0.272284, size = 886, normalized size = 2.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)*x^6/(x^6 - x^3 + 1),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(3)*(3*sqrt(3)*x^4 - 8*(sqrt(3)*cos(2/3*arctan(1/(sqrt(
3) - 2))) - sin(2/3*arctan(1/(sqrt(3) - 2))))*arctan(-(sqrt(3)*si
n(2/3*arctan(1/(sqrt(3) - 2))) + cos(2/3*arctan(1/(sqrt(3) - 2)))
)/(sqrt(3)*cos(2/3*arctan(1/(sqrt(3) - 2))) - 2*x - 2*sqrt(-sqrt(
3)*x*cos(2/3*arctan(1/(sqrt(3) - 2))) + x^2 + cos(2/3*arctan(1/(s
qrt(3) - 2)))^2 + x*sin(2/3*arctan(1/(sqrt(3) - 2))) + sin(2/3*ar
ctan(1/(sqrt(3) - 2)))^2) - sin(2/3*arctan(1/(sqrt(3) - 2))))) -
8*(sqrt(3)*cos(2/3*arctan(1/(sqrt(3) - 2))) + sin(2/3*arctan(1/(s
qrt(3) - 2))))*arctan((sqrt(3)*sin(2/3*arctan(1/(sqrt(3) - 2))) -
```

```
 cos(2/3*arctan(1/(sqrt(3) - 2))))/(sqrt(3)*cos(2/3*arctan(1/(sqr
t(3) - 2))) + 2*x + 2*sqrt(sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3) -
2))) + x^2 + cos(2/3*arctan(1/(sqrt(3) - 2)))^2 + x*sin(2/3*arcta
n(1/(sqrt(3) - 2))) + sin(2/3*arctan(1/(sqrt(3) - 2)))^2) + sin(2
/3*arctan(1/(sqrt(3) - 2))))) - 2*(sqrt(3)*sin(2/3*arctan(1/(sqrt
(3) - 2))) - cos(2/3*arctan(1/(sqrt(3) - 2))))*log(sqrt(3)*x*cos(
2/3*arctan(1/(sqrt(3) - 2))) + x^2 + cos(2/3*arctan(1/(sqrt(3) -
2)))^2 + x*sin(2/3*arctan(1/(sqrt(3) - 2))) + sin(2/3*arctan(1/(s
qrt(3) - 2)))^2) + 2*(sqrt(3)*sin(2/3*arctan(1/(sqrt(3) - 2))) +
cos(2/3*arctan(1/(sqrt(3) - 2))))*log(-sqrt(3)*x*cos(2/3*arctan(1
/(sqrt(3) - 2))) + x^2 + cos(2/3*arctan(1/(sqrt(3) - 2)))^2 + x*s
in(2/3*arctan(1/(sqrt(3) - 2))) + sin(2/3*arctan(1/(sqrt(3) - 2))
)^2) - 4*cos(2/3*arctan(1/(sqrt(3) - 2)))*log(x^2 + cos(2/3*arcta
n(1/(sqrt(3) - 2)))^2 - 2*x*sin(2/3*arctan(1/(sqrt(3) - 2))) + si
n(2/3*arctan(1/(sqrt(3) - 2)))^2) + 16*arctan(cos(2/3*arctan(1/(s
qrt(3) - 2)))/(x + sqrt(x^2 + cos(2/3*arctan(1/(sqrt(3) - 2)))^2
- 2*x*sin(2/3*arctan(1/(sqrt(3) - 2))) + sin(2/3*arctan(1/(sqrt(3
) - 2)))^2) - sin(2/3*arctan(1/(sqrt(3) - 2)))))*sin(2/3*arctan(1
/(sqrt(3) - 2))))
```

Sympy [A] time = 0.481544, size = 31, normalized size = 0.07

$$-\frac{x^4}{4} - \operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t \log\left(-1458t^4 + 9t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x**4/4 - RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(
-1458*_t**4 + 9*_t + x)))
```

GIAC/XCAS [A] time = 0.293495, size = 867, normalized size = 2.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)*x^6/(x^6 - x^3 + 1),x, algorithm="giac")
```

```
[Out] -1/4*x^4 - 1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^
2*sin(4/9*pi)^2 + 2*sqrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4
/9*pi) - 8*cos(4/9*pi)*sin(4/9*pi)^3 + sqrt(3)*cos(4/9*pi) + sin(
4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9*pi) - 2*x)/((sqrt(3)*i
+ 1)*sin(4/9*pi))) - 1/9*(2*sqrt(3)*cos(2/9*pi)^4 - 12*sqrt(3)*co
```

```
s(2/9*pi)^2*sin(2/9*pi)^2 + 2*sqrt(3)*sin(2/9*pi)^4 + 8*cos(2/9*p
i)^3*sin(2/9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)^3 + sqrt(3)*cos(2/9*
pi) + sin(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((
sqrt(3)*i + 1)*sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^4 - 12*
sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4 - 8
*cos(1/9*pi)^3*sin(1/9*pi) + 8*cos(1/9*pi)*sin(1/9*pi)^3 - sqrt(3
)*cos(1/9*pi) + sin(1/9*pi))*arctan(((sqrt(3)*i + 1)*cos(1/9*pi)
+ 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(8*sqrt(3)*cos(4/9*p
i)^3*sin(4/9*pi) - 8*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - 2*cos(4/
9*pi)^4 + 12*cos(4/9*pi)^2*sin(4/9*pi)^2 - 2*sin(4/9*pi)^4 + sqrt
(3)*sin(4/9*pi) - cos(4/9*pi))*ln(-(sqrt(3)*i*cos(4/9*pi) + cos(4
/9*pi))*x + x^2 + 1) - 1/18*(8*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)
- 8*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - 2*cos(2/9*pi)^4 + 12*cos(
2/9*pi)^2*sin(2/9*pi)^2 - 2*sin(2/9*pi)^4 + sqrt(3)*sin(2/9*pi) -
 cos(2/9*pi))*ln(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x + x^2 +
 1) + 1/18*(8*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 8*sqrt(3)*cos(1
/9*pi)*sin(1/9*pi)^3 + 2*cos(1/9*pi)^4 - 12*cos(1/9*pi)^2*sin(1/9
*pi)^2 + 2*sin(1/9*pi)^4 - sqrt(3)*sin(1/9*pi) - cos(1/9*pi))*ln(
(sqrt(3)*i*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

3.26 $\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=382

$$-\frac{x^2}{2}-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}+\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}$$

```
[Out] -x^2/2 + ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt
[3]])/((1 - I*Sqrt[3])/2)^(1/3) - ((I/3)*ArcTan[(1 + (2*x)/((1 +
I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1 + I*Sqrt[3])/2)^(1/3) + ((I/3)
*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(Sqrt[3]*((1 - I*Sqrt[3]
)/2)^(1/3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(Sqr
t[3]*((1 + I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 - I*Sqrt[3])^(2/3
) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*
(1 - I*Sqrt[3])^(1/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1
 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 + I*Sq
rt[3])^(1/3))
```

Rubi [A] time = 0.777977, antiderivative size = 382, normalized size of antiderivative = 1., number

of steps used = 15, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$

$$-\frac{x^2}{2}-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}+\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x^2/2 + ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt
[3]])/((1 - I*Sqrt[3])/2)^(1/3) - ((I/3)*ArcTan[(1 + (2*x)/((1 +
I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1 + I*Sqrt[3])/2)^(1/3) + ((I/3)
*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(Sqrt[3]*((1 - I*Sqrt[3]
)/2)^(1/3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(Sqr
t[3]*((1 + I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 - I*Sqrt[3])^(2/3
) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*
(1 - I*Sqrt[3])^(1/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1
 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 + I*Sq
rt[3])^(1/3))
```

Rubi in Sympy [A] time = 125.338, size = 340, normalized size = 0.89

$$-\frac{x^2}{2}+\frac{\sqrt[3]{2}\sqrt{3}i\log\left(\sqrt[3]{2}x-\sqrt[3]{1-\sqrt{3}i}\right)}{9\sqrt[3]{1-\sqrt{3}i}}-\frac{\sqrt[3]{2}\sqrt{3}i\log\left(\sqrt[3]{2}x-\sqrt[3]{1+\sqrt{3}i}\right)}{9\sqrt[3]{1+\sqrt{3}i}}$$
$$-\frac{\sqrt[3]{2}\sqrt{3}i\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1-\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{18\sqrt[3]{1-\sqrt{3}i}}+\frac{\sqrt[3]{2}\sqrt{3}i\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1+\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{18\sqrt[3]{1+\sqrt{3}i}}$$
$$+\frac{\sqrt[3]{2}i\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1-\sqrt{3}i}}+\frac{1}{3}\right)\right)}{3\sqrt[3]{1-\sqrt{3}i}}-\frac{\sqrt[3]{2}i\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1+\sqrt{3}i}}+\frac{1}{3}\right)\right)}{3\sqrt[3]{1+\sqrt{3}i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x**2/2 + 2**(1/3)*sqrt(3)*I*log(2**(1/3)*x - (1 - sqrt(3)*I)**(1
/3))/(9*(1 - sqrt(3)*I)**(1/3)) - 2**(1/3)*sqrt(3)*I*log(2**(1/3)
*x - (1 + sqrt(3)*I)**(1/3))/(9*(1 + sqrt(3)*I)**(1/3)) - 2**(1/3
)*sqrt(3)*I*log(x**2 + 2**(2/3)*x*(1 - sqrt(3)*I)**(1/3)/2 + 2**(
1/3)*(1 - sqrt(3)*I)**(2/3)/2)/(18*(1 - sqrt(3)*I)**(1/3)) + 2**(
1/3)*sqrt(3)*I*log(x**2 + 2**(2/3)*x*(1 + sqrt(3)*I)**(1/3)/2 + 2
**(1/3)*(1 + sqrt(3)*I)**(2/3)/2)/(18*(1 + sqrt(3)*I)**(1/3)) + 2
**(1/3)*I*atan(sqrt(3)*(2*2**(1/3)*x/(3*(1 - sqrt(3)*I)**(1/3)) +
 1/3))/(3*(1 - sqrt(3)*I)**(1/3)) - 2**(1/3)*I*atan(sqrt(3)*(2*2*
*(1/3)*x/(3*(1 + sqrt(3)*I)**(1/3)) + 1/3))/(3*(1 + sqrt(3)*I)**(
1/3))
```

Mathematica [C] time = 0.0202578, size = 48, normalized size = 0.13

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\log(x-\#1)}{2\#1^4-\#1}\&\right]-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x^2/2 + RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1 + 2*#1^4) &
 ]/3
```

Maple [C] time = 0.008, size = 44, normalized size = 0.1

$$-\frac{x^2}{2}+\frac{1}{3}\sum_{\_R=RootOf(\_Z^6-\_Z^3+1)}\frac{\_R\ln(x-\_R)}{2\_R^5-\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -1/2*x^2+1/3*sum(_R/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}x^2+\int\frac{x}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)*x^4/(x^6 - x^3 + 1),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + integrate(x/(x^6 - x^3 + 1), x)
```

Fricas [A] time = 0.282208, size = 1447, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)*x^4/(x^6 - x^3 + 1),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(3*sqrt(3)*x^2 + 4*(sqrt(3)*cos(2/3*arctan(1/(sqrt(
3) + 2))) + sin(2/3*arctan(1/(sqrt(3) + 2))))*arctan(-2*cos(2/3*a
rctan(1/(sqrt(3) + 2)))*sin(2/3*arctan(1/(sqrt(3) + 2)))/(cos(2/3
*arctan(1/(sqrt(3) + 2)))^2 - sin(2/3*arctan(1/(sqrt(3) + 2)))^2
- x - sqrt(cos(2/3*arctan(1/(sqrt(3) + 2)))^4 + sin(2/3*arctan(1/
(sqrt(3) + 2)))^4 - 2*x*cos(2/3*arctan(1/(sqrt(3) + 2)))^2 + 2*(c
os(2/3*arctan(1/(sqrt(3) + 2)))^2 + x)*sin(2/3*arctan(1/(sqrt(3)
+ 2)))^2 + x^2))) + 4*(sqrt(3)*cos(2/3*arctan(1/(sqrt(3) + 2))) -
 sin(2/3*arctan(1/(sqrt(3) + 2))))*arctan((2*sqrt(3)*cos(2/3*arct
```

```
an(1/(sqrt(3) + 2)))*sin(2/3*arctan(1/(sqrt(3) + 2))) - 3*cos(2/3
*arctan(1/(sqrt(3) + 2)))^2 + 3*sin(2/3*arctan(1/(sqrt(3) + 2)))^
2)/(sqrt(3)*cos(2/3*arctan(1/(sqrt(3) + 2)))^2 - sqrt(3)*sin(2/3*
arctan(1/(sqrt(3) + 2)))^2 + 6*cos(2/3*arctan(1/(sqrt(3) + 2)))*s
in(2/3*arctan(1/(sqrt(3) + 2))) + 2*sqrt(3)*x + 2*sqrt(3)*sqrt(co
s(2/3*arctan(1/(sqrt(3) + 2)))^4 + sin(2/3*arctan(1/(sqrt(3) + 2)
))^4 + 2*sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3) + 2)))*sin(2/3*arcta
n(1/(sqrt(3) + 2))) + x*cos(2/3*arctan(1/(sqrt(3) + 2)))^2 + (2*c
os(2/3*arctan(1/(sqrt(3) + 2)))^2 - x)*sin(2/3*arctan(1/(sqrt(3)
+ 2)))^2 + x^2))) + (sqrt(3)*sin(2/3*arctan(1/(sqrt(3) + 2))) + c
os(2/3*arctan(1/(sqrt(3) + 2))))*log(cos(2/3*arctan(1/(sqrt(3) +
2)))^4 + sin(2/3*arctan(1/(sqrt(3) + 2)))^4 + 2*sqrt(3)*x*cos(2/3
*arctan(1/(sqrt(3) + 2)))*sin(2/3*arctan(1/(sqrt(3) + 2))) + x*co
s(2/3*arctan(1/(sqrt(3) + 2)))^2 + (2*cos(2/3*arctan(1/(sqrt(3) +
 2)))^2 - x)*sin(2/3*arctan(1/(sqrt(3) + 2)))^2 + x^2) - 2*cos(2/
3*arctan(1/(sqrt(3) + 2)))*log(cos(2/3*arctan(1/(sqrt(3) + 2)))^4
 + sin(2/3*arctan(1/(sqrt(3) + 2)))^4 - 2*sqrt(3)*x*cos(2/3*arcta
n(1/(sqrt(3) + 2)))*sin(2/3*arctan(1/(sqrt(3) + 2))) + x*cos(2/3*
arctan(1/(sqrt(3) + 2)))^2 + (2*cos(2/3*arctan(1/(sqrt(3) + 2)))^
2 - x)*sin(2/3*arctan(1/(sqrt(3) + 2)))^2 + x^2) - (sqrt(3)*sin(2
/3*arctan(1/(sqrt(3) + 2))) - cos(2/3*arctan(1/(sqrt(3) + 2))))*l
og(cos(2/3*arctan(1/(sqrt(3) + 2)))^4 + sin(2/3*arctan(1/(sqrt(3)
 + 2)))^4 - 2*x*cos(2/3*arctan(1/(sqrt(3) + 2)))^2 + 2*(cos(2/3*a
rctan(1/(sqrt(3) + 2)))^2 + x)*sin(2/3*arctan(1/(sqrt(3) + 2)))^2
 + x^2) - 8*arctan(-(2*sqrt(3)*cos(2/3*arctan(1/(sqrt(3) + 2)))*s
in(2/3*arctan(1/(sqrt(3) + 2))) + 3*cos(2/3*arctan(1/(sqrt(3) + 2
)))^2 - 3*sin(2/3*arctan(1/(sqrt(3) + 2)))^2)/(sqrt(3)*cos(2/3*ar
ctan(1/(sqrt(3) + 2)))^2 - sqrt(3)*sin(2/3*arctan(1/(sqrt(3) + 2)
))^2 - 6*cos(2/3*arctan(1/(sqrt(3) + 2)))*sin(2/3*arctan(1/(sqrt(
3) + 2))) + 2*sqrt(3)*x + 2*sqrt(3)*sqrt(cos(2/3*arctan(1/(sqrt(3
) + 2)))^4 + sin(2/3*arctan(1/(sqrt(3) + 2)))^4 - 2*sqrt(3)*x*cos
(2/3*arctan(1/(sqrt(3) + 2)))*sin(2/3*arctan(1/(sqrt(3) + 2))) +
x*cos(2/3*arctan(1/(sqrt(3) + 2)))^2 + (2*cos(2/3*arctan(1/(sqrt(
3) + 2)))^2 - x)*sin(2/3*arctan(1/(sqrt(3) + 2)))^2 + x^2)))*sin(
2/3*arctan(1/(sqrt(3) + 2))))
```

Sympy [A] time = 0.492278, size = 32, normalized size = 0.08

$$-\frac{x^2}{2} - \operatorname{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(-6561t^5 - 27t^2 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x**2/2 - RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(
-6561*_t**5 - 27*_t**2 + x)))
```

GIAC/XCAS [A] time = 0.286148, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)*x^4/(x^6 - x^3 + 1),x, algorithm="giac")
```

```
[Out] Done
```

3.27 $\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=378

$$-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}} - x + \frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} - \frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}} - \frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} + \frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}$$

```
[Out] -x - ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]]
)/((1 - I*Sqrt[3])/2)^(2/3) + ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sq
rt[3])/2)^(1/3))/Sqrt[3]])/((1 + I*Sqrt[3])/2)^(2/3) + ((I/3)*Log
[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(Sqrt[3]*((1 - I*Sqrt[3])/2)
^(2/3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(Sqrt[3]
*((1 + I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 - I*Sqrt[3])^(2/3) +
(2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(1 -
 I*Sqrt[3])^(2/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I
*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(1 + I*Sqrt[3
])^(2/3))
```

Rubi [A] time = 0.635488, antiderivative size = 378, normalized size of antiderivative = 1., number

of steps used = 14, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348

$$-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}-x+\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x - ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]]
)/((1 - I*Sqrt[3])/2)^(2/3) + ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sq
rt[3])/2)^(1/3))/Sqrt[3]])/((1 + I*Sqrt[3])/2)^(2/3) + ((I/3)*Log
[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(Sqrt[3]*((1 - I*Sqrt[3])/2)
^(2/3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(Sqrt[3]
*((1 + I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 - I*Sqrt[3])^(2/3) +
(2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(1 -
 I*Sqrt[3])^(2/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I
*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(1 + I*Sqrt[3
])^(2/3))
```

Rubi in Sympy [A] time = 101.496, size = 337, normalized size = 0.89

$$-x+\frac{2^{\frac{2}{3}}\sqrt{3}i\log\left(\sqrt[3]{2}x-\sqrt[3]{1-\sqrt{3}i}\right)}{9\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}-\frac{2^{\frac{2}{3}}\sqrt{3}i\log\left(\sqrt[3]{2}x-\sqrt[3]{1+\sqrt{3}i}\right)}{9\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}-\frac{2^{\frac{2}{3}}\sqrt{3}i\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1-\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{18\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}+\frac{2^{\frac{2}{3}}\sqrt{3}i\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1+\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{18\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}-\frac{2^{\frac{2}{3}}i\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1-\sqrt{3}i}}+\frac{1}{3}\right)\right)}{3\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}+\frac{2^{\frac{2}{3}}i\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1+\sqrt{3}i}}+\frac{1}{3}\right)\right)}{3\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x + 2**(2/3)*sqrt(3)*I*log(2**(1/3)*x - (1 - sqrt(3)*I)**(1/3))/
(9*(1 - sqrt(3)*I)**(2/3)) - 2**(2/3)*sqrt(3)*I*log(2**(1/3)*x -
(1 + sqrt(3)*I)**(1/3))/(9*(1 + sqrt(3)*I)**(2/3)) - 2**(2/3)*sqr
t(3)*I*log(x**2 + 2**(2/3)*x*(1 - sqrt(3)*I)**(1/3)/2 + 2**(1/3)*
(1 - sqrt(3)*I)**(2/3)/2)/(18*(1 - sqrt(3)*I)**(2/3)) + 2**(2/3)*
sqrt(3)*I*log(x**2 + 2**(2/3)*x*(1 + sqrt(3)*I)**(1/3)/2 + 2**(1/
3)*(1 + sqrt(3)*I)**(2/3)/2)/(18*(1 + sqrt(3)*I)**(2/3)) - 2**(2/
3)*I*atan(sqrt(3)*(2*2**(1/3)*x/(3*(1 - sqrt(3)*I)**(1/3)) + 1/3)
)/(3*(1 - sqrt(3)*I)**(2/3)) + 2**(2/3)*I*atan(sqrt(3)*(2*2**(1/3
)*x/(3*(1 + sqrt(3)*I)**(1/3)) + 1/3))/(3*(1 + sqrt(3)*I)**(2/3))
```

Mathematica [C] time = 0.0194578, size = 46, normalized size = 0.12

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\log(x-\#1)}{2\#1^5-\#1^2}\&\right]-x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x + RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1^2 + 2*#1^5) & ]
/3
```

Maple [C] time = 0.007, size = 41, normalized size = 0.1

$$-x+\frac{1}{3}\left(\sum_{\_R=RootOf\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -x+1/3*sum(1/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-x+\int\frac{1}{x^{6}-x^{3}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)*x^3/(x^6 - x^3 + 1),x, algorithm="maxima")
```

```
[Out] -x + integrate(1/(x^6 - x^3 + 1), x)
```

Fricas [A] time = 0.276707, size = 884, normalized size = 2.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)*x^3/(x^6 - x^3 + 1),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(4*(sqrt(3)*cos(2/3*arctan(1/(sqrt(3) + 2))) - sin(2
/3*arctan(1/(sqrt(3) + 2))))*arctan(-(sqrt(3)*sin(2/3*arctan(1/(s
qrt(3) + 2))) - cos(2/3*arctan(1/(sqrt(3) + 2))))/(sqrt(3)*cos(2/
3*arctan(1/(sqrt(3) + 2))) - 2*x - 2*sqrt(-sqrt(3)*x*cos(2/3*arct
an(1/(sqrt(3) + 2))) + x^2 + cos(2/3*arctan(1/(sqrt(3) + 2)))^2 -
 x*sin(2/3*arctan(1/(sqrt(3) + 2))) + sin(2/3*arctan(1/(sqrt(3) +
 2)))^2) + sin(2/3*arctan(1/(sqrt(3) + 2))))) - 4*(sqrt(3)*cos(2/
3*arctan(1/(sqrt(3) + 2))) + sin(2/3*arctan(1/(sqrt(3) + 2))))*ar
ctan(cos(2/3*arctan(1/(sqrt(3) + 2)))/(x + sqrt(x^2 + cos(2/3*arc
tan(1/(sqrt(3) + 2)))^2 + 2*x*sin(2/3*arctan(1/(sqrt(3) + 2))) +
```

```
sin(2/3*arctan(1/(sqrt(3) + 2)))^2) + sin(2/3*arctan(1/(sqrt(3) +
 2))))) + 2*cos(2/3*arctan(1/(sqrt(3) + 2)))*log(sqrt(3)*x*cos(2/
3*arctan(1/(sqrt(3) + 2))) + x^2 + cos(2/3*arctan(1/(sqrt(3) + 2)
))^2 - x*sin(2/3*arctan(1/(sqrt(3) + 2))) + sin(2/3*arctan(1/(sqr
t(3) + 2)))^2) - (sqrt(3)*sin(2/3*arctan(1/(sqrt(3) + 2))) + cos(
2/3*arctan(1/(sqrt(3) + 2))))*log(-sqrt(3)*x*cos(2/3*arctan(1/(sq
rt(3) + 2))) + x^2 + cos(2/3*arctan(1/(sqrt(3) + 2)))^2 - x*sin(2
/3*arctan(1/(sqrt(3) + 2))) + sin(2/3*arctan(1/(sqrt(3) + 2)))^2)
 + (sqrt(3)*sin(2/3*arctan(1/(sqrt(3) + 2))) - cos(2/3*arctan(1/(
sqrt(3) + 2))))*log(x^2 + cos(2/3*arctan(1/(sqrt(3) + 2)))^2 + 2*
x*sin(2/3*arctan(1/(sqrt(3) + 2))) + sin(2/3*arctan(1/(sqrt(3) +
2)))^2) - 8*arctan((sqrt(3)*sin(2/3*arctan(1/(sqrt(3) + 2))) + co
s(2/3*arctan(1/(sqrt(3) + 2))))/(sqrt(3)*cos(2/3*arctan(1/(sqrt(3
) + 2))) + 2*x + 2*sqrt(sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3) + 2))
) + x^2 + cos(2/3*arctan(1/(sqrt(3) + 2)))^2 - x*sin(2/3*arctan(1
/(sqrt(3) + 2))) + sin(2/3*arctan(1/(sqrt(3) + 2)))^2) - sin(2/3*
arctan(1/(sqrt(3) + 2)))))*sin(2/3*arctan(1/(sqrt(3) + 2))) - 6*s
qrt(3)*x)
```

Sympy [A] time = 0.477629, size = 24, normalized size = 0.06

$$-x - \operatorname{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t \log\left(729t^4 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x - RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(729*_
t**4 + x)))
```

GIAC/XCAS [A] time = 0.286417, size = 853, normalized size = 2.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)*x^3/(x^6 - x^3 + 1),x, algorithm="giac")
```

```
[Out] -1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)
^2 + sqrt(3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(
4/9*pi)*sin(4/9*pi)^3 - sqrt(3)*cos(4/9*pi) - sin(4/9*pi))*arctan
(-((sqrt(3)*i + 1)*cos(4/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi
))) - 1/9*(sqrt(3)*cos(2/9*pi)^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin(2/
9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(2/9*pi)^3*sin(2/9*pi) - 4
```

```
*cos(2/9*pi)*sin(2/9*pi)^3 - sqrt(3)*cos(2/9*pi) - sin(2/9*pi))*a
rctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(2
/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9*pi)^2*s
in(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*pi
) + 4*cos(1/9*pi)*sin(1/9*pi)^3 + sqrt(3)*cos(1/9*pi) - sin(1/9*p
i))*arctan(((sqrt(3)*i + 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*s
in(1/9*pi))) - 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 4*sqrt
(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4/9*pi)^2*s
in(4/9*pi)^2 - sin(4/9*pi)^4 - sqrt(3)*sin(4/9*pi) + cos(4/9*pi))
*ln(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) - 1/18*(4
*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*cos(2/9*pi)*sin(2/
9*pi)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(2/9*pi)^2 - sin(2/9
*pi)^4 - sqrt(3)*sin(2/9*pi) + cos(2/9*pi))*ln(-(sqrt(3)*i*cos(2/
9*pi) + cos(2/9*pi))*x + x^2 + 1) + 1/18*(4*sqrt(3)*cos(1/9*pi)^3
*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^
4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 + sqrt(3)*sin(1
/9*pi) + cos(1/9*pi))*ln((sqrt(3)*i*cos(1/9*pi) + cos(1/9*pi))*x
+ x^2 + 1) - x
```

3.28 $\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=411

$$\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

```
[Out] ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[
3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I + Sqrt[3])*ArcTan[(1
 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*S
qrt[3])^(1/3)) - ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(
1/3)*x])/(9*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((3 + I*Sqrt[3])*Log
[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 + I*Sqrt[3])^(
1/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sq
rt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^(1/3)
) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3
]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rubi [A] time = 0.681898, antiderivative size = 411, normalized size of antiderivative = 1., number

of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[
3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I + Sqrt[3])*ArcTan[(1
 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*S
qrt[3])^(1/3)) - ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(
1/3)*x])/(9*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((3 + I*Sqrt[3])*Log
[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 + I*Sqrt[3])^(
1/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sq
rt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^(1/3)
) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3
]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rubi in Sympy [A] time = 105.392, size = 352, normalized size = 0.86

$$-\frac{\sqrt[3]{2}\left(3-\sqrt{3}i\right)\log\left(\sqrt[3]{2}x-\sqrt[3]{1-\sqrt{3}i}\right)}{18\sqrt[3]{1-\sqrt{3}i}}-\frac{\sqrt[3]{2}\left(3+\sqrt{3}i\right)\log\left(\sqrt[3]{2}x-\sqrt[3]{1+\sqrt{3}i}\right)}{18\sqrt[3]{1+\sqrt{3}i}}+\frac{\sqrt[3]{2}\left(3-\sqrt{3}i\right)\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1-\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{36\sqrt[3]{1-\sqrt{3}i}}+\frac{\sqrt[3]{2}\left(3+\sqrt{3}i\right)\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1+\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{36\sqrt[3]{1+\sqrt{3}i}}-\frac{\sqrt[3]{2}\left(\sqrt{3}-i\right)\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1-\sqrt{3}i}}+\frac{1}{3}\right)\right)}{6\sqrt[3]{1-\sqrt{3}i}}-\frac{\sqrt[3]{2}\left(\sqrt{3}+i\right)\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1+\sqrt{3}i}}+\frac{1}{3}\right)\right)}{6\sqrt[3]{1+\sqrt{3}i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -2**(1/3)*(3 - sqrt(3)*I)*log(2**(1/3)*x - (1 - sqrt(3)*I)**(1/3)
)/(18*(1 - sqrt(3)*I)**(1/3)) - 2**(1/3)*(3 + sqrt(3)*I)*log(2**(
1/3)*x - (1 + sqrt(3)*I)**(1/3))/(18*(1 + sqrt(3)*I)**(1/3)) + 2*
*(1/3)*(3 - sqrt(3)*I)*log(x**2 + 2**(2/3)*x*(1 - sqrt(3)*I)**(1/
3)/2 + 2**(1/3)*(1 - sqrt(3)*I)**(2/3)/2)/(36*(1 - sqrt(3)*I)**(1
/3)) + 2**(1/3)*(3 + sqrt(3)*I)*log(x**2 + 2**(2/3)*x*(1 + sqrt(3
)*I)**(1/3)/2 + 2**(1/3)*(1 + sqrt(3)*I)**(2/3)/2)/(36*(1 + sqrt(
3)*I)**(1/3)) - 2**(1/3)*(sqrt(3) - I)*atan(sqrt(3)*(2*2**(1/3)*x
/(3*(1 - sqrt(3)*I)**(1/3)) + 1/3))/(6*(1 - sqrt(3)*I)**(1/3)) -
2**(1/3)*(sqrt(3) + I)*atan(sqrt(3)*(2*2**(1/3)*x/(3*(1 + sqrt(3)
*I)**(1/3)) + 1/3))/(6*(1 + sqrt(3)*I)**(1/3))
```

Mathematica [C] time = 0.0184016, size = 55, normalized size = 0.13

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^4-\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-
#1 + 2*#1^4) & ]/3
```

Maple [C] time = 0.006, size = 44, normalized size = 0.1

$$-\frac{1}{3}\sum_{\_R=RootOf\left(\_Z^6-\_Z^3+1\right)}\frac{\left(\_R^4-\_R\right)\ln\left(x-\_R\right)}{2\_R^5-\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -1/3*sum((_R^4-_R)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(x^3-1\right)x}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)*x/(x^6 - x^3 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x^3 - 1)*x/(x^6 - x^3 + 1), x)
```

Fricas [A] time = 0.275555, size = 1415, normalized size = 3.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)*x/(x^6 - x^3 + 1),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(4*(sqrt(3)*cos(2/3*arctan(1/(sqrt(3) - 2))) + sin(2
/3*arctan(1/(sqrt(3) - 2))))*arctan(-(sqrt(3)*cos(2/3*arctan(1/(s
qrt(3) - 2)))^2 - sqrt(3)*sin(2/3*arctan(1/(sqrt(3) - 2)))^2 - 2*
```

```
cos(2/3*arctan(1/(sqrt(3) - 2)))*sin(2/3*arctan(1/(sqrt(3) - 2)))
)/(2*sqrt(3)*cos(2/3*arctan(1/(sqrt(3) - 2)))*sin(2/3*arctan(1/(s
qrt(3) - 2))) + cos(2/3*arctan(1/(sqrt(3) - 2)))^2 - sin(2/3*arct
an(1/(sqrt(3) - 2)))^2 + 2*x + 2*sqrt(cos(2/3*arctan(1/(sqrt(3) -
 2)))^4 + sin(2/3*arctan(1/(sqrt(3) - 2)))^4 + 2*sqrt(3)*x*cos(2/
3*arctan(1/(sqrt(3) - 2)))*sin(2/3*arctan(1/(sqrt(3) - 2))) + x*c
os(2/3*arctan(1/(sqrt(3) - 2)))^2 + (2*cos(2/3*arctan(1/(sqrt(3)
- 2)))^2 - x)*sin(2/3*arctan(1/(sqrt(3) - 2)))^2 + x^2))) + 4*(sq
rt(3)*cos(2/3*arctan(1/(sqrt(3) - 2))) - sin(2/3*arctan(1/(sqrt(3
) - 2))))*arctan((sqrt(3)*cos(2/3*arctan(1/(sqrt(3) - 2)))^2 - sq
rt(3)*sin(2/3*arctan(1/(sqrt(3) - 2)))^2 + 2*cos(2/3*arctan(1/(sq
rt(3) - 2)))*sin(2/3*arctan(1/(sqrt(3) - 2))))/(2*sqrt(3)*cos(2/3
*arctan(1/(sqrt(3) - 2)))*sin(2/3*arctan(1/(sqrt(3) - 2))) - cos(
2/3*arctan(1/(sqrt(3) - 2)))^2 + sin(2/3*arctan(1/(sqrt(3) - 2)))
^2 - 2*x - 2*sqrt(cos(2/3*arctan(1/(sqrt(3) - 2)))^4 + sin(2/3*ar
ctan(1/(sqrt(3) - 2)))^4 - 2*sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3)
- 2)))*sin(2/3*arctan(1/(sqrt(3) - 2))) + x*cos(2/3*arctan(1/(sqr
t(3) - 2)))^2 + (2*cos(2/3*arctan(1/(sqrt(3) - 2)))^2 - x)*sin(2/
3*arctan(1/(sqrt(3) - 2)))^2 + x^2))) + (sqrt(3)*sin(2/3*arctan(1
/(sqrt(3) - 2))) - cos(2/3*arctan(1/(sqrt(3) - 2))))*log(cos(2/3*
arctan(1/(sqrt(3) - 2)))^4 + sin(2/3*arctan(1/(sqrt(3) - 2)))^4 +
 2*sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3) - 2)))*sin(2/3*arctan(1/(s
qrt(3) - 2))) + x*cos(2/3*arctan(1/(sqrt(3) - 2)))^2 + (2*cos(2/3
*arctan(1/(sqrt(3) - 2)))^2 - x)*sin(2/3*arctan(1/(sqrt(3) - 2)))
^2 + x^2) - (sqrt(3)*sin(2/3*arctan(1/(sqrt(3) - 2))) + cos(2/3*a
rctan(1/(sqrt(3) - 2))))*log(cos(2/3*arctan(1/(sqrt(3) - 2)))^4 +
 sin(2/3*arctan(1/(sqrt(3) - 2)))^4 - 2*sqrt(3)*x*cos(2/3*arctan(
1/(sqrt(3) - 2)))*sin(2/3*arctan(1/(sqrt(3) - 2))) + x*cos(2/3*ar
ctan(1/(sqrt(3) - 2)))^2 + (2*cos(2/3*arctan(1/(sqrt(3) - 2)))^2
- x)*sin(2/3*arctan(1/(sqrt(3) - 2)))^2 + x^2) + 2*cos(2/3*arctan
(1/(sqrt(3) - 2)))*log(cos(2/3*arctan(1/(sqrt(3) - 2)))^4 + sin(2
/3*arctan(1/(sqrt(3) - 2)))^4 - 2*x*cos(2/3*arctan(1/(sqrt(3) - 2
)))^2 + 2*(cos(2/3*arctan(1/(sqrt(3) - 2)))^2 + x)*sin(2/3*arctan
(1/(sqrt(3) - 2)))^2 + x^2) + 8*arctan(-2*cos(2/3*arctan(1/(sqrt(
3) - 2)))*sin(2/3*arctan(1/(sqrt(3) - 2)))/(cos(2/3*arctan(1/(sqr
t(3) - 2)))^2 - sin(2/3*arctan(1/(sqrt(3) - 2)))^2 - x - sqrt(cos
(2/3*arctan(1/(sqrt(3) - 2)))^4 + sin(2/3*arctan(1/(sqrt(3) - 2))
)^4 - 2*x*cos(2/3*arctan(1/(sqrt(3) - 2)))^2 + 2*(cos(2/3*arctan(
1/(sqrt(3) - 2)))^2 + x)*sin(2/3*arctan(1/(sqrt(3) - 2)))^2 + x^2
)))*sin(2/3*arctan(1/(sqrt(3) - 2))))
```

Sympy [A] time = 0.4886, size = 22, normalized size = 0.05

$$-\operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t\log\left(-27t^2 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-27*_t**2
```

```
+ x)))
```

GIAC/XCAS [A] time = 0.285346, size = 1, normalized size = 0.

$$Done$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)*x/(x^6 - x^3 + 1),x, algorithm="giac")

[Out] Done
```

3.29 $\int \frac{1-x^3}{1-x^3+x^6} \, dx$

Optimal. Leaf size=411

$$\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

```
[Out] -((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt
[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I + Sqrt[3])*ArcTan[(
1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*
Sqrt[3])^(2/3)) - ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^
(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Lo
g[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 + I*Sqrt[3])^
(2/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*S
qrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(2/3
)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[
3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rubi [A] time = 0.69264, antiderivative size = 411, normalized size of antiderivative = 1., number of

steps used = 13, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$\frac{(3-i\sqrt{3})\log\left(2^{2/3}x^2+\sqrt[3]{2(1-i\sqrt{3})}\,x+(1-i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}\,(1-i\sqrt{3})^{2/3}} + \frac{(3+i\sqrt{3})\log\left(2^{2/3}x^2+\sqrt[3]{2(1+i\sqrt{3})}\,x+(1+i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}\,(1+i\sqrt{3})^{2/3}} - \frac{(3-i\sqrt{3})\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\,(1-i\sqrt{3})^{2/3}} - \frac{(3+i\sqrt{3})\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\,(1+i\sqrt{3})^{2/3}} - \frac{(-\sqrt{3}+i)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\,(1-i\sqrt{3})^{2/3}} + \frac{(\sqrt{3}+i)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\,(1+i\sqrt{3})^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^3)/(1 - x^3 + x^6),x]
```

```
[Out] -((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt
[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I + Sqrt[3])*ArcTan[(
1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*
Sqrt[3])^(2/3)) - ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^
(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Lo
g[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 + I*Sqrt[3])^
(2/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*S
qrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(2/3
)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[
3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rubi in Sympy [A] time = 103.32, size = 352, normalized size = 0.86

$$-\frac{2^{\frac{2}{3}}\left(3-\sqrt{3}i\right)\log\left(\sqrt[3]{2}x-\sqrt[3]{1-\sqrt{3}i}\right)}{18\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}-\frac{2^{\frac{2}{3}}\left(3+\sqrt{3}i\right)\log\left(\sqrt[3]{2}x-\sqrt[3]{1+\sqrt{3}i}\right)}{18\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}+\frac{2^{\frac{2}{3}}\left(3-\sqrt{3}i\right)\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1-\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{36\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}+\frac{2^{\frac{2}{3}}\left(3+\sqrt{3}i\right)\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1+\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{36\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}+\frac{2^{\frac{2}{3}}\left(\sqrt{3}-i\right)\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1-\sqrt{3}i}}+\frac{1}{3}\right)\right)}{6\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}+\frac{2^{\frac{2}{3}}\left(\sqrt{3}+i\right)\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1+\sqrt{3}i}}+\frac{1}{3}\right)\right)}{6\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -2**(2/3)*(3 - sqrt(3)*I)*log(2**(1/3)*x - (1 - sqrt(3)*I)**(1/3)
)/(18*(1 - sqrt(3)*I)**(2/3)) - 2**(2/3)*(3 + sqrt(3)*I)*log(2**(
1/3)*x - (1 + sqrt(3)*I)**(1/3))/(18*(1 + sqrt(3)*I)**(2/3)) + 2*
*(2/3)*(3 - sqrt(3)*I)*log(x**2 + 2**(2/3)*x*(1 - sqrt(3)*I)**(1/
3)/2 + 2**(1/3)*(1 - sqrt(3)*I)**(2/3)/2)/(36*(1 - sqrt(3)*I)**(2
/3)) + 2**(2/3)*(3 + sqrt(3)*I)*log(x**2 + 2**(2/3)*x*(1 + sqrt(3
)*I)**(1/3)/2 + 2**(1/3)*(1 + sqrt(3)*I)**(2/3)/2)/(36*(1 + sqrt(
3)*I)**(2/3)) + 2**(2/3)*(sqrt(3) - I)*atan(sqrt(3)*(2*2**(1/3)*x
/(3*(1 - sqrt(3)*I)**(1/3)) + 1/3))/(6*(1 - sqrt(3)*I)**(2/3)) +
2**(2/3)*(sqrt(3) + I)*atan(sqrt(3)*(2*2**(1/3)*x/(3*(1 + sqrt(3)
*I)**(1/3)) + 1/3))/(6*(1 + sqrt(3)*I)**(2/3))
```

Mathematica [C] time = 0.0171034, size = 57, normalized size = 0.14

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^5-\#1^2}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^3)/(1 - x^3 + x^6),x]
```

```
[Out] -RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-
#1^2 + 2*#1^5) & ]/3
```

Maple [C] time = 0.007, size = 44, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=RootOf\left(\_Z^6-\_Z^3+1\right)}\frac{\left(-\_R^3+1\right)\ln\left(x-\_R\right)}{2\_R^5-\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] 1/3*sum((-_R^3+1)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^3-1}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)/(x^6 - x^3 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

Fricas [A] time = 0.285585, size = 873, normalized size = 2.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)/(x^6 - x^3 + 1),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(4*(sqrt(3)*cos(2/3*arctan(1/(sqrt(3) - 2))) + sin(
2/3*arctan(1/(sqrt(3) - 2))))*arctan(-(sqrt(3)*sin(2/3*arctan(1/(
sqrt(3) - 2))) + cos(2/3*arctan(1/(sqrt(3) - 2))))/(sqrt(3)*cos(2
```

```
/3*arctan(1/(sqrt(3) - 2))) - 2*x - 2*sqrt(-sqrt(3)*x*cos(2/3*arc
tan(1/(sqrt(3) - 2))) + x^2 + cos(2/3*arctan(1/(sqrt(3) - 2)))^2
+ x*sin(2/3*arctan(1/(sqrt(3) - 2))) + sin(2/3*arctan(1/(sqrt(3)
- 2)))^2) - sin(2/3*arctan(1/(sqrt(3) - 2))))) + 4*(sqrt(3)*cos(2
/3*arctan(1/(sqrt(3) - 2))) - sin(2/3*arctan(1/(sqrt(3) - 2))))*a
rctan(cos(2/3*arctan(1/(sqrt(3) - 2)))/(x + sqrt(x^2 + cos(2/3*ar
ctan(1/(sqrt(3) - 2)))^2 - 2*x*sin(2/3*arctan(1/(sqrt(3) - 2))) +
 sin(2/3*arctan(1/(sqrt(3) - 2)))^2) - sin(2/3*arctan(1/(sqrt(3)
- 2))))) - 2*cos(2/3*arctan(1/(sqrt(3) - 2)))*log(sqrt(3)*x*cos(2
/3*arctan(1/(sqrt(3) - 2))) + x^2 + cos(2/3*arctan(1/(sqrt(3) - 2
)))^2 + x*sin(2/3*arctan(1/(sqrt(3) - 2))) + sin(2/3*arctan(1/(sq
rt(3) - 2)))^2) - (sqrt(3)*sin(2/3*arctan(1/(sqrt(3) - 2))) - cos
(2/3*arctan(1/(sqrt(3) - 2))))*log(-sqrt(3)*x*cos(2/3*arctan(1/(s
qrt(3) - 2))) + x^2 + cos(2/3*arctan(1/(sqrt(3) - 2)))^2 + x*sin(
2/3*arctan(1/(sqrt(3) - 2))) + sin(2/3*arctan(1/(sqrt(3) - 2)))^2
) + (sqrt(3)*sin(2/3*arctan(1/(sqrt(3) - 2))) + cos(2/3*arctan(1/
(sqrt(3) - 2))))*log(x^2 + cos(2/3*arctan(1/(sqrt(3) - 2)))^2 - 2
*x*sin(2/3*arctan(1/(sqrt(3) - 2))) + sin(2/3*arctan(1/(sqrt(3) -
 2)))^2) + 8*arctan((sqrt(3)*sin(2/3*arctan(1/(sqrt(3) - 2))) - c
os(2/3*arctan(1/(sqrt(3) - 2))))/(sqrt(3)*cos(2/3*arctan(1/(sqrt(
3) - 2))) + 2*x + 2*sqrt(sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3) - 2)
)) + x^2 + cos(2/3*arctan(1/(sqrt(3) - 2)))^2 + x*sin(2/3*arctan(
1/(sqrt(3) - 2))) + sin(2/3*arctan(1/(sqrt(3) - 2)))^2) + sin(2/3
*arctan(1/(sqrt(3) - 2)))))*sin(2/3*arctan(1/(sqrt(3) - 2))))
```

Sympy [A] time = 0.483475, size = 26, normalized size = 0.06

$$-\operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t\log\left(729t^4 - 9t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4
 - 9*_t + x)))
```

GIAC/XCAS [A] time = 0.315212, size = 860, normalized size = 2.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)/(x^6 - x^3 + 1),x, algorithm="giac")
```

```
[Out] 1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^
2 + sqrt(3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4
```

```
/9*pi)*sin(4/9*pi)^3 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*arc
tan(-((sqrt(3)*i + 1)*cos(4/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9
*pi))) + 1/9*(sqrt(3)*cos(2/9*pi)^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin
(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(2/9*pi)^3*sin(2/9*pi)
- 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*cos(2/9*pi) + 2*sin(2/9
*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1
)*sin(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9*
pi)^2*sin(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin
(1/9*pi) + 4*cos(1/9*pi)*sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) +
2*sin(1/9*pi))*arctan(((sqrt(3)*i + 1)*cos(1/9*pi) + 2*x)/((sqrt(
3)*i + 1)*sin(1/9*pi))) + 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*p
i) - 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(
4/9*pi)^2*sin(4/9*pi)^2 - sin(4/9*pi)^4 + 2*sqrt(3)*sin(4/9*pi) -
 2*cos(4/9*pi))*ln(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2
 + 1) + 1/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*cos
(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(2/9*
pi)^2 - sin(2/9*pi)^4 + 2*sqrt(3)*sin(2/9*pi) - 2*cos(2/9*pi))*ln
(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x + x^2 + 1) - 1/18*(4*sq
rt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*p
i)^3 + cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi
)^4 - 2*sqrt(3)*sin(1/9*pi) - 2*cos(1/9*pi))*ln((sqrt(3)*i*cos(1/
9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

3.30 $\int \frac{1-x^3}{x^2(1-x^3+x^6)}\,dx$

Optimal. Leaf size=416

$$\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{1}{x}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

```
[Out] -x^(-1) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1
/3))/Sqrt[3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + ((I - Sqrt[3])
*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3
)*(1 + I*Sqrt[3])^(1/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(
1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((3 - I*Sq
rt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 + I*
Sqrt[3])^(1/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2
*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt
[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1
+ I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])
^(1/3))
```

Rubi [A] time = 0.697243, antiderivative size = 416, normalized size of antiderivative = 1., number

of steps used = 14, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$

$$\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{1}{x} - \frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^3)/(x^2*(1 - x^3 + x^6)),x]
```

```
[Out] -x^(-1) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1
/3))/Sqrt[3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + ((I - Sqrt[3])
*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3
)*(1 + I*Sqrt[3])^(1/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(
1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((3 - I*Sq
rt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 + I*
Sqrt[3])^(1/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2
*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt
[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1
+ I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])
^(1/3))
```

Rubi in Sympy [A] time = 108.539, size = 338, normalized size = 0.81

$$-\frac{\sqrt[3]{2}\sqrt{3}i\left(1-\sqrt{3}i\right)^{\frac{2}{3}}\log\left(\sqrt[3]{2}x-\sqrt[3]{1-\sqrt{3}i}\right)}{18}+\frac{\sqrt[3]{2}\sqrt{3}i\left(1+\sqrt{3}i\right)^{\frac{2}{3}}\log\left(\sqrt[3]{2}x-\sqrt[3]{1+\sqrt{3}i}\right)}{18}$$
$$+\frac{\sqrt[3]{2}\sqrt{3}i\left(1-\sqrt{3}i\right)^{\frac{2}{3}}\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1-\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{36}$$
$$-\frac{\sqrt[3]{2}\sqrt{3}i\left(1+\sqrt{3}i\right)^{\frac{2}{3}}\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1+\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{36}$$
$$-\frac{\sqrt[3]{2}i\left(1-\sqrt{3}i\right)^{\frac{2}{3}}\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1-\sqrt{3}i}}+\frac{1}{3}\right)\right)}{6}$$
$$+\frac{\sqrt[3]{2}i\left(1+\sqrt{3}i\right)^{\frac{2}{3}}\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1+\sqrt{3}i}}+\frac{1}{3}\right)\right)}{6}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**3+1)/x**2/(x**6-x**3+1),x)
```

```
[Out] -2**(1/3)*sqrt(3)*I*(1 - sqrt(3)*I)**(2/3)*log(2**(1/3)*x - (1 -
sqrt(3)*I)**(1/3))/18 + 2**(1/3)*sqrt(3)*I*(1 + sqrt(3)*I)**(2/3)
*log(2**(1/3)*x - (1 + sqrt(3)*I)**(1/3))/18 + 2**(1/3)*sqrt(3)*I
*(1 - sqrt(3)*I)**(2/3)*log(x**2 + 2**(2/3)*x*(1 - sqrt(3)*I)**(1
/3)/2 + 2**(1/3)*(1 - sqrt(3)*I)**(2/3)/2)/36 - 2**(1/3)*sqrt(3)*
I*(1 + sqrt(3)*I)**(2/3)*log(x**2 + 2**(2/3)*x*(1 + sqrt(3)*I)**(
1/3)/2 + 2**(1/3)*(1 + sqrt(3)*I)**(2/3)/2)/36 - 2**(1/3)*I*(1 -
sqrt(3)*I)**(2/3)*atan(sqrt(3)*(2*2**(1/3)*x/(3*(1 - sqrt(3)*I)**
(1/3)) + 1/3))/6 + 2**(1/3)*I*(1 + sqrt(3)*I)**(2/3)*atan(sqrt(3)
*(2*2**(1/3)*x/(3*(1 + sqrt(3)*I)**(1/3)) + 1/3))/6 - 1/x
```

Mathematica [C] time = 0.0203759, size = 47, normalized size = 0.11

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^2\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^3)/(x^2*(1 - x^3 + x^6)),x]
```

```
[Out] -x^(-1) - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 + 2*
#1^3) & ]/3
```

Maple [C] time = 0.01, size = 46, normalized size = 0.1

$$-\frac{1}{3}\sum_{\_R = RootOf\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^4 \ln\left(x-\_R\right)}{2\,\_R^5-\_R^2}-x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^3+1)/x^2/(x^6-x^3+1),x)
```

```
[Out] -1/3*sum(_R^4/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))-1/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{x}-\int \frac{x^4}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)/((x^6 - x^3 + 1)*x^2),x, algorithm="maxima")
```

```
[Out] -1/x - integrate(x^4/(x^6 - x^3 + 1), x)
```

Fricas [A] time = 0.282379, size = 1463, normalized size = 3.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)/((x^6 - x^3 + 1)*x^2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(2*x*cos(2/3*arctan(1/(sqrt(3) + 2)))*log(cos(2/3*ar
ctan(1/(sqrt(3) + 2)))^4 + sin(2/3*arctan(1/(sqrt(3) + 2)))^4 + 2
*sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3) + 2)))*sin(2/3*arctan(1/(sqr
```

```
t(3) + 2))) + x*cos(2/3*arctan(1/(sqrt(3) + 2)))^2 + (2*cos(2/3*a
rctan(1/(sqrt(3) + 2)))^2 - x)*sin(2/3*arctan(1/(sqrt(3) + 2)))^2
 + x^2) - 8*x*arctan((2*sqrt(3)*cos(2/3*arctan(1/(sqrt(3) + 2)))*
sin(2/3*arctan(1/(sqrt(3) + 2))) - 3*cos(2/3*arctan(1/(sqrt(3) +
2)))^2 + 3*sin(2/3*arctan(1/(sqrt(3) + 2)))^2)/(sqrt(3)*cos(2/3*a
rctan(1/(sqrt(3) + 2)))^2 - sqrt(3)*sin(2/3*arctan(1/(sqrt(3) + 2
)))^2 + 6*cos(2/3*arctan(1/(sqrt(3) + 2)))*sin(2/3*arctan(1/(sqrt
(3) + 2))) + 2*sqrt(3)*x + 2*sqrt(3)*sqrt(cos(2/3*arctan(1/(sqrt(
3) + 2)))^4 + sin(2/3*arctan(1/(sqrt(3) + 2)))^4 + 2*sqrt(3)*x*co
s(2/3*arctan(1/(sqrt(3) + 2)))*sin(2/3*arctan(1/(sqrt(3) + 2))) +
 x*cos(2/3*arctan(1/(sqrt(3) + 2)))^2 + (2*cos(2/3*arctan(1/(sqrt
(3) + 2)))^2 - x)*sin(2/3*arctan(1/(sqrt(3) + 2)))^2 + x^2)))*sin
(2/3*arctan(1/(sqrt(3) + 2))) + 4*(sqrt(3)*x*cos(2/3*arctan(1/(sq
rt(3) + 2))) - x*sin(2/3*arctan(1/(sqrt(3) + 2))))*arctan(-2*cos(
2/3*arctan(1/(sqrt(3) + 2)))*sin(2/3*arctan(1/(sqrt(3) + 2)))/(co
s(2/3*arctan(1/(sqrt(3) + 2)))^2 - sin(2/3*arctan(1/(sqrt(3) + 2)
))^2 - x - sqrt(cos(2/3*arctan(1/(sqrt(3) + 2)))^4 + sin(2/3*arct
an(1/(sqrt(3) + 2)))^4 - 2*x*cos(2/3*arctan(1/(sqrt(3) + 2)))^2 +
 2*(cos(2/3*arctan(1/(sqrt(3) + 2)))^2 + x)*sin(2/3*arctan(1/(sqr
t(3) + 2)))^2 + x^2))) - 4*(sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3) +
 2))) + x*sin(2/3*arctan(1/(sqrt(3) + 2))))*arctan(-(2*sqrt(3)*co
s(2/3*arctan(1/(sqrt(3) + 2)))*sin(2/3*arctan(1/(sqrt(3) + 2))) +
 3*cos(2/3*arctan(1/(sqrt(3) + 2)))^2 - 3*sin(2/3*arctan(1/(sqrt(
3) + 2)))^2)/(sqrt(3)*cos(2/3*arctan(1/(sqrt(3) + 2)))^2 - sqrt(3
)*sin(2/3*arctan(1/(sqrt(3) + 2)))^2 - 6*cos(2/3*arctan(1/(sqrt(3
) + 2)))*sin(2/3*arctan(1/(sqrt(3) + 2))) + 2*sqrt(3)*x + 2*sqrt(
3)*sqrt(cos(2/3*arctan(1/(sqrt(3) + 2)))^4 + sin(2/3*arctan(1/(sq
rt(3) + 2)))^4 - 2*sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3) + 2)))*sin
(2/3*arctan(1/(sqrt(3) + 2))) + x*cos(2/3*arctan(1/(sqrt(3) + 2))
)^2 + (2*cos(2/3*arctan(1/(sqrt(3) + 2)))^2 - x)*sin(2/3*arctan(1
/(sqrt(3) + 2)))^2 + x^2))) + (sqrt(3)*x*sin(2/3*arctan(1/(sqrt(3
) + 2))) - x*cos(2/3*arctan(1/(sqrt(3) + 2))))*log(cos(2/3*arctan
(1/(sqrt(3) + 2)))^4 + sin(2/3*arctan(1/(sqrt(3) + 2)))^4 - 2*sqr
t(3)*x*cos(2/3*arctan(1/(sqrt(3) + 2)))*sin(2/3*arctan(1/(sqrt(3)
 + 2))) + x*cos(2/3*arctan(1/(sqrt(3) + 2)))^2 + (2*cos(2/3*arcta
n(1/(sqrt(3) + 2)))^2 - x)*sin(2/3*arctan(1/(sqrt(3) + 2)))^2 + x
^2) - (sqrt(3)*x*sin(2/3*arctan(1/(sqrt(3) + 2))) + x*cos(2/3*arc
tan(1/(sqrt(3) + 2))))*log(cos(2/3*arctan(1/(sqrt(3) + 2)))^4 + s
in(2/3*arctan(1/(sqrt(3) + 2)))^4 - 2*x*cos(2/3*arctan(1/(sqrt(3)
 + 2)))^2 + 2*(cos(2/3*arctan(1/(sqrt(3) + 2)))^2 + x)*sin(2/3*ar
ctan(1/(sqrt(3) + 2)))^2 + x^2) - 6*sqrt(3))/x
```

Sympy [A] time = 0.548836, size = 31, normalized size = 0.07

$$-\operatorname{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(6561t^5 + 54t^2 + x\right)\right)\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**3+1)/x**2/(x**6-x**3+1),x)
```

```
[Out] -RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**
5 + 54*_t**2 + x))) - 1/x
```

GIAC/XCAS [A] time = 0.294574, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)/((x^6 - x^3 + 1)*x^2),x, algorithm="giac")
```

```
[Out] Done
```

3.31 $\int \frac{1-x^3}{x^3(1-x^3+x^6)}\,dx$

Optimal. Leaf size=418

$$-\frac{1}{2x^2}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

```
[Out] -1/(2*x^2) + ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)
^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((I - Sqrt[
3])*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(
1/3)*(1 + I*Sqrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3]
)^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((3 - I
*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 +
 I*Sqrt[3])^(2/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) +
 (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*S
qrt[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*
(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[
3])^(2/3))
```

Rubi [A] time = 0.854118, antiderivative size = 418, normalized size of antiderivative = 1., number

of steps used = 15, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$

$$-\frac{1}{2x^2}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(-\sqrt{3}+i\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^3)/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(2*x^2) + ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)
^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((I - Sqrt[
3])*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(
1/3)*(1 + I*Sqrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3]
)^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((3 - I
*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 +
 I*Sqrt[3])^(2/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) +
 (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*S
qrt[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*
(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[
3])^(2/3))
```

Rubi in Sympy [A] time = 129.123, size = 342, normalized size = 0.82

$$-\frac{2^{\frac{2}{3}}\sqrt{3}i\sqrt[3]{1-\sqrt{3}i}\log\left(\sqrt[3]{2}x-\sqrt[3]{1-\sqrt{3}i}\right)}{18}+\frac{2^{\frac{2}{3}}\sqrt{3}i\sqrt[3]{1+\sqrt{3}i}\log\left(\sqrt[3]{2}x-\sqrt[3]{1+\sqrt{3}i}\right)}{18}+\frac{2^{\frac{2}{3}}\sqrt{3}i\sqrt[3]{1-\sqrt{3}i}\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1-\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1-\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{36}-\frac{2^{\frac{2}{3}}\sqrt{3}i\sqrt[3]{1+\sqrt{3}i}\log\left(x^2+\frac{2^{\frac{2}{3}}x\sqrt[3]{1+\sqrt{3}i}}{2}+\frac{\sqrt[3]{2}\left(1+\sqrt{3}i\right)^{\frac{2}{3}}}{2}\right)}{36}+\frac{2^{\frac{2}{3}}i\sqrt[3]{1-\sqrt{3}i}\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1-\sqrt{3}i}}+\frac{1}{3}\right)\right)}{6}-\frac{2^{\frac{2}{3}}i\sqrt[3]{1+\sqrt{3}i}\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[3]{2}x}{3\sqrt[3]{1+\sqrt{3}i}}+\frac{1}{3}\right)\right)}{6}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**3+1)/x**3/(x**6-x**3+1),x)
```

```
[Out] -2**(2/3)*sqrt(3)*I*(1 - sqrt(3)*I)**(1/3)*log(2**(1/3)*x - (1 -
sqrt(3)*I)**(1/3))/18 + 2**(2/3)*sqrt(3)*I*(1 + sqrt(3)*I)**(1/3)
*log(2**(1/3)*x - (1 + sqrt(3)*I)**(1/3))/18 + 2**(2/3)*sqrt(3)*I
*(1 - sqrt(3)*I)**(1/3)*log(x**2 + 2**(2/3)*x*(1 - sqrt(3)*I)**(1
/3)/2 + 2**(1/3)*(1 - sqrt(3)*I)**(2/3)/2)/36 - 2**(2/3)*sqrt(3)*
I*(1 + sqrt(3)*I)**(1/3)*log(x**2 + 2**(2/3)*x*(1 + sqrt(3)*I)**(
1/3)/2 + 2**(1/3)*(1 + sqrt(3)*I)**(2/3)/2)/36 + 2**(2/3)*I*(1 -
sqrt(3)*I)**(1/3)*atan(sqrt(3)*(2*2**(1/3)*x/(3*(1 - sqrt(3)*I)**
(1/3)) + 1/3))/6 - 2**(2/3)*I*(1 + sqrt(3)*I)**(1/3)*atan(sqrt(3)
*(2*2**(1/3)*x/(3*(1 + sqrt(3)*I)**(1/3)) + 1/3))/6 - 1/(2*x**2)
```

Mathematica [C] time = 0.0204511, size = 47, normalized size = 0.11

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^3)/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(2*x^2) - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2
*#1^3) & ]/3
```

Maple [C] time = 0.009, size = 46, normalized size = 0.1

$$-\frac{1}{3}\sum_{\_R=RootOf\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\,\_R^5-\_R^2}-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^3+1)/x^3/(x^6-x^3+1),x)
```

```
[Out] -1/3*sum(_R^3/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))-1/2/
x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2\,x^2}-\int\frac{x^3}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)/((x^6 - x^3 + 1)*x^3),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate(x^3/(x^6 - x^3 + 1), x)
```

Fricas [A] time = 0.279619, size = 930, normalized size = 2.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)/((x^6 - x^3 + 1)*x^3),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(2*x^2*cos(2/3*arctan(1/(sqrt(3) + 2)))*log(x^2 + co
s(2/3*arctan(1/(sqrt(3) + 2)))^2 + 2*x*sin(2/3*arctan(1/(sqrt(3)
+ 2))) + sin(2/3*arctan(1/(sqrt(3) + 2)))^2) + 8*x^2*arctan(cos(2
/3*arctan(1/(sqrt(3) + 2)))/(x + sqrt(x^2 + cos(2/3*arctan(1/(sqr
t(3) + 2)))^2 + 2*x*sin(2/3*arctan(1/(sqrt(3) + 2))) + sin(2/3*ar
ctan(1/(sqrt(3) + 2)))^2) + sin(2/3*arctan(1/(sqrt(3) + 2)))))*si
n(2/3*arctan(1/(sqrt(3) + 2))) - 4*(sqrt(3)*x^2*cos(2/3*arctan(1/
(sqrt(3) + 2))) - x^2*sin(2/3*arctan(1/(sqrt(3) + 2))))*arctan((s
qrt(3)*sin(2/3*arctan(1/(sqrt(3) + 2))) + cos(2/3*arctan(1/(sqrt(
```

```
3) + 2))))/(sqrt(3)*cos(2/3*arctan(1/(sqrt(3) + 2))) + 2*x + 2*sq
rt(sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3) + 2))) + x^2 + cos(2/3*arc
tan(1/(sqrt(3) + 2)))^2 - x*sin(2/3*arctan(1/(sqrt(3) + 2))) + si
n(2/3*arctan(1/(sqrt(3) + 2)))^2) - sin(2/3*arctan(1/(sqrt(3) + 2
))))) - 4*(sqrt(3)*x^2*cos(2/3*arctan(1/(sqrt(3) + 2))) + x^2*sin
(2/3*arctan(1/(sqrt(3) + 2))))*arctan(-(sqrt(3)*sin(2/3*arctan(1/
(sqrt(3) + 2))) - cos(2/3*arctan(1/(sqrt(3) + 2))))/(sqrt(3)*cos(
2/3*arctan(1/(sqrt(3) + 2))) - 2*x - 2*sqrt(-sqrt(3)*x*cos(2/3*ar
ctan(1/(sqrt(3) + 2))) + x^2 + cos(2/3*arctan(1/(sqrt(3) + 2)))^2
 - x*sin(2/3*arctan(1/(sqrt(3) + 2))) + sin(2/3*arctan(1/(sqrt(3)
 + 2)))^2) + sin(2/3*arctan(1/(sqrt(3) + 2))))) - (sqrt(3)*x^2*si
n(2/3*arctan(1/(sqrt(3) + 2))) + x^2*cos(2/3*arctan(1/(sqrt(3) +
2))))*log(sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3) + 2))) + x^2 + cos(
2/3*arctan(1/(sqrt(3) + 2)))^2 - x*sin(2/3*arctan(1/(sqrt(3) + 2)
)) + sin(2/3*arctan(1/(sqrt(3) + 2)))^2) + (sqrt(3)*x^2*sin(2/3*a
rctan(1/(sqrt(3) + 2))) - x^2*cos(2/3*arctan(1/(sqrt(3) + 2))))*l
og(-sqrt(3)*x*cos(2/3*arctan(1/(sqrt(3) + 2))) + x^2 + cos(2/3*ar
ctan(1/(sqrt(3) + 2)))^2 - x*sin(2/3*arctan(1/(sqrt(3) + 2))) + s
in(2/3*arctan(1/(sqrt(3) + 2)))^2) - 3*sqrt(3))/x^2
```

Sympy [A] time = 0.562082, size = 32, normalized size = 0.08

$$-\operatorname{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(-1458t^4 - 9t + x\right)\right)\right) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**3+1)/x**3/(x**6-x**3+1),x)
```

```
[Out] -RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t*
*4 - 9*_t + x))) - 1/(2*x**2)
```

GIAC/XCAS [A] time = 0.297666, size = 867, normalized size = 2.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)/((x^6 - x^3 + 1)*x^3),x, algorithm="giac")
```

```
[Out] 1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^2*sin(4/9*p
i)^2 + 2*sqrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4/9*pi) - 8*
cos(4/9*pi)*sin(4/9*pi)^3 + sqrt(3)*cos(4/9*pi) + sin(4/9*pi))*ar
ctan(-((sqrt(3)*i + 1)*cos(4/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/
9*pi))) + 1/9*(2*sqrt(3)*cos(2/9*pi)^4 - 12*sqrt(3)*cos(2/9*pi)^2
```

```
*sin(2/9*pi)^2 + 2*sqrt(3)*sin(2/9*pi)^4 + 8*cos(2/9*pi)^3*sin(2/
9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)^3 + sqrt(3)*cos(2/9*pi) + sin(2
/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i +
 1)*sin(2/9*pi))) + 1/9*(2*sqrt(3)*cos(1/9*pi)^4 - 12*sqrt(3)*cos
(1/9*pi)^2*sin(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4 - 8*cos(1/9*pi
)^3*sin(1/9*pi) + 8*cos(1/9*pi)*sin(1/9*pi)^3 - sqrt(3)*cos(1/9*p
i) + sin(1/9*pi))*arctan(((sqrt(3)*i + 1)*cos(1/9*pi) + 2*x)/((sq
rt(3)*i + 1)*sin(1/9*pi))) + 1/18*(8*sqrt(3)*cos(4/9*pi)^3*sin(4/
9*pi) - 8*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - 2*cos(4/9*pi)^4 + 1
2*cos(4/9*pi)^2*sin(4/9*pi)^2 - 2*sin(4/9*pi)^4 + sqrt(3)*sin(4/9
*pi) - cos(4/9*pi))*ln(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x +
 x^2 + 1) + 1/18*(8*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 8*sqrt(3)
*cos(2/9*pi)*sin(2/9*pi)^3 - 2*cos(2/9*pi)^4 + 12*cos(2/9*pi)^2*s
in(2/9*pi)^2 - 2*sin(2/9*pi)^4 + sqrt(3)*sin(2/9*pi) - cos(2/9*pi
))*ln(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x + x^2 + 1) - 1/18*
(8*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 8*sqrt(3)*cos(1/9*pi)*sin(
1/9*pi)^3 + 2*cos(1/9*pi)^4 - 12*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*
sin(1/9*pi)^4 - sqrt(3)*sin(1/9*pi) - cos(1/9*pi))*ln((sqrt(3)*i*
cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) - 1/2/x^2
```

3.32 $\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=36

$$\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

```
[Out] ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3] + Log[1 - x^3 + x^6]/6
```

Rubi [A] time = 0.0909392, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(-2 + x^3))/(1 - x^3 + x^6),x]
```

```
[Out] ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3] + Log[1 - x^3 + x^6]/6
```

Rubi in Sympy [A] time = 11.4389, size = 34, normalized size = 0.94

$$\frac{\log\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^3}{3}-\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(x**3-2)/(x**6-x**3+1),x)
```

```
[Out] log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(sqrt(3)*(2*x**3/3 - 1/3))/3
```

Mathematica [A] time = 0.0139765, size = 37, normalized size = 1.03

$$\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(-2 + x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -(ArcTan[(-1 + 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Maple [A] time = 0.003, size = 33, normalized size = 0.9

$$\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}-\frac{\sqrt{3}}{3}\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(x^3-2)/(x^6-x^3+1),x)
```

```
[Out] 1/6*ln(x^6-x^3+1)-1/3*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A] time = 0.827459, size = 43, normalized size = 1.19

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{3}-1\right)\right)+\frac{1}{6}\log\left(x^{6}-x^{3}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 2)*x^2/(x^6 - x^3 + 1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3
+ 1)
```

Fricas [A] time = 0.258586, size = 49, normalized size = 1.36

$$\frac{1}{18}\sqrt{3}\left(\sqrt{3}\log\left(x^{6}-x^{3}+1\right)-6\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{3}-1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 2)*x^2/(x^6 - x^3 + 1),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(sqrt(3)*log(x^6 - x^3 + 1) - 6*arctan(1/3*sqrt(3)*(
2*x^3 - 1)))
```

Sympy [A] time = 0.284947, size = 37, normalized size = 1.03

$$\frac{\log\left(x^6 - x^3 + 1\right)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(x**3-2)/(x**6-x**3+1),x)
```

```
[Out] log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/
3)/3
```

GIAC/XCAS [A] time = 0.279371, size = 43, normalized size = 1.19

$$-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^3 - 1\right)\right) + \frac{1}{6}\ln\left(x^6 - x^3 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - 2)*x^2/(x^6 - x^3 + 1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*ln(x^6 - x^3 +
 1)
```

3.33 $\int \frac{1+x^3}{x(1-x^3+x^6)}\,dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

```
[Out] -(ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 - x^3 + x
^6]/6
```

Rubi [A] time = 0.116528, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^3)/(x*(1 - x^3 + x^6)),x]
```

```
[Out] -(ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 - x^3 + x
^6]/6
```

Rubi in Sympy [A] time = 16.6819, size = 41, normalized size = 1.05

$$\frac{\log\left(x^3\right)}{3}-\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^3}{3}-\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+1)/x/(x**6-x**3+1),x)
```

```
[Out] log(x**3)/3 - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(sqrt(3)*(2*x*
*3/3 - 1/3))/3
```

Mathematica [C] time = 0.0210754, size = 55, normalized size = 1.41

$$\log(x)-\frac{1}{3}\mathrm{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-2\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^3)/(x*(1 - x^3 + x^6)),x]
```

```
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (-2*Log[x - #1] + Log[x - #1
]*#1^3)/(-1 + 2*#1^3) & ]/3
```

Maple [A] time = 0.008, size = 35, normalized size = 0.9

$$\ln(x) - \frac{\ln(x^6 - x^3 + 1)}{6} + \frac{\sqrt{3}}{3}\arctan\left(\frac{(2x^3 - 1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+1)/x/(x^6-x^3+1),x)
```

```
[Out] ln(x)-1/6*ln(x^6-x^3+1)+1/3*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A] time = 0.820072, size = 51, normalized size = 1.31

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3 - 1)\right) - \frac{1}{6}\log(x^6 - x^3 + 1) + \frac{1}{3}\log(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)/((x^6 - x^3 + 1)*x),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 +
 1) + 1/3*log(x^3)
```

Fricas [A] time = 0.275647, size = 58, normalized size = 1.49

$$-\frac{1}{18}\sqrt{3}\left(\sqrt{3}\log(x^6 - x^3 + 1) - 6\sqrt{3}\log(x) - 6\arctan\left(\frac{1}{3}\sqrt{3}(2x^3 - 1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)/((x^6 - x^3 + 1)*x),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*log(x^6 - x^3 + 1) - 6*sqrt(3)*log(x) - 6*
arctan(1/3*sqrt(3)*(2*x^3 - 1)))
```

Sympy [A] time = 0.328135, size = 41, normalized size = 1.05

$$\log(x) - \frac{\log(x^6 - x^3 + 1)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+1)/x/(x**6-x**3+1),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 -
 sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.271527, size = 47, normalized size = 1.21

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3 - 1)\right) - \frac{1}{6}\ln(x^6 - x^3 + 1) + \ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)/((x^6 - x^3 + 1)*x),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*ln(x^6 - x^3 +
1) + ln(abs(x))
```

3.34 $\int \frac{1+x^3}{x-x^4+x^7} \, dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

```
[Out] -(ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 - x^3 + x
^6]/6
```

Rubi [A] time = 0.11416, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^3)/(x - x^4 + x^7),x]
```

```
[Out] -(ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 - x^3 + x
^6]/6
```

Rubi in Sympy [A] time = 18.0126, size = 41, normalized size = 1.05

$$\frac{\log\left(x^3\right)}{3}-\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x^3}{3}-\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+1)/(x**7-x**4+x),x)
```

```
[Out] log(x**3)/3 - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(sqrt(3)*(2*x*
*3/3 - 1/3))/3
```

Mathematica [C] time = 0.0161054, size = 55, normalized size = 1.41

$$\log(x)-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-2\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^3)/(x - x^4 + x^7),x]
```

```
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (-2*Log[x - #1] + Log[x - #1
]*#1^3)/(-1 + 2*#1^3) & ]/3
```

Maple [A] time = 0.006, size = 35, normalized size = 0.9

$$\ln(x) - \frac{\ln(x^6 - x^3 + 1)}{6} + \frac{\sqrt{3}}{3}\arctan\left(\frac{(2x^3 - 1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+1)/(x^7-x^4+x),x)
```

```
[Out] ln(x)-1/6*ln(x^6-x^3+1)+1/3*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^5 - 2x^2}{x^6 - x^3 + 1}\,dx + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)/(x^7 - x^4 + x),x, algorithm="maxima")
```

```
[Out] -integrate((x^5 - 2*x^2)/(x^6 - x^3 + 1), x) + log(x)
```

Fricas [A] time = 0.252582, size = 58, normalized size = 1.49

$$-\frac{1}{18}\sqrt{3}\left(\sqrt{3}\log(x^6 - x^3 + 1) - 6\sqrt{3}\log(x) - 6\arctan\left(\frac{1}{3}\sqrt{3}(2x^3 - 1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)/(x^7 - x^4 + x),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*log(x^6 - x^3 + 1) - 6*sqrt(3)*log(x) - 6*
arctan(1/3*sqrt(3)*(2*x^3 - 1)))
```

Sympy [A] time = 0.32551, size = 41, normalized size = 1.05

$$\log(x) - \frac{\log(x^6 - x^3 + 1)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+1)/(x**7-x**4+x),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 -
 sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.273239, size = 47, normalized size = 1.21

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3 - 1)\right) - \frac{1}{6}\ln(x^6 - x^3 + 1) + \ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)/(x^7 - x^4 + x),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*ln(x^6 - x^3 +
1) + ln(abs(x))
```

3.35 $\int \left(d+ex^3\right)^{5/2} \left(a+bx^3+cx^6\right) \, dx$

Optimal. Leaf size=396

$$\frac{2x\left(d+ex^3\right)^{5/2}\left(667ae^2-58bde+16cd^2\right)}{11339e^2}+\frac{30dx\left(d+ex^3\right)^{3/2}\left(667ae^2-58bde+16cd^2\right)}{124729e^2}$$
$$+\frac{54d^2x\sqrt{d+ex^3}\left(667ae^2-58bde+16cd^2\right)}{124729e^2}$$
$$+\frac{54\, 3^{3/4}\sqrt{2+\sqrt{3}}d^3\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(667ae^2-58bde+16cd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)|-7-4\sqrt{3}\right)}{124729e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$
$$-\frac{2x\left(d+ex^3\right)^{7/2}(8cd-29be)}{667e^2}+\frac{2cx^4\left(d+ex^3\right)^{7/2}}{29e}$$

```
[Out] (54*d^2*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*Sqrt[d + e*x^3])/(124
729*e^2) + (30*d*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*(d + e*x^3)^
(3/2))/(124729*e^2) + (2*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*(d +
 e*x^3)^(5/2))/(11339*e^2) - (2*(8*c*d - 29*b*e)*x*(d + e*x^3)^(7
/2))/(667*e^2) + (2*c*x^4*(d + e*x^3)^(7/2))/(29*e) + (54*3^(3/4)
*Sqrt[2 + Sqrt[3]]*d^3*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*(d^(1/3)
 + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((
1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[
3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7
 - 4*Sqrt[3]])/(124729*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x
))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rubi [A] time = 0.774189, antiderivative size = 396, normalized size of antiderivative = 1., number

of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{2x(d+ex^3)^{5/2}(667ae^2-58bde+16cd^2)}{11339e^2}+\frac{30dx(d+ex^3)^{3/2}(667ae^2-58bde+16cd^2)}{124729e^2}+\frac{54d^2x\sqrt{d+ex^3}(667ae^2-58bde+16cd^2)}{124729e^2}+\frac{54\,3^{3/4}\sqrt{2+\sqrt{3}}d^3\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}(667ae^2-58bde+16cd^2)F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)|-7-4\sqrt{3}\right)}{124729e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}-\frac{2x(d+ex^3)^{7/2}(8cd-29be)}{667e^2}+\frac{2cx^4(d+ex^3)^{7/2}}{29e}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6),x]
```

```
[Out] (54*d^2*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*Sqrt[d + e*x^3])/(124
729*e^2) + (30*d*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*(d + e*x^3)^
(3/2))/(124729*e^2) + (2*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*(d +
 e*x^3)^(5/2))/(11339*e^2) - (2*(8*c*d - 29*b*e)*x*(d + e*x^3)^(7
/2))/(667*e^2) + (2*c*x^4*(d + e*x^3)^(7/2))/(29*e) + (54*3^(3/4)
*Sqrt[2 + Sqrt[3]]*d^3*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*(d^(1/3)
 + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((
1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[
3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7
 - 4*Sqrt[3]])/(124729*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x
))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rubi in Sympy [A] time = 46.9048, size = 379, normalized size = 0.96

$$\frac{2cx^4\left(d+ex^3\right)^{\frac{7}{2}}}{29e} + \frac{54\cdot 3^{\frac{3}{4}}d^3\sqrt{\frac{d^{\frac{2}{3}}-\sqrt[3]{d}\sqrt[3]{e}x+e^{\frac{2}{3}}x^2}{\left(\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(667ae^2-58bde+16cd^2\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{d}\left(-1+\sqrt{3}\right)+\sqrt[3]{e}x}{\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x}\right)\middle|-7-4\sqrt{3}\right)}{124729e^{\frac{7}{3}}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}} + \frac{54d^2x\sqrt{d+ex^3}\left(667ae^2-58bde+16cd^2\right)}{124729e^2} + \frac{30dx\left(d+ex^3\right)^{\frac{3}{2}}\left(667ae^2-58bde+16cd^2\right)}{124729e^2} + \frac{2x\left(d+ex^3\right)^{\frac{7}{2}}(29be-8cd)}{667e^2} + \frac{2x\left(d+ex^3\right)^{\frac{5}{2}}\left(667ae^2-58bde+16cd^2\right)}{11339e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**3+d)**(5/2)*(c*x**6+b*x**3+a),x)
```

```
[Out] 2*c*x**4*(d + e*x**3)**(7/2)/(29*e) + 54*3**(3/4)*d**3*sqrt((d**(
2/3) - d**(1/3)*e**(1/3)*x + e**(2/3)*x**2)/(d**(1/3)*(1 + sqrt(3
)) + e**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(d**(1/3) + e**(1/3)*x)*(6
67*a*e**2 - 58*b*d*e + 16*c*d**2)*elliptic_f(asin((-d**(1/3)*(-1
+ sqrt(3)) + e**(1/3)*x)/(d**(1/3)*(1 + sqrt(3)) + e**(1/3)*x)),
-7 - 4*sqrt(3))/(124729*e**(7/3)*sqrt(d**(1/3)*(d**(1/3) + e**(1/
3)*x)/(d**(1/3)*(1 + sqrt(3)) + e**(1/3)*x)**2)*sqrt(d + e*x**3))
 + 54*d**2*x*sqrt(d + e*x**3)*(667*a*e**2 - 58*b*d*e + 16*c*d**2)
/(124729*e**2) + 30*d*x*(d + e*x**3)**(3/2)*(667*a*e**2 - 58*b*d*
e + 16*c*d**2)/(124729*e**2) + 2*x*(d + e*x**3)**(7/2)*(29*b*e -
8*c*d)/(667*e**2) + 2*x*(d + e*x**3)**(5/2)*(667*a*e**2 - 58*b*d*
e + 16*c*d**2)/(11339*e**2)
```

Mathematica [C] time = 0.581605, size = 279, normalized size = 0.7

$$-\frac{2\left(\sqrt[3]{-e}\left(d+ex^3\right)\left(-11e^2x^7\left(29e(23ae+49bd)+781cd^2\right)-dex^4\left(29e(851ae+487bd)+405cd^2\right)+d^2x\left(648cd^2-29e(121\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6),x]
```

```
[Out] (-2*((-e)^(1/3)*(d + e*x^3)*(d^2*(648*c*d^2 - 29*e*(81*b*d + 1219
*a*e))*x - d*e*(405*c*d^2 + 29*e*(487*b*d + 851*a*e))*x^4 - 11*e^
2*(781*c*d^2 + 29*e*(49*b*d + 23*a*e))*x^7 - 187*e^3*(61*c*d + 29
*b*e)*x^10 - 4301*c*e^4*x^13) - (27*I)*3^(3/4)*d^(10/3)*(16*c*d^2
 + 29*e*(-2*b*d + 23*a*e))*Sqrt[(-1)^(5/6)*(-1 + ((-e)^(1/3)*x)/d
^(1/3))]*Sqrt[1 + ((-e)^(1/3)*x)/d^(1/3) + ((-e)^(2/3)*x^2)/d^(2/
3)]*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-e)^(1/3)*x)/d^(1/3)]
/3^(1/4)], (-1)^(1/3)]))/(124729*(-e)^(7/3)*Sqrt[d + e*x^3])
```

Maple [B] time = 0.203, size = 1070, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x)
```

```
[Out] a*(2/17*e^2*x^7*(e*x^3+d)^(1/2)+74/187*d*e*x^4*(e*x^3+d)^(1/2)+10
6/187*d^2*x*(e*x^3+d)^(1/2)-54/187*I*d^3*3^(1/2)/e*(-e^2*d)^(1/3)
*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/
2)*e/(-e^2*d)^(1/3))^(1/2)*((x-1/e*(-e^2*d)^(1/3))/(-3/2/e*(-e^2*
d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-e^
2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/
3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-e^2
*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3
))^(1/2),(I*3^(1/2)/e*(-e^2*d)^(1/3)/(-3/2/e*(-e^2*d)^(1/3)+1/2*I
*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)))+b*(2/23*e^2*x^10*(e*x^3+d)^(1
/2)+98/391*d*e*x^7*(e*x^3+d)^(1/2)+974/4301*d^2*x^4*(e*x^3+d)^(1/
2)+162/4301*d^3/e*x*(e*x^3+d)^(1/2)+108/4301*I*d^4/e^2*3^(1/2)*(-
e^2*d)^(1/3)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^
(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)*((x-1/e*(-e^2*d)^(1/3))/(-
3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)*(-I*(
x+1/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/
(-e^2*d)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(
-e^2*d)^(1/3))^(1/2),(I*3^(1/2)/e*(-e^2*d)^(1/3)/(-3/2/e*(-e^2*d)
^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)))+c*(2/29*e^2*x^13*
(e*x^3+d)^(1/2)+122/667*d*e*x^10*(e*x^3+d)^(1/2)+1562/11339*d^2*x
^7*(e*x^3+d)^(1/2)+810/124729*d^3/e*x^4*(e*x^3+d)^(1/2)-1296/1247
29*d^4/e^2*x*(e*x^3+d)^(1/2)-864/124729*I*d^5/e^3*3^(1/2)*(-e^2*d
)^(1/3)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)
)*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)*((x-1/e*(-e^2*d)^(1/3))/(-3/2/e
*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)*(-I*(x+1/2
/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2
*d)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*
d)^(1/3))^(1/2),(I*3^(1/2)/e*(-e^2*d)^(1/3)/(-3/2/e*(-e^2*d)^(1/3
)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)\left(ex^3 + d\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ce^2x^{12} + \left(2\,cde + be^2\right)x^9 + \left(cd^2 + 2\,bde + ae^2\right)x^6 + \left(bd^2 + 2\,ade\right)x^3 + ad^2\right)\sqrt{ex^3 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((c*e^2*x^12 + (2*c*d*e + b*e^2)*x^9 + (c*d^2 + 2*b*d*e +
 a*e^2)*x^6 + (b*d^2 + 2*a*d*e)*x^3 + a*d^2)*sqrt(e*x^3 + d), x)
```

Sympy [A] time = 25.6834, size = 400, normalized size = 1.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**3+d)**(5/2)*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**(5/2)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), e*x**3*exp_pol
ar(I*pi)/d)/(3*gamma(4/3)) + 2*a*d**(3/2)*e*x**4*gamma(4/3)*hyper
((-1/2, 4/3), (7/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) +
a*sqrt(d)*e**2*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e*x**3
*exp_polar(I*pi)/d)/(3*gamma(10/3)) + b*d**(5/2)*x**4*gamma(4/3)*
hyper((-1/2, 4/3), (7/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3
)) + 2*b*d**(3/2)*e*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e
*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3)) + b*sqrt(d)*e**2*x**10*g
amma(10/3)*hyper((-1/2, 10/3), (13/3,), e*x**3*exp_polar(I*pi)/d)
/(3*gamma(13/3)) + c*d**(5/2)*x**7*gamma(7/3)*hyper((-1/2, 7/3),
```

```
(10/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3)) + 2*c*d**(3/2)
*e*x**10*gamma(10/3)*hyper((-1/2, 10/3), (13/3,), e*x**3*exp_pola
r(I*pi)/d)/(3*gamma(13/3)) + c*sqrt(d)*e**2*x**13*gamma(13/3)*hyp
er((-1/2, 13/3), (16/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(16/3
))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)\left(ex^3 + d\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(5/2), x)
```

3.36 $\int \left(d+ex^3\right)^{3/2} \left(a+bx^3+cx^6\right) \, dx$

Optimal. Leaf size=356

$$\frac{2x\left(d+ex^3\right)^{3/2}\left(391ae^2-46bde+16cd^2\right)}{4301e^2}+\frac{18dx\sqrt{d+ex^3}\left(391ae^2-46bde+16cd^2\right)}{21505e^2}+\frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}d^2\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(391ae^2-46bde+16cd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)|-7-4\sqrt{3}\right)}{21505e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}-\frac{2x\left(d+ex^3\right)^{5/2}(8cd-23be)}{391e^2}+\frac{2cx^4\left(d+ex^3\right)^{5/2}}{23e}$$

```
[Out] (18*d*(16*c*d^2 - 46*b*d*e + 391*a*e^2)*x*Sqrt[d + e*x^3])/(21505
*e^2) + (2*(16*c*d^2 - 46*b*d*e + 391*a*e^2)*x*(d + e*x^3)^(3/2))
/(4301*e^2) - (2*(8*c*d - 23*b*e)*x*(d + e*x^3)^(5/2))/(391*e^2)
+ (2*c*x^4*(d + e*x^3)^(5/2))/(23*e) + (18*3^(3/4)*Sqrt[2 + Sqrt[
3]]*d^2*(16*c*d^2 - 46*b*d*e + 391*a*e^2)*(d^(1/3) + e^(1/3)*x)*S
qrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^
(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e
^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/
(21505*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3]
)*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rubi [A] time = 0.601849, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{2x\left(d+ex^3\right)^{3/2}\left(391ae^2-46bde+16cd^2\right)}{4301e^2}+\frac{18dx\sqrt{d+ex^3}\left(391ae^2-46bde+16cd^2\right)}{21505e^2}+\frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}d^2\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(391ae^2-46bde+16cd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)|-7-4\sqrt{3}\right)}{21505e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}-\frac{2x\left(d+ex^3\right)^{5/2}(8cd-23be)}{391e^2}+\frac{2cx^4\left(d+ex^3\right)^{5/2}}{23e}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6),x]
```

```
[Out] (18*d*(16*c*d^2 - 46*b*d*e + 391*a*e^2)*x*Sqrt[d + e*x^3])/(21505
*e^2) + (2*(16*c*d^2 - 46*b*d*e + 391*a*e^2)*x*(d + e*x^3)^(3/2))
/(4301*e^2) - (2*(8*c*d - 23*b*e)*x*(d + e*x^3)^(5/2))/(391*e^2)
+ (2*c*x^4*(d + e*x^3)^(5/2))/(23*e) + (18*3^(3/4)*Sqrt[2 + Sqrt[
3]]*d^2*(16*c*d^2 - 46*b*d*e + 391*a*e^2)*(d^(1/3) + e^(1/3)*x)*S
qrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^
(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e
^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/
(21505*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3]
)*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rubi in Sympy [A] time = 39.8086, size = 337, normalized size = 0.95

$$\frac{2cx^4\left(d+ex^3\right)^{\frac{5}{2}}}{23e}+\frac{18\cdot 3^{\frac{3}{4}}d^2\sqrt{\frac{d^{\frac{2}{3}}-\sqrt[3]{d}\sqrt[3]{e}x+e^{\frac{2}{3}}x^2}{\left(\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(391ae^2-46bde+16cd^2\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{d}\left(-1+\sqrt{3}\right)+\sqrt[3]{e}x}{\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x}\right)\middle|-7-4\sqrt{3}\right)}{21505e^{\frac{7}{3}}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}+\frac{18dx\sqrt{d+ex^3}\left(391ae^2-46bde+16cd^2\right)}{21505e^2}+\frac{2x\left(d+ex^3\right)^{\frac{5}{2}}\left(23be-8cd\right)}{391e^2}+\frac{2x\left(d+ex^3\right)^{\frac{3}{2}}\left(391ae^2-46bde+16cd^2\right)}{4301e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**3+d)**(3/2)*(c*x**6+b*x**3+a),x)
```

```
[Out] 2*c*x**4*(d + e*x**3)**(5/2)/(23*e) + 18*3**(3/4)*d**2*sqrt((d**(
2/3) - d**(1/3)*e**(1/3)*x + e**(2/3)*x**2)/(d**(1/3)*(1 + sqrt(3
)) + e**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(d**(1/3) + e**(1/3)*x)*(3
91*a*e**2 - 46*b*d*e + 16*c*d**2)*elliptic_f(asin((-d**(1/3)*(-1
+ sqrt(3)) + e**(1/3)*x)/(d**(1/3)*(1 + sqrt(3)) + e**(1/3)*x)),
-7 - 4*sqrt(3))/(21505*e**(7/3)*sqrt(d**(1/3)*(d**(1/3) + e**(1/3
)*x)/(d**(1/3)*(1 + sqrt(3)) + e**(1/3)*x)**2)*sqrt(d + e*x**3))
+ 18*d*x*sqrt(d + e*x**3)*(391*a*e**2 - 46*b*d*e + 16*c*d**2)/(21
505*e**2) + 2*x*(d + e*x**3)**(5/2)*(23*b*e - 8*c*d)/(391*e**2) +
 2*x*(d + e*x**3)**(3/2)*(391*a*e**2 - 46*b*d*e + 16*c*d**2)/(430
1*e**2)
```

Mathematica [C] time = 0.523604, size = 249, normalized size = 0.7

$$-\frac{2\left(\sqrt[3]{-e}\left(d+ex^3\right)\left(-5ex^4\left(23e(17ae+20bd)+27cd^2\right)+dx\left(216cd^2-23e(238ae+27bd)\right)-55e^2x^7(23be+26cd)-935ce^3\right.\right.}{21505(-}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6),x]
```

```
[Out] (-2*((-e)^(1/3)*(d + e*x^3)*(d*(216*c*d^2 - 23*e*(27*b*d + 238*a*
e))*x - 5*e*(27*c*d^2 + 23*e*(20*b*d + 17*a*e))*x^4 - 55*e^2*(26*
c*d + 23*b*e)*x^7 - 935*c*e^3*x^10) - (9*I)*3^(3/4)*d^(7/3)*(16*c
*d^2 + 23*e*(-2*b*d + 17*a*e))*Sqrt[(-1)^(5/6)*(-1 + ((-e)^(1/3)*
x)/d^(1/3))]*Sqrt[1 + ((-e)^(1/3)*x)/d^(1/3) + ((-e)^(2/3)*x^2)/d
^(2/3)]*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-e)^(1/3)*x)/d^(1
/3)]/3^(1/4)], (-1)^(1/3)]))/(21505*(-e)^(7/3)*Sqrt[d + e*x^3])
```

Maple [B] time = 0.045, size = 1010, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x)
```

```
[Out] a*(2/11*e*x^4*(e*x^3+d)^(1/2)+28/55*d*x*(e*x^3+d)^(1/2)-18/55*I*d
^2*3^(1/2)/e*(-e^2*d)^(1/3)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1
/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)*((x-1/e*(-e
^2*d)^(1/3))/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3
)))^(1/2)*(-I*(x+1/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1
/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/
3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2),(I*3^(1/2)/e*(-e^2*d)^(1/3)/(
-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)))+b*
(2/17*e*x^7*(e*x^3+d)^(1/2)+40/187*d*x^4*(e*x^3+d)^(1/2)+54/935*d
^2/e*x*(e*x^3+d)^(1/2)+36/935*I*d^3/e^2*3^(1/2)*(-e^2*d)^(1/3)*(I
*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*
e/(-e^2*d)^(1/3))^(1/2)*((x-1/e*(-e^2*d)^(1/3))/(-3/2/e*(-e^2*d)^
(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-e^2*d
)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))
^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-e^2*d)
^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^
(1/2),(I*3^(1/2)/e*(-e^2*d)^(1/3)/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^
```

```
(1/2)/e*(-e^2*d)^(1/3)))^(1/2)))+c*(2/23*e*x^10*(e*x^3+d)^(1/2)+5
2/391*d*x^7*(e*x^3+d)^(1/2)+54/4301*d^2/e*x^4*(e*x^3+d)^(1/2)-432
/21505*d^3/e^2*x*(e*x^3+d)^(1/2)-288/21505*I*d^4/e^3*3^(1/2)*(-e^
2*d)^(1/3)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1
/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)*((x-1/e*(-e^2*d)^(1/3))/(-3/
2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)*(-I*(x+
1/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-
e^2*d)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e
^2*d)^(1/3))^(1/2),(I*3^(1/2)/e*(-e^2*d)^(1/3)/(-3/2/e*(-e^2*d)^(
1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6+bx^3+a\right)\left(ex^3+d\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cex^9+(cd+be)x^6+(bd+ae)x^3+ad\right)\sqrt{ex^3+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*e*x^9 + (c*d + b*e)*x^6 + (b*d + a*e)*x^3 + a*d)*sqrt
(e*x^3 + d), x)
```

Sympy [A] time = 13.5335, size = 257, normalized size = 0.72

$$\frac{ad^{\frac{3}{2}}x\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\,\Gamma\left(\frac{4}{3}\right)}+\frac{a\sqrt{d}ex^4\,\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\,\Gamma\left(\frac{7}{3}\right)}+\frac{bd^{\frac{3}{2}}x^4\,\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\,\Gamma\left(\frac{7}{3}\right)}+\frac{b\sqrt{d}ex^7\,\Gamma\left(\frac{7}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\,\Gamma\left(\frac{10}{3}\right)}+\frac{cd^{\frac{3}{2}}x^7\,\Gamma\left(\frac{7}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\,\Gamma\left(\frac{10}{3}\right)}+\frac{c\sqrt{d}ex^{10}\,\Gamma\left(\frac{10}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{10}{3}\\ \frac{13}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\,\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**3+d)**(3/2)*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**(3/2)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), e*x**3*exp_pol
ar(I*pi)/d)/(3*gamma(4/3)) + a*sqrt(d)*e*x**4*gamma(4/3)*hyper((-
1/2, 4/3), (7/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + b*d
**(3/2)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), e*x**3*exp_pol
ar(I*pi)/d)/(3*gamma(7/3)) + b*sqrt(d)*e*x**7*gamma(7/3)*hyper((-
1/2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3)) + c
*d**(3/2)*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e*x**3*exp_
polar(I*pi)/d)/(3*gamma(10/3)) + c*sqrt(d)*e*x**10*gamma(10/3)*hy
per((-1/2, 10/3), (13/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(13/
3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6+bx^3+a\right)\left(ex^3+d\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(3/2), x)
```

3.37 $\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)\,dx$

Optimal. Leaf size=316

$$\frac{2x\sqrt{d+ex^3}\left(187ae^2-34bde+16cd^2\right)}{935e^2}+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}d\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(187ae^2-34bde+16cd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)|-7-4\sqrt{3}\right)}{935e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}-\frac{2x\left(d+ex^3\right)^{3/2}(8cd-17be)}{187e^2}+\frac{2cx^4\left(d+ex^3\right)^{3/2}}{17e}$$

```
[Out] (2*(16*c*d^2 - 34*b*d*e + 187*a*e^2)*x*Sqrt[d + e*x^3])/(935*e^2)
 - (2*(8*c*d - 17*b*e)*x*(d + e*x^3)^(3/2))/(187*e^2) + (2*c*x^4*
(d + e*x^3)^(3/2))/(17*e) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*d*(16*c*
d^2 - 34*b*d*e + 187*a*e^2)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) -
 d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1
+ Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(935*e^(7/3)*S
qrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1
/3)*x)^2]*Sqrt[d + e*x^3])
```

Rubi [A] time = 0.504449, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{2x\sqrt{d+ex^3}\left(187ae^2-34bde+16cd^2\right)}{935e^2}+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}d\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(187ae^2-34bde+16cd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)|-7-4\sqrt{3}\right)}{935e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}-\frac{2x\left(d+ex^3\right)^{3/2}(8cd-17be)}{187e^2}+\frac{2cx^4\left(d+ex^3\right)^{3/2}}{17e}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d + e*x^3]*(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*(16*c*d^2 - 34*b*d*e + 187*a*e^2)*x*Sqrt[d + e*x^3])/(935*e^2)
 - (2*(8*c*d - 17*b*e)*x*(d + e*x^3)^(3/2))/(187*e^2) + (2*c*x^4*
(d + e*x^3)^(3/2))/(17*e) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*d*(16*c*
d^2 - 34*b*d*e + 187*a*e^2)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) -
 d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1
+ Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(935*e^(7/3)*S
qrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1
/3)*x)^2]*Sqrt[d + e*x^3])
```

Rubi in Sympy [A] time = 33.4453, size = 294, normalized size = 0.93

$$\frac{2cx^4\left(d+ex^3\right)^{\frac{3}{2}}}{17e} + \frac{2\cdot 3^{\frac{3}{4}}d\sqrt{\frac{d^{\frac{2}{3}}-\sqrt[3]{d}\sqrt[3]{e}x+e^{\frac{2}{3}}x^2}{\left(\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(187ae^2-34bde+16cd^2\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{d}\left(-1+\sqrt{3}\right)+\sqrt[3]{e}x}{\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x}\right)\middle|-7-4\sqrt{3}\right)}{935e^{\frac{7}{3}}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}} + \frac{2x\left(d+ex^3\right)^{\frac{3}{2}}\left(17be-8cd\right)}{187e^2} + \frac{2x\sqrt{d+ex^3}\left(187ae^2-34bde+16cd^2\right)}{935e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**3+d)**(1/2)*(c*x**6+b*x**3+a),x)
```

```
[Out] 2*c*x**4*(d + e*x**3)**(3/2)/(17*e) + 2*3**(3/4)*d*sqrt((d**(2/3)
 - d**(1/3)*e**(1/3)*x + e**(2/3)*x**2)/(d**(1/3)*(1 + sqrt(3)) +
 e**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(d**(1/3) + e**(1/3)*x)*(187*a
*e**2 - 34*b*d*e + 16*c*d**2)*elliptic_f(asin((-d**(1/3)*(-1 + sq
rt(3)) + e**(1/3)*x)/(d**(1/3)*(1 + sqrt(3)) + e**(1/3)*x)), -7 -
 4*sqrt(3))/(935*e**(7/3)*sqrt(d**(1/3)*(d**(1/3) + e**(1/3)*x)/(
d**(1/3)*(1 + sqrt(3)) + e**(1/3)*x)**2)*sqrt(d + e*x**3)) + 2*x*
(d + e*x**3)**(3/2)*(17*b*e - 8*c*d)/(187*e**2) + 2*x*sqrt(d + e*
x**3)*(187*a*e**2 - 34*b*d*e + 16*c*d**2)/(935*e**2)
```

Mathematica [C] time = 0.47553, size = 219, normalized size = 0.69

$$-\frac{2\left(\sqrt[3]{-e}x\left(d+ex^3\right)\left(c\left(24d^2-15dex^3-55e^2x^6\right)-17e\left(11ae+3bd+5bex^3\right)\right)-i3^{3/4}d^{4/3}\sqrt{(-1)^{5/6}\left(\frac{\sqrt[3]{-e}x}{\sqrt[3]{d}}-1\right)}\sqrt{\frac{(-e)^{2/3}x}{d^{2/3}}}\right.}{935(-e)^{7/3}\sqrt{d+ex^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[d + e*x^3]*(a + b*x^3 + c*x^6),x]
```

```
[Out] (-2*((-e)^(1/3)*x*(d + e*x^3)*(-17*e*(3*b*d + 11*a*e + 5*b*e*x^3)
 + c*(24*d^2 - 15*d*e*x^3 - 55*e^2*x^6)) - I*3^(3/4)*d^(4/3)*(16*
c*d^2 + 17*e*(-2*b*d + 11*a*e))*Sqrt[(-1)^(5/6)*(-1 + ((-e)^(1/3)
*x)/d^(1/3))]*Sqrt[1 + ((-e)^(1/3)*x)/d^(1/3) + ((-e)^(2/3)*x^2)/
d^(2/3)]*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-e)^(1/3)*x)/d^(
1/3)]/3^(1/4)], (-1)^(1/3)]))/(935*(-e)^(7/3)*Sqrt[d + e*x^3])
```

Maple [B] time = 0.044, size = 956, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x)
```

```
[Out] a*(2/5*x*(e*x^3+d)^(1/2)-2/5*I*d*3^(1/2)/e*(-e^2*d)^(1/3)*(I*(x+1
/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e
^2*d)^(1/3))^(1/2)*((x-1/e*(-e^2*d)^(1/3))/(-3/2/e*(-e^2*d)^(1/3)
+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-e^2*d)^(1/
3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2
)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-e^2*d)^(1/3
)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)
,(I*3^(1/2)/e*(-e^2*d)^(1/3)/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)
/e*(-e^2*d)^(1/3)))^(1/2)))+b*(2/11*x^4*(e*x^3+d)^(1/2)+6/55*d/e*
x*(e*x^3+d)^(1/2)+4/55*I*d^2/e^2*3^(1/2)*(-e^2*d)^(1/3)*(I*(x+1/2
/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2
*d)^(1/3))^(1/2)*((x-1/e*(-e^2*d)^(1/3))/(-3/2/e*(-e^2*d)^(1/3)+1
/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-e^2*d)^(1/3)
+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)/
(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-e^2*d)^(1/3)-
1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2),(
I*3^(1/2)/e*(-e^2*d)^(1/3)/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e
*(-e^2*d)^(1/3)))^(1/2)))+c*(2/17*x^7*(e*x^3+d)^(1/2)+6/187*d/e*x
^4*(e*x^3+d)^(1/2)-48/935*d^2/e^2*x*(e*x^3+d)^(1/2)-32/935*I*d^3/
e^3*3^(1/2)*(-e^2*d)^(1/3)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/
2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)*((x-1/e*(-e^
2*d)^(1/3))/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)
))^(1/2)*(-I*(x+1/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/
3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3
))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2),(I*3^(1/2)/e*(-e^2*d)^(1/3)/(-
3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^6 + bx^3 + a)\sqrt{ex^3 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((cx^6 + bx^3 + a)\sqrt{ex^3 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d), x)
```

Sympy [A] time = 6.816, size = 124, normalized size = 0.39

$$\frac{a\sqrt{d}x\,\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{b\sqrt{d}x^4\,\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{c\sqrt{d}x^7\,\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**3+d)**(1/2)*(c*x**6+b*x**3+a),x)
```

```
[Out] a*sqrt(d)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), e*x**3*exp_pola
r(I*pi)/d)/(3*gamma(4/3)) + b*sqrt(d)*x**4*gamma(4/3)*hyper((-1/2
, 4/3), (7/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + c*sqrt
(d)*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e*x**3*exp_polar(
I*pi)/d)/(3*gamma(10/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^6 + bx^3 + a)\sqrt{ex^3 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d), x)
```

3.38 $\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx$

Optimal. Leaf size=278

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(55ae^2-22bde+16cd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)|-7-4\sqrt{3}\right)}{55\sqrt[4]{3}e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$
$$-\frac{2x\sqrt{d+ex^3}(8cd-11be)}{55e^2}+\frac{2cx^4\sqrt{d+ex^3}}{11e}$$

```
[Out] (-2*(8*c*d - 11*b*e)*x*Sqrt[d + e*x^3])/(55*e^2) + (2*c*x^4*Sqrt[
d + e*x^3])/(11*e) + (2*Sqrt[2 + Sqrt[3]]*(16*c*d^2 - 22*b*d*e +
55*a*e^2)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x
 + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3)
 + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(55*3^(1/4)*e^(7/3)*Sqrt[(d^(1/3
)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*S
qrt[d + e*x^3])
```

Rubi [A] time = 0.370475, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(55ae^2-22bde+16cd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)|-7-4\sqrt{3}\right)}{55\sqrt[4]{3}e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$
$$-\frac{2x\sqrt{d+ex^3}(8cd-11be)}{55e^2}+\frac{2cx^4\sqrt{d+ex^3}}{11e}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)/Sqrt[d + e*x^3],x]
```


```
[Out] (-2*(8*c*d - 11*b*e)*x*Sqrt[d + e*x^3])/(55*e^2) + (2*c*x^4*Sqrt[
d + e*x^3])/(11*e) + (2*Sqrt[2 + Sqrt[3]]*(16*c*d^2 - 22*b*d*e +
55*a*e^2)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x
 + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3)
```

```
 + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(55*3^(1/4)*e^(7/3)*Sqrt[(d^(1/3
)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*S
qrt[d + e*x^3])
```

Rubi in Sympy [A] time = 27.4319, size = 253, normalized size = 0.91

$$\frac{2cx^4\sqrt{d+ex^3}}{11e}+\frac{2x\sqrt{d+ex^3}\,(11be-8cd)}{55e^2}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{d^{\frac{2}{3}}-\sqrt[3]{d}\sqrt[3]{e}x+e^{\frac{2}{3}}x^2}{\left(\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(55ae^2-22bde+16cd^2\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{d}\left(-1+\sqrt{3}\right)+\sqrt[3]{e}x}{\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x}\right)\middle|-7-4\sqrt{3}\right)}{165e^{\frac{7}{3}}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(1/2),x)
```

```
[Out] 2*c*x**4*sqrt(d + e*x**3)/(11*e) + 2*x*sqrt(d + e*x**3)*(11*b*e -
 8*c*d)/(55*e**2) + 2*3**(3/4)*sqrt((d**(2/3) - d**(1/3)*e**(1/3)
*x + e**(2/3)*x**2)/(d**(1/3)*(1 + sqrt(3)) + e**(1/3)*x)**2)*sqr
t(sqrt(3) + 2)*(d**(1/3) + e**(1/3)*x)*(55*a*e**2 - 22*b*d*e + 16
*c*d**2)*elliptic_f(asin((-d**(1/3)*(-1 + sqrt(3)) + e**(1/3)*x)/
(d**(1/3)*(1 + sqrt(3)) + e**(1/3)*x)), -7 - 4*sqrt(3))/(165*e**(
7/3)*sqrt(d**(1/3)*(d**(1/3) + e**(1/3)*x)/(d**(1/3)*(1 + sqrt(3)
) + e**(1/3)*x)**2)*sqrt(d + e*x**3))
```

Mathematica [C] time = 0.606227, size = 194, normalized size = 0.7

$$\frac{2\sqrt{d+ex^3}\left(11bex-8cdx+5cex^4\right)}{55e^2}+\frac{2i\sqrt[3]{d}\sqrt{(-1)^{5/6}\left(\frac{\sqrt[3]{-e}x}{\sqrt[3]{d}}-1\right)}\sqrt{\frac{(-e)^{2/3}x^2}{d^{2/3}}+\frac{\sqrt[3]{-e}x}{\sqrt[3]{d}}+1}\left(11e(5ae-2bd)+16cd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-e}x}{\sqrt[3]{d}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\middle|\sqrt[3]{-1}\right)}{55\sqrt[4]{3}(-e)^{7/3}\sqrt{d+ex^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3 + c*x^6)/Sqrt[d + e*x^3],x]
```

```
[Out] (2*Sqrt[d + e*x^3]*(-8*c*d*x + 11*b*e*x + 5*c*e*x^4))/(55*e^2) +
(((2*I)/55)*d^(1/3)*(16*c*d^2 + 11*e*(-2*b*d + 5*a*e))*Sqrt[(-1)^
(5/6)*(-1 + ((-e)^(1/3)*x)/d^(1/3))]*Sqrt[1 + ((-e)^(1/3)*x)/d^(1
/3) + ((-e)^(2/3)*x^2)/d^(2/3)]*EllipticF[ArcSin[Sqrt[-(-1)^(5/6)
 - (I*(-e)^(1/3)*x)/d^(1/3)]/3^(1/4)], (-1)^(1/3)])/(3^(1/4)*(-e)
^(7/3)*Sqrt[d + e*x^3])
```

Maple [B] time = 0.044, size = 907, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x)
```

```
[Out] -2/3*I*a*3^(1/2)/e*(-e^2*d)^(1/3)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*
I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)*((x-1
/e*(-e^2*d)^(1/3))/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d
)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2
*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*
d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2),(I*3^(1/2)/e*(-e^2*d)^(
1/3)/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2
))+b*(2/5/e*x*(e*x^3+d)^(1/2)+4/15*I*d/e^2*3^(1/2)*(-e^2*d)^(1/3)
*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/
2)*e/(-e^2*d)^(1/3))^(1/2)*((x-1/e*(-e^2*d)^(1/3))/(-3/2/e*(-e^2*
d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-e^
2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/
3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-e^2
*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3
))^(1/2),(I*3^(1/2)/e*(-e^2*d)^(1/3)/(-3/2/e*(-e^2*d)^(1/3)+1/2*I
*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)))+c*(2/11/e*x^4*(e*x^3+d)^(1/2)
-16/55*d/e^2*x*(e*x^3+d)^(1/2)-32/165*I*d^2/e^3*3^(1/2)*(-e^2*d)^
(1/3)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*
3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)*((x-1/e*(-e^2*d)^(1/3))/(-3/2/e*(
-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)*(-I*(x+1/2/e
*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d
)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*
(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)
^(1/3))^(1/2),(I*3^(1/2)/e*(-e^2*d)^(1/3)/(-3/2/e*(-e^2*d)^(1/3)+
1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^6 + bx^3 + a}{\sqrt{ex^3 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)/sqrt(e*x^3 + d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/sqrt(e*x^3 + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{cx^6 + bx^3 + a}{\sqrt{ex^3 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)/sqrt(e*x^3 + d),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)/sqrt(e*x^3 + d), x)
```

Sympy [A] time = 6.30519, size = 119, normalized size = 0.43

$$\frac{ax\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\,\Gamma\left(\frac{4}{3}\right)} + \frac{bx^4\,\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\,\Gamma\left(\frac{7}{3}\right)} + \frac{cx^7\,\Gamma\left(\frac{7}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\,\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(1/2),x)
```

```
[Out] a*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), e*x**3*exp_polar(I*pi)/d
)/(3*sqrt(d)*gamma(4/3)) + b*x**4*gamma(4/3)*hyper((1/2, 4/3), (7
/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqrt(d)*gamma(7/3)) + c*x**7*g
amma(7/3)*hyper((1/2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3
*sqrt(d)*gamma(10/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^6 + bx^3 + a}{\sqrt{ex^3 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)/sqrt(e*x^3 + d),x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)/sqrt(e*x^3 + d), x)
```

3.39 $\int \frac{a+bx^3+cx^6}{(d+ex^3)^{3/2}}\,dx$

Optimal. Leaf size=289

$$\frac{2x\left(ae^2-bde+cd^2\right)}{3de^2\sqrt{d+ex^3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(16cd^2-5e(ae+2bd)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)\middle|-7-4\sqrt{3}\right)}{15\sqrt[4]{3}de^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}+\frac{2cx\sqrt{d+ex^3}}{5e^2}$$

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(3*d*e^2*Sqrt[d + e*x^3]) + (2*c*x*
Sqrt[d + e*x^3])/(5*e^2) - (2*Sqrt[2 + Sqrt[3]]*(16*c*d^2 - 5*e*(
2*b*d + a*e))*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/
3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(
1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(15*3^(1/4)*d*e^(7/3)*Sqrt[(
d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x
)^2]*Sqrt[d + e*x^3])
```

Rubi [A] time = 0.402462, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{2x\left(ae^2-bde+cd^2\right)}{3de^2\sqrt{d+ex^3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(16cd^2-5e(ae+2bd)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)\middle|-7-4\sqrt{3}\right)}{15\sqrt[4]{3}de^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}+\frac{2cx\sqrt{d+ex^3}}{5e^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2),x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(3*d*e^2*Sqrt[d + e*x^3]) + (2*c*x*
Sqrt[d + e*x^3])/(5*e^2) - (2*Sqrt[2 + Sqrt[3]]*(16*c*d^2 - 5*e*(
2*b*d + a*e))*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/
3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(
1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(15*3^(1/4)*d*e^(7/3)*Sqrt[(
d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x
)^2]*Sqrt[d + e*x^3])
```

Rubi in Sympy [A] time = 42.235, size = 262, normalized size = 0.91

$$\frac{2cx\sqrt{d+ex^3}}{5e^2}+\frac{2x\left(ae^2-bde+cd^2\right)}{3de^2\sqrt{d+ex^3}}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{d^{\frac{2}{3}}-\sqrt[3]{d}\sqrt[3]{e}x+e^{\frac{2}{3}}x^2}{\left(\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(5ae^2+10bde-16cd^2\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{d}\left(-1+\sqrt{3}\right)+\sqrt[3]{e}x}{\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x}\right)\middle|-7-4\sqrt{3}\right)}{45de^{\frac{7}{3}}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(3/2),x)
```

```
[Out] 2*c*x*sqrt(d + e*x**3)/(5*e**2) + 2*x*(a*e**2 - b*d*e + c*d**2)/(
3*d*e**2*sqrt(d + e*x**3)) + 2*3**(3/4)*sqrt((d**(2/3) - d**(1/3)
*e**(1/3)*x + e**(2/3)*x**2)/(d**(1/3)*(1 + sqrt(3)) + e**(1/3)*x
)**2)*sqrt(sqrt(3) + 2)*(d**(1/3) + e**(1/3)*x)*(5*a*e**2 + 10*b*
d*e - 16*c*d**2)*elliptic_f(asin((-d**(1/3)*(-1 + sqrt(3)) + e**(
1/3)*x)/(d**(1/3)*(1 + sqrt(3)) + e**(1/3)*x)), -7 - 4*sqrt(3))/(
45*d*e**(7/3)*sqrt(d**(1/3)*(d**(1/3) + e**(1/3)*x)/(d**(1/3)*(1
+ sqrt(3)) + e**(1/3)*x)**2)*sqrt(d + e*x**3))
```

Mathematica [C] time = 0.455728, size = 197, normalized size = 0.68

$$\frac{2\left(3\sqrt[3]{-e}x\left(5e(ae-bd)+cd\left(8d+3ex^3\right)\right)-i3^{3/4}\sqrt[3]{d}\sqrt{(-1)^{5/6}\left(\frac{\sqrt[3]{-e}x}{\sqrt[3]{d}}-1\right)}\sqrt{\frac{(-e)^{2/3}x^2}{d^{2/3}}+\frac{\sqrt[3]{-e}x}{\sqrt[3]{d}}+1}\left(16cd^2-5e(ae+2bd)\right)F\right.}{45d(-e)^{7/3}\sqrt{d+ex^3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2),x]
```

```
[Out] (2*(3*(-e)^(1/3)*x*(5*e*(-(b*d) + a*e) + c*d*(8*d + 3*e*x^3)) - I
*3^(3/4)*d^(1/3)*(16*c*d^2 - 5*e*(2*b*d + a*e))*Sqrt[(-1)^(5/6)*(
-1 + ((-e)^(1/3)*x)/d^(1/3))]*Sqrt[1 + ((-e)^(1/3)*x)/d^(1/3) + (
(-e)^(2/3)*x^2)/d^(2/3)]*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(
-e)^(1/3)*x)/d^(1/3)]/3^(1/4)], (-1)^(1/3)]))/(45*d*(-e)^(7/3)*Sq
rt[d + e*x^3])
```

Maple [B] time = 0.06, size = 934, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2),x)
```

```
[Out] a*(2/3/d*x/((x^3+d/e)*e)^(1/2)-2/9*I/d*3^(1/2)/e*(-e^2*d)^(1/3)*(
I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)
*e/(-e^2*d)^(1/3))^(1/2)*((x-1/e*(-e^2*d)^(1/3))/(-3/2/e*(-e^2*d)
^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-e^2*
d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3)
)^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-e^2*d
)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))
^(1/2),(I*3^(1/2)/e*(-e^2*d)^(1/3)/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3
^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)))+b*(-2/3/e*x/((x^3+d/e)*e)^(1/2)
-4/9*I/e^2*3^(1/2)*(-e^2*d)^(1/3)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*
I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)*((x-1
/e*(-e^2*d)^(1/3))/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d
)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2
*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*
d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2),(I*3^(1/2)/e*(-e^2*d)^(
1/3)/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2
)))+c*(2/3/e^2*d*x/((x^3+d/e)*e)^(1/2)+2/5/e^2*x*(e*x^3+d)^(1/2)+
32/45*I/e^3*d*3^(1/2)*(-e^2*d)^(1/3)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1
/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)*((
x-1/e*(-e^2*d)^(1/3))/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^
2*d)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-
e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e
^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2),(I*3^(1/2)/e*(-e^2*d
)^(1/3)/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(
1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^6 + bx^3 + a}{(ex^3 + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{cx^6 + bx^3 + a}{(ex^3 + d)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(3/2), x)
```

Sympy [A] time = 59.4504, size = 119, normalized size = 0.41

$$\frac{ax\,\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3}, \frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{3}{2}}\,\Gamma\left(\frac{4}{3}\right)} + \frac{bx^4\,\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{4}{3}, \frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{3}{2}}\,\Gamma\left(\frac{7}{3}\right)} + \frac{cx^7\,\Gamma\left(\frac{7}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2}, \frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{3}{2}}\,\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(3/2),x)
```

```
[Out] a*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), e*x**3*exp_polar(I*pi)/d
)/(3*d**(3/2)*gamma(4/3)) + b*x**4*gamma(4/3)*hyper((4/3, 3/2), (
7/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(3/2)*gamma(7/3)) + c*x**7
*gamma(7/3)*hyper((3/2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/
(3*d**(3/2)*gamma(10/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^6 + bx^3 + a}{(ex^3 + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(3/2),x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(3/2), x)
```

3.40 $\int \frac{a+bx^3+cx^6}{(d+ex^3)^{5/2}}\,dx$

Optimal. Leaf size=309

$$-\frac{2x\left(-7ae^2-2bde+11cd^2\right)}{27d^2e^2\sqrt{d+ex^3}}+\frac{2x\left(ae^2-bde+cd^2\right)}{9de^2\left(d+ex^3\right)^{3/2}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(e(7ae+2bd)+16cd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}d^2e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(9*d*e^2*(d + e*x^3)^(3/2)) - (2*(1
1*c*d^2 - 2*b*d*e - 7*a*e^2)*x)/(27*d^2*e^2*Sqrt[d + e*x^3]) + (2
*Sqrt[2 + Sqrt[3]]*(16*c*d^2 + e*(2*b*d + 7*a*e))*(d^(1/3) + e^(1
/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqr
t[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(
1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sq
rt[3]])/(27*3^(1/4)*d^2*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*
x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rubi [A] time = 0.438426, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{2x\left(-7ae^2-2bde+11cd^2\right)}{27d^2e^2\sqrt{d+ex^3}}+\frac{2x\left(ae^2-bde+cd^2\right)}{9de^2\left(d+ex^3\right)^{3/2}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(e(7ae+2bd)+16cd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}d^2e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(5/2),x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(9*d*e^2*(d + e*x^3)^(3/2)) - (2*(1
1*c*d^2 - 2*b*d*e - 7*a*e^2)*x)/(27*d^2*e^2*Sqrt[d + e*x^3]) + (2
*Sqrt[2 + Sqrt[3]]*(16*c*d^2 + e*(2*b*d + 7*a*e))*(d^(1/3) + e^(1
/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqr
t[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(
```

```
1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sq
rt[3]])/(27*3^(1/4)*d^2*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*
x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rubi in Sympy [A] time = 43.7627, size = 286, normalized size = 0.93

$$\frac{2x\left(ae^2-bde+cd^2\right)}{9de^2\left(d+ex^3\right)^{\frac{3}{2}}}+\frac{2x\left(7ae^2+2bde-11cd^2\right)}{27d^2e^2\sqrt{d+ex^3}}+\frac{4\cdot 3^{\frac{3}{4}}\sqrt{\frac{d^{\frac{2}{3}}-\sqrt[3]{d}\sqrt[3]{e}x+e^{\frac{2}{3}}x^2}{\left(\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(\frac{7ae^2}{2}+bde+8cd^2\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{d}\left(-1+\sqrt{3}\right)+\sqrt[3]{e}x}{\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x}\right)\middle|-7-4\sqrt{3}\right)}{81d^2e^{\frac{7}{3}}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(5/2),x)
```

```
[Out] 2*x*(a*e**2 - b*d*e + c*d**2)/(9*d*e**2*(d + e*x**3)**(3/2)) + 2*
x*(7*a*e**2 + 2*b*d*e - 11*c*d**2)/(27*d**2*e**2*sqrt(d + e*x**3)
) + 4*3**(3/4)*sqrt((d**(2/3) - d**(1/3)*e**(1/3)*x + e**(2/3)*x*
*2)/(d**(1/3)*(1 + sqrt(3)) + e**(1/3)*x)**2)*sqrt(sqrt(3) + 2)*(
d**(1/3) + e**(1/3)*x)*(7*a*e**2/2 + b*d*e + 8*c*d**2)*elliptic_f
(asin((-d**(1/3)*(-1 + sqrt(3)) + e**(1/3)*x)/(d**(1/3)*(1 + sqrt
(3)) + e**(1/3)*x)), -7 - 4*sqrt(3))/(81*d**2*e**(7/3)*sqrt(d**(1
/3)*(d**(1/3) + e**(1/3)*x)/(d**(1/3)*(1 + sqrt(3)) + e**(1/3)*x)
**2)*sqrt(d + e*x**3))
```

Mathematica [C] time = 0.673866, size = 224, normalized size = 0.72

$$\frac{2\left(3\sqrt[3]{-e}x\left(e\left(ae\left(10d+7ex^3\right)-bd\left(d-2ex^3\right)\right)-cd^2\left(8d+11ex^3\right)\right)+i3^{3/4}\sqrt[3]{d}\sqrt{(-1)^{5/6}\left(\frac{\sqrt[3]{-e}x}{\sqrt[3]{d}}-1\right)}\sqrt{\frac{(-e)^{2/3}x^2}{d^{2/3}}+\frac{\sqrt[3]{-e}x}{\sqrt[3]{d}}+\right.}{81d^2(-e)^{7/3}\left(d+ex^3\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(5/2),x]
```

```
[Out] (2*(3*(-e)^(1/3)*x*(-(c*d^2*(8*d + 11*e*x^3)) + e*(-(b*d*(d - 2*e
*x^3)) + a*e*(10*d + 7*e*x^3))) + I*3^(3/4)*d^(1/3)*(16*c*d^2 + e
```

```
*(2*b*d + 7*a*e))*Sqrt[(-1)^(5/6)*(-1 + ((-e)^(1/3)*x)/d^(1/3))]*
Sqrt[1 + ((-e)^(1/3)*x)/d^(1/3) + ((-e)^(2/3)*x^2)/d^(2/3)]*(d +
e*x^3)*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-e)^(1/3)*x)/d^(1/
3)]/3^(1/4)], (-1)^(1/3)]))/(81*d^2*(-e)^(7/3)*(d + e*x^3)^(3/2))
```

Maple [B] time = 0.07, size = 1005, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2),x)
```

```
[Out] a*(2/9/d*x/e^2*(e*x^3+d)^(1/2)/(x^3+d/e)^2+14/27/d^2*x/((x^3+d/e)
*e)^(1/2)-14/81*I/d^2*3^(1/2)/e*(-e^2*d)^(1/3)*(I*(x+1/2/e*(-e^2*
d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3)
)^(1/2)*((x-1/e*(-e^2*d)^(1/3))/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1
/2)/e*(-e^2*d)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-e^2*d)^(1/3)+1/2*I*3^
(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)/(e*x^3+d)
^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(
1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2),(I*3^(1/2)
/e*(-e^2*d)^(1/3)/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)
^(1/3)))^(1/2)))+b*(-2/9*x/e^3*(e*x^3+d)^(1/2)/(x^3+d/e)^2+4/27/e
/d*x/((x^3+d/e)*e)^(1/2)-4/81*I/d/e^2*3^(1/2)*(-e^2*d)^(1/3)*(I*(
x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/
(-e^2*d)^(1/3))^(1/2)*((x-1/e*(-e^2*d)^(1/3))/(-3/2/e*(-e^2*d)^(1
/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-e^2*d)^
(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(
1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-e^2*d)^(
1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1
/2),(I*3^(1/2)/e*(-e^2*d)^(1/3)/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1
/2)/e*(-e^2*d)^(1/3)))^(1/2)))+c*(2/9*d*x/e^4*(e*x^3+d)^(1/2)/(x^
3+d/e)^2-22/27/e^2*x/((x^3+d/e)*e)^(1/2)-32/81*I/e^3*3^(1/2)*(-e^
2*d)^(1/3)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1
/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)*((x-1/e*(-e^2*d)^(1/3))/(-3/
2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)*(-I*(x+
1/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-
e^2*d)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e
^2*d)^(1/3))^(1/2),(I*3^(1/2)/e*(-e^2*d)^(1/3)/(-3/2/e*(-e^2*d)^(
1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^6 + bx^3 + a}{(ex^3 + d)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{cx^6 + bx^3 + a}{(e^2x^6 + 2\,dex^3 + d^2)\sqrt{ex^3 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)/((e^2*x^6 + 2*d*e*x^3 + d^2)*sqrt(e*
x^3 + d)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^6 + bx^3 + a}{(ex^3 + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(5/2), x)
```

3.41 $\int \frac{a+bx^3+cx^6}{(d+ex^3)^{7/2}}\,dx$

Optimal. Leaf size=349

$$-\frac{2x\left(-13ae^2-2bde+17cd^2\right)}{135d^2e^2\left(d+ex^3\right)^{3/2}}+\frac{2x\left(ae^2-bde+cd^2\right)}{15de^2\left(d+ex^3\right)^{5/2}}+\frac{2x\left(91ae^2+14bde+16cd^2\right)}{405d^3e^2\sqrt{d+ex^3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(91ae^2+14bde+16cd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)|-7-4\sqrt{3}\right)}{405\sqrt[4]{3}d^3e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(15*d*e^2*(d + e*x^3)^(5/2)) - (2*(
17*c*d^2 - 2*b*d*e - 13*a*e^2)*x)/(135*d^2*e^2*(d + e*x^3)^(3/2))
 + (2*(16*c*d^2 + 14*b*d*e + 91*a*e^2)*x)/(405*d^3*e^2*Sqrt[d + e
*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(16*c*d^2 + 14*b*d*e + 91*a*e^2)*(d
^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x
^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x
)], -7 - 4*Sqrt[3]])/(405*3^(1/4)*d^3*e^(7/3)*Sqrt[(d^(1/3)*(d^(1
/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d +
 e*x^3])
```

Rubi [A] time = 0.641623, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{2x\left(-13ae^2-2bde+17cd^2\right)}{135d^2e^2\left(d+ex^3\right)^{3/2}}+\frac{2x\left(ae^2-bde+cd^2\right)}{15de^2\left(d+ex^3\right)^{5/2}}+\frac{2x\left(91ae^2+14bde+16cd^2\right)}{405d^3e^2\sqrt{d+ex^3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(91ae^2+14bde+16cd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)|-7-4\sqrt{3}\right)}{405\sqrt[4]{3}d^3e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2),x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(15*d*e^2*(d + e*x^3)^(5/2)) - (2*(
17*c*d^2 - 2*b*d*e - 13*a*e^2)*x)/(135*d^2*e^2*(d + e*x^3)^(3/2))
 + (2*(16*c*d^2 + 14*b*d*e + 91*a*e^2)*x)/(405*d^3*e^2*Sqrt[d + e
```

```
*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(16*c*d^2 + 14*b*d*e + 91*a*e^2)*(d
^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x
^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x
)], -7 - 4*Sqrt[3]])/(405*3^(1/4)*d^3*e^(7/3)*Sqrt[(d^(1/3)*(d^(1
/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d +
 e*x^3])
```

Rubi in Sympy [A] time = 50.5503, size = 328, normalized size = 0.94

$$\frac{2x\left(ae^2-bde+cd^2\right)}{15de^2\left(d+ex^3\right)^{\frac{5}{2}}}+\frac{2x\left(13ae^2+2bde-17cd^2\right)}{135d^2e^2\left(d+ex^3\right)^{\frac{3}{2}}}+\frac{2x\left(91ae^2+14bde+16cd^2\right)}{405d^3e^2\sqrt{d+ex^3}}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{d^{\frac{2}{3}}-\sqrt[3]{d}\sqrt[3]{e}x+e^{\frac{2}{3}}x^2}{\left(\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(91ae^2+14bde+16cd^2\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{d}\left(-1+\sqrt{3}\right)+\sqrt[3]{e}x}{\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x}\right)\middle|-7-4\sqrt{3}\right)}{1215d^3e^{\frac{7}{3}}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(7/2),x)
```

```
[Out] 2*x*(a*e**2 - b*d*e + c*d**2)/(15*d*e**2*(d + e*x**3)**(5/2)) + 2
*x*(13*a*e**2 + 2*b*d*e - 17*c*d**2)/(135*d**2*e**2*(d + e*x**3)*
*(3/2)) + 2*x*(91*a*e**2 + 14*b*d*e + 16*c*d**2)/(405*d**3*e**2*s
qrt(d + e*x**3)) + 2*3**(3/4)*sqrt((d**(2/3) - d**(1/3)*e**(1/3)*
x + e**(2/3)*x**2)/(d**(1/3)*(1 + sqrt(3)) + e**(1/3)*x)**2)*sqrt
(sqrt(3) + 2)*(d**(1/3) + e**(1/3)*x)*(91*a*e**2 + 14*b*d*e + 16*
c*d**2)*elliptic_f(asin((-d**(1/3)*(-1 + sqrt(3)) + e**(1/3)*x)/(
d**(1/3)*(1 + sqrt(3)) + e**(1/3)*x)), -7 - 4*sqrt(3))/(1215*d**3
*e**(7/3)*sqrt(d**(1/3)*(d**(1/3) + e**(1/3)*x)/(d**(1/3)*(1 + sq
rt(3)) + e**(1/3)*x)**2)*sqrt(d + e*x**3))
```

Mathematica [C] time = 0.837752, size = 262, normalized size = 0.75

$$\frac{2\left(3\sqrt[3]{-e}x\left(-3d\left(d+ex^3\right)\left(17cd^2-e(13ae+2bd)\right)+\left(d+ex^3\right)^2\left(7e(13ae+2bd)+16cd^2\right)+27d^2\left(e(ae-bd)+cd^2\right)\right)+i3^{3/\cdots}\right.}{1215d^3(-e)^{7/\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2),x]
```

```
[Out] (2*(3*(-e)^(1/3)*x*(27*d^2*(c*d^2 + e*(-(b*d) + a*e)) - 3*d*(17*c
*d^2 - e*(2*b*d + 13*a*e))*(d + e*x^3) + (16*c*d^2 + 7*e*(2*b*d +
 13*a*e))*(d + e*x^3)^2) + I*3^(3/4)*d^(1/3)*(16*c*d^2 + 7*e*(2*b
*d + 13*a*e))*Sqrt[(-1)^(5/6)*(-1 + ((-e)^(1/3)*x)/d^(1/3))]*Sqrt
[1 + ((-e)^(1/3)*x)/d^(1/3) + ((-e)^(2/3)*x^2)/d^(2/3)]*(d + e*x^
3)^2*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-e)^(1/3)*x)/d^(1/3)
]/3^(1/4)], (-1)^(1/3)]))/(1215*d^3*(-e)^(7/3)*(d + e*x^3)^(5/2))
```

Maple [B] time = 0.074, size = 1095, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x)
```

```
[Out] a*(2/15/d*x/e^3*(e*x^3+d)^(1/2)/(x^3+d/e)^3+26/135/d^2*x/e^2*(e*x
^3+d)^(1/2)/(x^3+d/e)^2+182/405/d^3*x/((x^3+d/e)*e)^(1/2)-182/121
5*I/d^3*3^(1/2)/e*(-e^2*d)^(1/3)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I
*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)*((x-1/
e*(-e^2*d)^(1/3))/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)
^(1/3)))^(1/2)*(-I*(x+1/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*
d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d
)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2),(I*3^(1/2)/e*(-e^2*d)^(1
/3)/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)
))+b*(-2/15*x/e^4*(e*x^3+d)^(1/2)/(x^3+d/e)^3+4/135/d*x/e^3*(e*x^
3+d)^(1/2)/(x^3+d/e)^2+28/405/e/d^2*x/((x^3+d/e)*e)^(1/2)-28/1215
*I/d^2/e^2*3^(1/2)*(-e^2*d)^(1/3)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*
I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)*((x-1
/e*(-e^2*d)^(1/3))/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d
)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2
*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*
d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2),(I*3^(1/2)/e*(-e^2*d)^(
1/3)/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2
)))+c*(2/15*d*x/e^5*(e*x^3+d)^(1/2)/(x^3+d/e)^3-34/135*x/e^4*(e*x
^3+d)^(1/2)/(x^3+d/e)^2+32/405/e^2/d*x/((x^3+d/e)*e)^(1/2)-32/121
5*I/d/e^3*3^(1/2)*(-e^2*d)^(1/3)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I
*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)*((x-1/
e*(-e^2*d)^(1/3))/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)
^(1/3)))^(1/2)*(-I*(x+1/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*
d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d
)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2),(I*3^(1/2)/e*(-e^2*d)^(1
/3)/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)
```

```
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^6 + bx^3 + a}{(ex^3 + d)^{\frac{7}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{cx^6 + bx^3 + a}{(e^3x^9 + 3\,de^2x^6 + 3\,d^2ex^3 + d^3)\sqrt{ex^3 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)/((e^3*x^9 + 3*d*e^2*x^6 + 3*d^2*e*x^
3 + d^3)*sqrt(e*x^3 + d)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(7/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^6 + bx^3 + a}{(ex^3 + d)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(7/2),x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(7/2), x)
```

3.42 $\int \frac{a+bx^3+cx^6}{(d+ex^3)^{9/2}}\,dx$

Optimal. Leaf size=389

$$-\frac{2x\left(-19ae^2-2bde+23cd^2\right)}{315d^2e^2\left(d+ex^3\right)^{5/2}}+\frac{2x\left(ae^2-bde+cd^2\right)}{21de^2\left(d+ex^3\right)^{7/2}}+\frac{2x\left(247ae^2+26bde+16cd^2\right)}{1215d^4e^2\sqrt{d+ex^3}}+\frac{2x\left(247ae^2+26bde+16cd^2\right)}{2835d^3e^2\left(d+ex^3\right)^{3/2}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(247ae^2+26bde+16cd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)|-7-4\sqrt{3}\right)}{1215\sqrt[4]{3}d^4e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(21*d*e^2*(d + e*x^3)^(7/2)) - (2*(
23*c*d^2 - 2*b*d*e - 19*a*e^2)*x)/(315*d^2*e^2*(d + e*x^3)^(5/2))
 + (2*(16*c*d^2 + 26*b*d*e + 247*a*e^2)*x)/(2835*d^3*e^2*(d + e*x
^3)^(3/2)) + (2*(16*c*d^2 + 26*b*d*e + 247*a*e^2)*x)/(1215*d^4*e^
2*Sqrt[d + e*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(16*c*d^2 + 26*b*d*e +
247*a*e^2)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*
x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF
[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3
) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(1215*3^(1/4)*d^4*e^(7/3)*Sqrt[
(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*
x)^2]*Sqrt[d + e*x^3])
```

Rubi [A] time = 0.737356, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{2x\left(-19ae^2-2bde+23cd^2\right)}{315d^2e^2\left(d+ex^3\right)^{5/2}}+\frac{2x\left(ae^2-bde+cd^2\right)}{21de^2\left(d+ex^3\right)^{7/2}}+\frac{2x\left(247ae^2+26bde+16cd^2\right)}{1215d^4e^2\sqrt{d+ex^3}}+\frac{2x\left(247ae^2+26bde+16cd^2\right)}{2835d^3e^2\left(d+ex^3\right)^{3/2}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(247ae^2+26bde+16cd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)|-7-4\sqrt{3}\right)}{1215\sqrt[4]{3}d^4e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2),x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(21*d*e^2*(d + e*x^3)^(7/2)) - (2*(
23*c*d^2 - 2*b*d*e - 19*a*e^2)*x)/(315*d^2*e^2*(d + e*x^3)^(5/2))
 + (2*(16*c*d^2 + 26*b*d*e + 247*a*e^2)*x)/(2835*d^3*e^2*(d + e*x
^3)^(3/2)) + (2*(16*c*d^2 + 26*b*d*e + 247*a*e^2)*x)/(1215*d^4*e^
2*Sqrt[d + e*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(16*c*d^2 + 26*b*d*e +
247*a*e^2)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*
x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF
[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3
) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(1215*3^(1/4)*d^4*e^(7/3)*Sqrt[
(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*
x)^2]*Sqrt[d + e*x^3])
```

Rubi in Sympy [A] time = 57.4961, size = 371, normalized size = 0.95

$$\frac{2x\left(ae^2-bde+cd^2\right)}{21de^2\left(d+ex^3\right)^{\frac{7}{2}}}+\frac{2x\left(19ae^2+2bde-23cd^2\right)}{315d^2e^2\left(d+ex^3\right)^{\frac{5}{2}}}+\frac{2x\left(247ae^2+26bde+16cd^2\right)}{2835d^3e^2\left(d+ex^3\right)^{\frac{3}{2}}}+\frac{2x\left(247ae^2+26bde+16cd^2\right)}{1215d^4e^2\sqrt{d+ex^3}}+\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{d^{\frac{2}{3}}-\sqrt[3]{d}\sqrt[3]{e}x+e^{\frac{2}{3}}x^2}{\left(\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(247ae^2+26bde+16cd^2\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{d}\left(-1+\sqrt{3}\right)+\sqrt[3]{e}x}{\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x}\right)\middle|-7-4\sqrt{3}\right)}{3645d^4e^{\frac{7}{3}}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt[3]{d}\left(1+\sqrt{3}\right)+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(9/2),x)
```

```
[Out] 2*x*(a*e**2 - b*d*e + c*d**2)/(21*d*e**2*(d + e*x**3)**(7/2)) + 2
*x*(19*a*e**2 + 2*b*d*e - 23*c*d**2)/(315*d**2*e**2*(d + e*x**3)*
*(5/2)) + 2*x*(247*a*e**2 + 26*b*d*e + 16*c*d**2)/(2835*d**3*e**2
*(d + e*x**3)**(3/2)) + 2*x*(247*a*e**2 + 26*b*d*e + 16*c*d**2)/(
1215*d**4*e**2*sqrt(d + e*x**3)) + 2*3**(3/4)*sqrt((d**(2/3) - d*
*(1/3)*e**(1/3)*x + e**(2/3)*x**2)/(d**(1/3)*(1 + sqrt(3)) + e**(
1/3)*x)**2)*sqrt(sqrt(3) + 2)*(d**(1/3) + e**(1/3)*x)*(247*a*e**2
 + 26*b*d*e + 16*c*d**2)*elliptic_f(asin((-d**(1/3)*(-1 + sqrt(3)
) + e**(1/3)*x)/(d**(1/3)*(1 + sqrt(3)) + e**(1/3)*x)), -7 - 4*sq
rt(3))/(3645*d**4*e**(7/3)*sqrt(d**(1/3)*(d**(1/3) + e**(1/3)*x)/
(d**(1/3)*(1 + sqrt(3)) + e**(1/3)*x)**2)*sqrt(d + e*x**3))
```

Mathematica [C] time = 1.03152, size = 296, normalized size = 0.76

$$2\left(3\sqrt[3]{-e}x\left(-27d^2\left(d+ex^3\right)\left(23cd^2-e(19ae+2bd)\right)+3d\left(d+ex^3\right)^2\left(13e(19ae+2bd)+16cd^2\right)+7\left(d+ex^3\right)^3\left(13e(19ae+\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2),x]
```

```
[Out] (2*(3*(-e)^(1/3)*x*(405*d^3*(c*d^2 + e*(-(b*d) + a*e)) - 27*d^2*(
23*c*d^2 - e*(2*b*d + 19*a*e))*(d + e*x^3) + 3*d*(16*c*d^2 + 13*e
*(2*b*d + 19*a*e))*(d + e*x^3)^2 + 7*(16*c*d^2 + 13*e*(2*b*d + 19
*a*e))*(d + e*x^3)^3) + (7*I)*3^(3/4)*d^(1/3)*(16*c*d^2 + 13*e*(2
*b*d + 19*a*e))*Sqrt[(-1)^(5/6)*(-1 + ((-e)^(1/3)*x)/d^(1/3))]*Sq
rt[1 + ((-e)^(1/3)*x)/d^(1/3) + ((-e)^(2/3)*x^2)/d^(2/3)]*(d + e*
x^3)^3*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-e)^(1/3)*x)/d^(1/
3)]/3^(1/4)], (-1)^(1/3)]))/(25515*d^4*(-e)^(7/3)*(d + e*x^3)^(7/
2))
```

Maple [B] time = 0.078, size = 1182, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x)
```

```
[Out] a*(2/21/d*x/e^4*(e*x^3+d)^(1/2)/(x^3+d/e)^4+38/315/d^2*x/e^3*(e*x
^3+d)^(1/2)/(x^3+d/e)^3+494/2835/d^3*x/e^2*(e*x^3+d)^(1/2)/(x^3+d
/e)^2+494/1215/d^4*x/((x^3+d/e)*e)^(1/2)-494/3645*I/d^4*3^(1/2)/e
*(-e^2*d)^(1/3)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*
d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)*((x-1/e*(-e^2*d)^(1/3))
/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)*(-
I*(x+1/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)
*e/(-e^2*d)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*
e/(-e^2*d)^(1/3))^(1/2),(I*3^(1/2)/e*(-e^2*d)^(1/3)/(-3/2/e*(-e^2
*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)))+b*(-2/21*x/e^5
*(e*x^3+d)^(1/2)/(x^3+d/e)^4+4/315/d*x/e^4*(e*x^3+d)^(1/2)/(x^3+d
/e)^3+52/2835/d^2*x/e^3*(e*x^3+d)^(1/2)/(x^3+d/e)^2+52/1215/e/d^3
*x/((x^3+d/e)*e)^(1/2)-52/3645*I/d^3/e^2*3^(1/2)*(-e^2*d)^(1/3)*(
I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)
*e/(-e^2*d)^(1/3))^(1/2)*((x-1/e*(-e^2*d)^(1/3))/(-3/2/e*(-e^2*d)
```

```
^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-e^2*
d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3)
)^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-e^2*d
)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))
^(1/2),(I*3^(1/2)/e*(-e^2*d)^(1/3)/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3
^(1/2)/e*(-e^2*d)^(1/3)))^(1/2)))+c*(2/21*d*x/e^6*(e*x^3+d)^(1/2)
/(x^3+d/e)^4-46/315*x/e^5*(e*x^3+d)^(1/2)/(x^3+d/e)^3+32/2835/d*x
/e^4*(e*x^3+d)^(1/2)/(x^3+d/e)^2+32/1215/e^2/d^2*x/((x^3+d/e)*e)^
(1/2)-32/3645*I/d^2/e^3*3^(1/2)*(-e^2*d)^(1/3)*(I*(x+1/2/e*(-e^2*
d)^(1/3)-1/2*I*3^(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3)
)^(1/2)*((x-1/e*(-e^2*d)^(1/3))/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1
/2)/e*(-e^2*d)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-e^2*d)^(1/3)+1/2*I*3^
(1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2)/(e*x^3+d)
^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-e^2*d)^(1/3)-1/2*I*3^(
1/2)/e*(-e^2*d)^(1/3))*3^(1/2)*e/(-e^2*d)^(1/3))^(1/2),(I*3^(1/2)
/e*(-e^2*d)^(1/3)/(-3/2/e*(-e^2*d)^(1/3)+1/2*I*3^(1/2)/e*(-e^2*d)
^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^6 + bx^3 + a}{(ex^3 + d)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{cx^6 + bx^3 + a}{(e^4x^{12} + 4\,de^3x^9 + 6\,d^2e^2x^6 + 4\,d^3ex^3 + d^4)\sqrt{ex^3 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)/((e^4*x^12 + 4*d*e^3*x^9 + 6*d^2*e^2
*x^6 + 4*d^3*e*x^3 + d^4)*sqrt(e*x^3 + d)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^6 + bx^3 + a}{(ex^3 + d)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(9/2), x)
```

3.43 $\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=433

$$-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{ex}{c}$$

```
[Out] (e*x)/c - ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*
c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(
2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - ((c*d - b*e -
 (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(
1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b +
 Sqrt[b^2 - 4*a*c])^(3/4)) - ((c*d - b*e + (b*c*d - b^2*e + 2*a*c
*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2
 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3
/4)) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])
*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*
2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 2.40237, antiderivative size = 433, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{ex}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] (e*x)/c - ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*
c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(
2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - ((c*d - b*e -
 (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(
1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b +
 Sqrt[b^2 - 4*a*c])^(3/4)) - ((c*d - b*e + (b*c*d - b^2*e + 2*a*c
*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2
 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3
/4)) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])
*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*
2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi in Sympy [A] time = 169.92, size = 432, normalized size = 1.

$$\frac{ex}{c} - \frac{2^{\frac{3}{4}}\left(-2ace + b(be - cd) - \sqrt{-4ac + b^2}\,(be - cd)\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b + \sqrt{-4ac + b^2}}}\right)}{4c^{\frac{5}{4}}\left(-b + \sqrt{-4ac + b^2}\right)^{\frac{3}{4}}\sqrt{-4ac + b^2}} - \frac{2^{\frac{3}{4}}\left(-2ace + b(be - cd) - \sqrt{-4ac + b^2}\,(be - cd)\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b + \sqrt{-4ac + b^2}}}\right)}{4c^{\frac{5}{4}}\left(-b + \sqrt{-4ac + b^2}\right)^{\frac{3}{4}}\sqrt{-4ac + b^2}} + \frac{2^{\frac{3}{4}}\left(-2ace + b(be - cd) + \sqrt{-4ac + b^2}\,(be - cd)\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b - \sqrt{-4ac + b^2}}}\right)}{4c^{\frac{5}{4}}\left(-b - \sqrt{-4ac + b^2}\right)^{\frac{3}{4}}\sqrt{-4ac + b^2}} + \frac{2^{\frac{3}{4}}\left(-2ace + b(be - cd) + \sqrt{-4ac + b^2}\,(be - cd)\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b - \sqrt{-4ac + b^2}}}\right)}{4c^{\frac{5}{4}}\left(-b - \sqrt{-4ac + b^2}\right)^{\frac{3}{4}}\sqrt{-4ac + b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] e*x/c - 2**(3/4)*(-2*a*c*e + b*(b*e - c*d) - sqrt(-4*a*c + b**2)*
(b*e - c*d))*atan(2**(1/4)*c**(1/4)*x/(-b + sqrt(-4*a*c + b**2))*
*(1/4))/(4*c**(5/4)*(-b + sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4*a*c
 + b**2)) - 2**(3/4)*(-2*a*c*e + b*(b*e - c*d) - sqrt(-4*a*c + b*
*2)*(b*e - c*d))*atanh(2**(1/4)*c**(1/4)*x/(-b + sqrt(-4*a*c + b*
*2))**(1/4))/(4*c**(5/4)*(-b + sqrt(-4*a*c + b**2))**(3/4)*sqrt(-
4*a*c + b**2)) + 2**(3/4)*(-2*a*c*e + b*(b*e - c*d) + sqrt(-4*a*c
 + b**2)*(b*e - c*d))*atan(2**(1/4)*c**(1/4)*x/(-b - sqrt(-4*a*c
+ b**2))**(1/4))/(4*c**(5/4)*(-b - sqrt(-4*a*c + b**2))**(3/4)*sq
rt(-4*a*c + b**2)) + 2**(3/4)*(-2*a*c*e + b*(b*e - c*d) + sqrt(-4
*a*c + b**2)*(b*e - c*d))*atanh(2**(1/4)*c**(1/4)*x/(-b - sqrt(-4
*a*c + b**2))**(1/4))/(4*c**(5/4)*(-b - sqrt(-4*a*c + b**2))**(3/
4)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.104407, size = 88, normalized size = 0.2

$$\frac{ex}{c}-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4be\log(x-\#1)+\#1^4(-c)d\log(x-\#1)+ae\log(x-\#1)}{2\#1^7c+\#1^3b}\&\right]}{4c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] (e*x)/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*e*Log[x - #1] - c*d*
Log[x - #1]*#1^4 + b*e*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/
(4*c)
```

Maple [C] time = 0.007, size = 67, normalized size = 0.2

$$\frac{ex}{c}+\frac{1}{4c}\sum_{\_R=RootOf(c\_Z^8+\_Z^4b+a)}\frac{((-be+cd)\_R^4-ae)\ln(x-\_R)}{2\_R^7c+\_R^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

```
[Out] e*x/c+1/4/c*sum(((-b*e+c*d)*_R^4-a*e)/(2*_R^7*c+_R^3*b)*ln(x-_R),
_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{ex}{c}-\frac{-\int\frac{(cd-be)x^4-ae}{cx^8+bx^4+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)*x^4/(c*x^8 + b*x^4 + a),x, algorithm="maxima")
```

```
[Out] e*x/c - integrate(-((c*d - b*e)*x^4 - a*e)/(c*x^8 + b*x^4 + a), x
)/c
```

Fricas [A] time = 4.4671, size = 17662, normalized size = 40.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)*x^4/(c*x^8 + b*x^4 + a),x, algorithm="fricas")
```

```
[Out] 1/4*(4*c*sqrt(sqrt(1/2)*sqrt(-(b*c^4*d^4 - 4*(b^2*c^3 - 2*a*c^4)*
d^3*e + 6*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b^4*c - 4*a*b^2*c^2
+ 2*a^2*c^3)*d*e^3 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^4 + (b^4*c
^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((c^8*d^8 - 8*b*c^7*d^7*e + 4*
(7*b^2*c^6 - 3*a*c^7)*d^6*e^2 - 8*(7*b^3*c^5 - 8*a*b*c^6)*d^5*e^3
 + 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 - 8*(7*b^5*
c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^3*e^5 + 4*(7*b^6*c^2 - 28*a*
b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^2*e^6 - 8*(b^7*c - 5*a*b^
5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^7 + (b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^8)/(b^6*c^10 - 12*a*b^
4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6
+ 16*a^2*c^7)))*arctan(-1/2*((b^2*c^4 - 4*a*c^5)*d^4*e - 4*(b^3*c
^3 - 4*a*b*c^4)*d^3*e^2 + 6*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d
^2*e^3 - 4*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d*e^4 + (b^6 - 7*a
*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^5 + (2*(b^4*c^6 - 8*a*b^2*
c^7 + 16*a^2*c^8)*d - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*e)*s
qrt((c^8*d^8 - 8*b*c^7*d^7*e + 4*(7*b^2*c^6 - 3*a*c^7)*d^6*e^2 -
8*(7*b^3*c^5 - 8*a*b*c^6)*d^5*e^3 + 2*(35*b^4*c^4 - 71*a*b^2*c^5
+ 19*a^2*c^6)*d^4*e^4 - 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^
5)*d^3*e^5 + 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3
*c^5)*d^2*e^6 - 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*
c^4)*d*e^7 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
a^4*c^4)*e^8)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^
3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b*c^4*d^4 - 4*(b^2*c^3 - 2*a*c^4)
*d^3*e + 6*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b^4*c - 4*a*b^2*c^2
 + 2*a^2*c^3)*d*e^3 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^4 + (b^4*
c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((c^8*d^8 - 8*b*c^7*d^7*e + 4
*(7*b^2*c^6 - 3*a*c^7)*d^6*e^2 - 8*(7*b^3*c^5 - 8*a*b*c^6)*d^5*e^
3 + 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 - 8*(7*b^5
*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^3*e^5 + 4*(7*b^6*c^2 - 28*a
*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^2*e^6 - 8*(b^7*c - 5*a*b
^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^7 + (b^8 - 6*a*b^6*c +
11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^8)/(b^6*c^10 - 12*a*b
^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6
 + 16*a^2*c^7)))/((c^5*d^6 - 5*b*c^4*d^5*e + 5*(2*b^2*c^3 - a*c^4
)*d^4*e^2 - 10*(b^3*c^2 - a*b*c^3)*d^3*e^3 + 5*(b^4*c - a*b^2*c^2
 - a^2*c^3)*d^2*e^4 - (b^5 + a*b^3*c - 7*a^2*b*c^2)*d*e^5 + (a*b^
4 - 3*a^2*b^2*c + a^3*c^2)*e^6)*x + sqrt(1/2)*(c^5*d^6 - 5*b*c^4*
d^5*e + 5*(2*b^2*c^3 - a*c^4)*d^4*e^2 - 10*(b^3*c^2 - a*b*c^3)*d^
3*e^3 + 5*(b^4*c - a*b^2*c^2 - a^2*c^3)*d^2*e^4 - (b^5 + a*b^3*c
- 7*a^2*b*c^2)*d*e^5 + (a*b^4 - 3*a^2*b^2*c + a^3*c^2)*e^6)*sqrt(
(2*(c^6*d^8 - 6*b*c^5*d^7*e + (15*b^2*c^4 - 4*a*c^5)*d^6*e^2 - 10
*(2*b^3*c^3 - a*b*c^4)*d^5*e^3 + 5*(3*b^4*c^2 - a*b^2*c^3 - 2*a^2
*c^4)*d^4*e^4 - 2*(3*b^5*c + 3*a*b^3*c^2 - 11*a^2*b*c^3)*d^3*e^5
+ (b^6 + 7*a*b^4*c - 15*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^6 - 2*(a*b
^5 - a^2*b^3*c - 3*a^3*b*c^2)*d*e^7 + (a^2*b^4 - 3*a^3*b^2*c + a^
4*c^2)*e^8)*x^2 + sqrt(1/2)*(2*(b^2*c^6 - 4*a*c^7)*d^6 - 10*(b^3*
c^5 - 4*a*b*c^6)*d^5*e + 7*(3*b^4*c^4 - 14*a*b^2*c^5 + 8*a^2*c^6)
*d^4*e^2 - 12*(2*b^5*c^3 - 11*a*b^3*c^4 + 12*a^2*b*c^5)*d^3*e^3 +
```

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 2*(8*b^6*c^2 - 52*a*b^4*c^3 + 87*a^2*b^2*c^4 - 28*a^3*c^5)*d^2*e
^4 - 2*(3*b^7*c - 23*a*b^5*c^2 + 53*a^2*b^3*c^3 - 36*a^3*b*c^4)*d
*e^5 + (b^8 - 9*a*b^6*c + 27*a^2*b^4*c^2 - 30*a^3*b^2*c^3 + 8*a^4
*c^4)*e^6 + (2*(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*
c^9)*d*e - (b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^
8)*e^2)*sqrt((c^8*d^8 - 8*b*c^7*d^7*e + 4*(7*b^2*c^6 - 3*a*c^7)*d
^6*e^2 - 8*(7*b^3*c^5 - 8*a*b*c^6)*d^5*e^3 + 2*(35*b^4*c^4 - 71*a
*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 - 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13
*a^2*b*c^5)*d^3*e^5 + 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^
4 - 3*a^3*c^5)*d^2*e^6 - 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 -
 2*a^3*b*c^4)*d*e^7 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b
^2*c^3 + a^4*c^4)*e^8)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^1
2 - 64*a^3*c^13)))*sqrt(-(b*c^4*d^4 - 4*(b^2*c^3 - 2*a*c^4)*d^3*e
 + 6*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b^4*c - 4*a*b^2*c^2 + 2*a
^2*c^3)*d*e^3 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^4 + (b^4*c^5 -
8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((c^8*d^8 - 8*b*c^7*d^7*e + 4*(7*b^
2*c^6 - 3*a*c^7)*d^6*e^2 - 8*(7*b^3*c^5 - 8*a*b*c^6)*d^5*e^3 + 2*
(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 - 8*(7*b^5*c^3 -
 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^3*e^5 + 4*(7*b^6*c^2 - 28*a*b^4*c
^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^2*e^6 - 8*(b^7*c - 5*a*b^5*c^2
 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^7 + (b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^8)/(b^6*c^10 - 12*a*b^4*c^1
1 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*
a^2*c^7)))/(c^6*d^8 - 6*b*c^5*d^7*e + (15*b^2*c^4 - 4*a*c^5)*d^6*
e^2 - 10*(2*b^3*c^3 - a*b*c^4)*d^5*e^3 + 5*(3*b^4*c^2 - a*b^2*c^3
 - 2*a^2*c^4)*d^4*e^4 - 2*(3*b^5*c + 3*a*b^3*c^2 - 11*a^2*b*c^3)*
d^3*e^5 + (b^6 + 7*a*b^4*c - 15*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^6
- 2*(a*b^5 - a^2*b^3*c - 3*a^3*b*c^2)*d*e^7 + (a^2*b^4 - 3*a^3*b^
2*c + a^4*c^2)*e^8)))) - 4*c*sqrt(sqrt(1/2)*sqrt(-(b*c^4*d^4 - 4*
(b^2*c^3 - 2*a*c^4)*d^3*e + 6*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(
b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^3 + (b^5 - 5*a*b^3*c + 5*a^2
*b*c^2)*e^4 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((c^8*d^8
- 8*b*c^7*d^7*e + 4*(7*b^2*c^6 - 3*a*c^7)*d^6*e^2 - 8*(7*b^3*c^5
- 8*a*b*c^6)*d^5*e^3 + 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)
*d^4*e^4 - 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^3*e^5 +
4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^2*e^6
 - 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^7 +
(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^8)
/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^
4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(1/2*((b^2*c^4 - 4*a*c^
5)*d^4*e - 4*(b^3*c^3 - 4*a*b*c^4)*d^3*e^2 + 6*(b^4*c^2 - 5*a*b^2
*c^3 + 4*a^2*c^4)*d^2*e^3 - 4*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)
*d*e^4 + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^5 - (2*
(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*d - (b^5*c^5 - 8*a*b^3*c^6 +
 16*a^2*b*c^7)*e)*sqrt((c^8*d^8 - 8*b*c^7*d^7*e + 4*(7*b^2*c^6 -
3*a*c^7)*d^6*e^2 - 8*(7*b^3*c^5 - 8*a*b*c^6)*d^5*e^3 + 2*(35*b^4*
c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 - 8*(7*b^5*c^3 - 21*a*b^
3*c^4 + 13*a^2*b*c^5)*d^3*e^5 + 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*
a^2*b^2*c^4 - 3*a^3*c^5)*d^2*e^6 - 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2
*b^3*c^3 - 2*a^3*b*c^4)*d*e^7 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
 - 6*a^3*b^2*c^3 + a^4*c^4)*e^8)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a
^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b*c^4*d^4 - 4*
(b^2*c^3 - 2*a*c^4)*d^3*e + 6*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(
b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^3 + (b^5 - 5*a*b^3*c + 5*a^2
```

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*b*c^2)*e^4 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((c^8*d^8
- 8*b*c^7*d^7*e + 4*(7*b^2*c^6 - 3*a*c^7)*d^6*e^2 - 8*(7*b^3*c^5
- 8*a*b*c^6)*d^5*e^3 + 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)
*d^4*e^4 - 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^3*e^5 +
4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^2*e^6
 - 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^7 +
(b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^8)
/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^
4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/((c^5*d^6 - 5*b*c^4*d^5*e + 5
*(2*b^2*c^3 - a*c^4)*d^4*e^2 - 10*(b^3*c^2 - a*b*c^3)*d^3*e^3 + 5
*(b^4*c - a*b^2*c^2 - a^2*c^3)*d^2*e^4 - (b^5 + a*b^3*c - 7*a^2*b
*c^2)*d*e^5 + (a*b^4 - 3*a^2*b^2*c + a^3*c^2)*e^6)*x + sqrt(1/2)*
(c^5*d^6 - 5*b*c^4*d^5*e + 5*(2*b^2*c^3 - a*c^4)*d^4*e^2 - 10*(b^
3*c^2 - a*b*c^3)*d^3*e^3 + 5*(b^4*c - a*b^2*c^2 - a^2*c^3)*d^2*e^
4 - (b^5 + a*b^3*c - 7*a^2*b*c^2)*d*e^5 + (a*b^4 - 3*a^2*b^2*c +
a^3*c^2)*e^6)*sqrt((2*(c^6*d^8 - 6*b*c^5*d^7*e + (15*b^2*c^4 - 4*
a*c^5)*d^6*e^2 - 10*(2*b^3*c^3 - a*b*c^4)*d^5*e^3 + 5*(3*b^4*c^2
- a*b^2*c^3 - 2*a^2*c^4)*d^4*e^4 - 2*(3*b^5*c + 3*a*b^3*c^2 - 11*
a^2*b*c^3)*d^3*e^5 + (b^6 + 7*a*b^4*c - 15*a^2*b^2*c^2 - 4*a^3*c^
3)*d^2*e^6 - 2*(a*b^5 - a^2*b^3*c - 3*a^3*b*c^2)*d*e^7 + (a^2*b^4
 - 3*a^3*b^2*c + a^4*c^2)*e^8)*x^2 + sqrt(1/2)*(2*(b^2*c^6 - 4*a*
c^7)*d^6 - 10*(b^3*c^5 - 4*a*b*c^6)*d^5*e + 7*(3*b^4*c^4 - 14*a*b
^2*c^5 + 8*a^2*c^6)*d^4*e^2 - 12*(2*b^5*c^3 - 11*a*b^3*c^4 + 12*a
^2*b*c^5)*d^3*e^3 + 2*(8*b^6*c^2 - 52*a*b^4*c^3 + 87*a^2*b^2*c^4
- 28*a^3*c^5)*d^2*e^4 - 2*(3*b^7*c - 23*a*b^5*c^2 + 53*a^2*b^3*c^
3 - 36*a^3*b*c^4)*d*e^5 + (b^8 - 9*a*b^6*c + 27*a^2*b^4*c^2 - 30*
a^3*b^2*c^3 + 8*a^4*c^4)*e^6 - (2*(b^6*c^6 - 12*a*b^4*c^7 + 48*a^
2*b^2*c^8 - 64*a^3*c^9)*d*e - (b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^
3*c^7 - 64*a^3*b*c^8)*e^2)*sqrt((c^8*d^8 - 8*b*c^7*d^7*e + 4*(7*b
^2*c^6 - 3*a*c^7)*d^6*e^2 - 8*(7*b^3*c^5 - 8*a*b*c^6)*d^5*e^3 + 2
*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 - 8*(7*b^5*c^3
- 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^3*e^5 + 4*(7*b^6*c^2 - 28*a*b^4*
c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^2*e^6 - 8*(b^7*c - 5*a*b^5*c^
2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^7 + (b^8 - 6*a*b^6*c + 11*a^
2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^8)/(b^6*c^10 - 12*a*b^4*c^
11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b*c^4*d^4 - 4*(b^2*c
^3 - 2*a*c^4)*d^3*e + 6*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b^4*c
- 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^3 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2
)*e^4 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((c^8*d^8 - 8*b*
c^7*d^7*e + 4*(7*b^2*c^6 - 3*a*c^7)*d^6*e^2 - 8*(7*b^3*c^5 - 8*a*
b*c^6)*d^5*e^3 + 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e
^4 - 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^3*e^5 + 4*(7*b
^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^2*e^6 - 8*(
b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^7 + (b^8 -
 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^8)/(b^6*
c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5
- 8*a*b^2*c^6 + 16*a^2*c^7)))/(c^6*d^8 - 6*b*c^5*d^7*e + (15*b^2*
c^4 - 4*a*c^5)*d^6*e^2 - 10*(2*b^3*c^3 - a*b*c^4)*d^5*e^3 + 5*(3*
b^4*c^2 - a*b^2*c^3 - 2*a^2*c^4)*d^4*e^4 - 2*(3*b^5*c + 3*a*b^3*c
^2 - 11*a^2*b*c^3)*d^3*e^5 + (b^6 + 7*a*b^4*c - 15*a^2*b^2*c^2 -
4*a^3*c^3)*d^2*e^6 - 2*(a*b^5 - a^2*b^3*c - 3*a^3*b*c^2)*d*e^7 +
(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e^8)))) + c*sqrt(sqrt(1/2)*sqrt
(-(b*c^4*d^4 - 4*(b^2*c^3 - 2*a*c^4)*d^3*e + 6*(b^3*c^2 - 3*a*b*c
^3)*d^2*e^2 - 4*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^3 + (b^5 -
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5*a*b^3*c + 5*a^2*b*c^2)*e^4 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^
7)*sqrt((c^8*d^8 - 8*b*c^7*d^7*e + 4*(7*b^2*c^6 - 3*a*c^7)*d^6*e^
2 - 8*(7*b^3*c^5 - 8*a*b*c^6)*d^5*e^3 + 2*(35*b^4*c^4 - 71*a*b^2*
c^5 + 19*a^2*c^6)*d^4*e^4 - 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*
b*c^5)*d^3*e^5 + 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3
*a^3*c^5)*d^2*e^6 - 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^
3*b*c^4)*d*e^7 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^
3 + a^4*c^4)*e^8)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 6
4*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((c^5*d^6
 - 5*b*c^4*d^5*e + 5*(2*b^2*c^3 - a*c^4)*d^4*e^2 - 10*(b^3*c^2 -
a*b*c^3)*d^3*e^3 + 5*(b^4*c - a*b^2*c^2 - a^2*c^3)*d^2*e^4 - (b^5
 + a*b^3*c - 7*a^2*b*c^2)*d*e^5 + (a*b^4 - 3*a^2*b^2*c + a^3*c^2)
*e^6)*x + 1/2*((b^2*c^4 - 4*a*c^5)*d^4*e - 4*(b^3*c^3 - 4*a*b*c^4
)*d^3*e^2 + 6*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d^2*e^3 - 4*(b^
5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d*e^4 + (b^6 - 7*a*b^4*c + 13*a^
2*b^2*c^2 - 4*a^3*c^3)*e^5 + (2*(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c
^8)*d - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*e)*sqrt((c^8*d^8 -
 8*b*c^7*d^7*e + 4*(7*b^2*c^6 - 3*a*c^7)*d^6*e^2 - 8*(7*b^3*c^5 -
 8*a*b*c^6)*d^5*e^3 + 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*
d^4*e^4 - 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^3*e^5 + 4
*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^2*e^6
- 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^7 + (
b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^8)/
(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt
(sqrt(1/2)*sqrt(-(b*c^4*d^4 - 4*(b^2*c^3 - 2*a*c^4)*d^3*e + 6*(b^
3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*
d*e^3 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^4 + (b^4*c^5 - 8*a*b^2*
c^6 + 16*a^2*c^7)*sqrt((c^8*d^8 - 8*b*c^7*d^7*e + 4*(7*b^2*c^6 -
3*a*c^7)*d^6*e^2 - 8*(7*b^3*c^5 - 8*a*b*c^6)*d^5*e^3 + 2*(35*b^4*
c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 - 8*(7*b^5*c^3 - 21*a*b^
3*c^4 + 13*a^2*b*c^5)*d^3*e^5 + 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*
a^2*b^2*c^4 - 3*a^3*c^5)*d^2*e^6 - 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2
*b^3*c^3 - 2*a^3*b*c^4)*d*e^7 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
 - 6*a^3*b^2*c^3 + a^4*c^4)*e^8)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a
^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)
))) - c*sqrt(sqrt(1/2)*sqrt(-(b*c^4*d^4 - 4*(b^2*c^3 - 2*a*c^4)*d
^3*e + 6*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b^4*c - 4*a*b^2*c^2 +
 2*a^2*c^3)*d*e^3 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^4 + (b^4*c^
5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((c^8*d^8 - 8*b*c^7*d^7*e + 4*(
7*b^2*c^6 - 3*a*c^7)*d^6*e^2 - 8*(7*b^3*c^5 - 8*a*b*c^6)*d^5*e^3
+ 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 - 8*(7*b^5*c
^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^3*e^5 + 4*(7*b^6*c^2 - 28*a*b
^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^2*e^6 - 8*(b^7*c - 5*a*b^5
*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^7 + (b^8 - 6*a*b^6*c + 11
*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^8)/(b^6*c^10 - 12*a*b^4
*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 +
 16*a^2*c^7)))*log((c^5*d^6 - 5*b*c^4*d^5*e + 5*(2*b^2*c^3 - a*c^
4)*d^4*e^2 - 10*(b^3*c^2 - a*b*c^3)*d^3*e^3 + 5*(b^4*c - a*b^2*c^
2 - a^2*c^3)*d^2*e^4 - (b^5 + a*b^3*c - 7*a^2*b*c^2)*d*e^5 + (a*b
^4 - 3*a^2*b^2*c + a^3*c^2)*e^6)*x - 1/2*((b^2*c^4 - 4*a*c^5)*d^4
*e - 4*(b^3*c^3 - 4*a*b*c^4)*d^3*e^2 + 6*(b^4*c^2 - 5*a*b^2*c^3 +
 4*a^2*c^4)*d^2*e^3 - 4*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d*e^4
 + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^5 + (2*(b^4*c
^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*d - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^
```

```
2*b*c^7)*e)*sqrt((c^8*d^8 - 8*b*c^7*d^7*e + 4*(7*b^2*c^6 - 3*a*c^
7)*d^6*e^2 - 8*(7*b^3*c^5 - 8*a*b*c^6)*d^5*e^3 + 2*(35*b^4*c^4 -
71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 - 8*(7*b^5*c^3 - 21*a*b^3*c^4
+ 13*a^2*b*c^5)*d^3*e^5 + 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^
2*c^4 - 3*a^3*c^5)*d^2*e^6 - 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c
^3 - 2*a^3*b*c^4)*d*e^7 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a
^3*b^2*c^3 + a^4*c^4)*e^8)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2
*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b*c^4*d^4 - 4*(b^2*c
^3 - 2*a*c^4)*d^3*e + 6*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b^4*c
- 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^3 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2
)*e^4 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((c^8*d^8 - 8*b*
c^7*d^7*e + 4*(7*b^2*c^6 - 3*a*c^7)*d^6*e^2 - 8*(7*b^3*c^5 - 8*a*
b*c^6)*d^5*e^3 + 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e
^4 - 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^3*e^5 + 4*(7*b
^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^2*e^6 - 8*(
b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^7 + (b^8 -
 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^8)/(b^6*
c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5
- 8*a*b^2*c^6 + 16*a^2*c^7)))) + c*sqrt(sqrt(1/2)*sqrt(-(b*c^4*d^
4 - 4*(b^2*c^3 - 2*a*c^4)*d^3*e + 6*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2
 - 4*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^3 + (b^5 - 5*a*b^3*c +
 5*a^2*b*c^2)*e^4 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((c^
8*d^8 - 8*b*c^7*d^7*e + 4*(7*b^2*c^6 - 3*a*c^7)*d^6*e^2 - 8*(7*b^
3*c^5 - 8*a*b*c^6)*d^5*e^3 + 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^
2*c^6)*d^4*e^4 - 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^3*
e^5 + 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d
^2*e^6 - 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*
e^7 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4
)*e^8)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log((c^5*d^6 - 5*b*c^4*
d^5*e + 5*(2*b^2*c^3 - a*c^4)*d^4*e^2 - 10*(b^3*c^2 - a*b*c^3)*d^
3*e^3 + 5*(b^4*c - a*b^2*c^2 - a^2*c^3)*d^2*e^4 - (b^5 + a*b^3*c
- 7*a^2*b*c^2)*d*e^5 + (a*b^4 - 3*a^2*b^2*c + a^3*c^2)*e^6)*x + 1
/2*((b^2*c^4 - 4*a*c^5)*d^4*e - 4*(b^3*c^3 - 4*a*b*c^4)*d^3*e^2 +
 6*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d^2*e^3 - 4*(b^5*c - 6*a*b
^3*c^2 + 8*a^2*b*c^3)*d*e^4 + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 -
 4*a^3*c^3)*e^5 - (2*(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*d - (b^
5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*e)*sqrt((c^8*d^8 - 8*b*c^7*d^
7*e + 4*(7*b^2*c^6 - 3*a*c^7)*d^6*e^2 - 8*(7*b^3*c^5 - 8*a*b*c^6)
*d^5*e^3 + 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 - 8
*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^3*e^5 + 4*(7*b^6*c^2
 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^2*e^6 - 8*(b^7*c
- 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^7 + (b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^8)/(b^6*c^10 -
 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*
sqrt(-(b*c^4*d^4 - 4*(b^2*c^3 - 2*a*c^4)*d^3*e + 6*(b^3*c^2 - 3*a
*b*c^3)*d^2*e^2 - 4*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^3 + (b^
5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^4 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^
2*c^7)*sqrt((c^8*d^8 - 8*b*c^7*d^7*e + 4*(7*b^2*c^6 - 3*a*c^7)*d^
6*e^2 - 8*(7*b^3*c^5 - 8*a*b*c^6)*d^5*e^3 + 2*(35*b^4*c^4 - 71*a*
b^2*c^5 + 19*a^2*c^6)*d^4*e^4 - 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*
a^2*b*c^5)*d^3*e^5 + 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4
 - 3*a^3*c^5)*d^2*e^6 - 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 -
2*a^3*b*c^4)*d*e^7 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
```

```
2*c^3 + a^4*c^4)*e^8)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12
 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - c*sqr
t(sqrt(1/2)*sqrt(-(b*c^4*d^4 - 4*(b^2*c^3 - 2*a*c^4)*d^3*e + 6*(b
^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)
*d*e^3 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^4 - (b^4*c^5 - 8*a*b^2
*c^6 + 16*a^2*c^7)*sqrt((c^8*d^8 - 8*b*c^7*d^7*e + 4*(7*b^2*c^6 -
 3*a*c^7)*d^6*e^2 - 8*(7*b^3*c^5 - 8*a*b*c^6)*d^5*e^3 + 2*(35*b^4
*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 - 8*(7*b^5*c^3 - 21*a*b
^3*c^4 + 13*a^2*b*c^5)*d^3*e^5 + 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28
*a^2*b^2*c^4 - 3*a^3*c^5)*d^2*e^6 - 8*(b^7*c - 5*a*b^5*c^2 + 7*a^
2*b^3*c^3 - 2*a^3*b*c^4)*d*e^7 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^
2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^8)/(b^6*c^10 - 12*a*b^4*c^11 + 48*
a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7
)))*log((c^5*d^6 - 5*b*c^4*d^5*e + 5*(2*b^2*c^3 - a*c^4)*d^4*e^2
- 10*(b^3*c^2 - a*b*c^3)*d^3*e^3 + 5*(b^4*c - a*b^2*c^2 - a^2*c^3
)*d^2*e^4 - (b^5 + a*b^3*c - 7*a^2*b*c^2)*d*e^5 + (a*b^4 - 3*a^2*
b^2*c + a^3*c^2)*e^6)*x - 1/2*((b^2*c^4 - 4*a*c^5)*d^4*e - 4*(b^3
*c^3 - 4*a*b*c^4)*d^3*e^2 + 6*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)
*d^2*e^3 - 4*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d*e^4 + (b^6 - 7
*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^5 - (2*(b^4*c^6 - 8*a*b^
2*c^7 + 16*a^2*c^8)*d - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*e)
*sqrt((c^8*d^8 - 8*b*c^7*d^7*e + 4*(7*b^2*c^6 - 3*a*c^7)*d^6*e^2
- 8*(7*b^3*c^5 - 8*a*b*c^6)*d^5*e^3 + 2*(35*b^4*c^4 - 71*a*b^2*c^
5 + 19*a^2*c^6)*d^4*e^4 - 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*
c^5)*d^3*e^5 + 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a
^3*c^5)*d^2*e^6 - 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*
b*c^4)*d*e^7 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3
+ a^4*c^4)*e^8)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*
a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b*c^4*d^4 - 4*(b^2*c^3 - 2*a*c^
4)*d^3*e + 6*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b^4*c - 4*a*b^2*c
^2 + 2*a^2*c^3)*d*e^3 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^4 - (b^
4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((c^8*d^8 - 8*b*c^7*d^7*e +
 4*(7*b^2*c^6 - 3*a*c^7)*d^6*e^2 - 8*(7*b^3*c^5 - 8*a*b*c^6)*d^5*
e^3 + 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 - 8*(7*b
^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^3*e^5 + 4*(7*b^6*c^2 - 28
*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^2*e^6 - 8*(b^7*c - 5*a
*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^7 + (b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^8)/(b^6*c^10 - 12*a
*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c
^6 + 16*a^2*c^7)))) + 4*e*x)/c
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^4 + d)x^4}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)*x^4/(c*x^8 + b*x^4 + a),x, algorithm="giac")

[Out] integrate((e*x^4 + d)*x^4/(c*x^8 + b*x^4 + a), x)
```

3.44 $\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=72

$$\frac{e\log\left(a+bx^4+cx^8\right)}{8c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}$$

```
[Out] -((2*c*d - b*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*c*Sq
rt[b^2 - 4*a*c]) + (e*Log[a + b*x^4 + c*x^8])/(8*c)
```

Rubi [A] time = 0.17663, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{e\log\left(a+bx^4+cx^8\right)}{8c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] -((2*c*d - b*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*c*Sq
rt[b^2 - 4*a*c]) + (e*Log[a + b*x^4 + c*x^8])/(8*c)
```

Rubi in Sympy [A] time = 24.6482, size = 63, normalized size = 0.88

$$\frac{e\log\left(a+bx^4+cx^8\right)}{8c}+\frac{(be-2cd)\operatorname{atanh}\left(\frac{b+2cx^4}{\sqrt{-4ac+b^2}}\right)}{4c\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] e*log(a + b*x**4 + c*x**8)/(8*c) + (b*e - 2*c*d)*atanh((b + 2*c*x
**4)/sqrt(-4*a*c + b**2))/(4*c*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.0979036, size = 71, normalized size = 0.99

$$\frac{e\log\left(a+bx^4+cx^8\right)-\frac{2(be-2cd)\tan^{-1}\left(\frac{b+2cx^4}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{8c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((-2*(-2*c*d + b*e)*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqr
t[-b^2 + 4*a*c] + e*Log[a + b*x^4 + c*x^8])/(8*c)
```

Maple [A] time = 0.004, size = 99, normalized size = 1.4

$$\frac{e\ln\left(cx^8+bx^4+a\right)}{8\,c}+\frac{d}{2}\arctan\left((2\,cx^4+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}-\frac{be}{4\,c}\arctan\left((2\,cx^4+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/8*e*ln(c*x^8+b*x^4+a)/c+1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b
)/(4*a*c-b^2)^(1/2))*d-1/4/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(
4*a*c-b^2)^(1/2))/c*b*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)*x^3/(c*x^8 + b*x^4 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.394829, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{b^2-4\,ac}e\log\left(cx^8+bx^4+a\right)-(2\,cd-be)\log\left(\frac{2\,(b^2c-4\,ac^2)\,x^4+b^3-4\,abc+(2\,c^2x^8+2\,bcx^4+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)}{8\,\sqrt{b^2-4\,ac}c}, \frac{\sqrt{-b^2+4\,ac}e\log\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)*x^3/(c*x^8 + b*x^4 + a),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(b^2 - 4*a*c)*e*log(c*x^8 + b*x^4 + a) - (2*c*d - b*e)*
log((2*(b^2*c - 4*a*c^2)*x^4 + b^3 - 4*a*b*c + (2*c^2*x^8 + 2*b*c
*x^4 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)))/(sqr
t(b^2 - 4*a*c)*c), 1/8*(sqrt(-b^2 + 4*a*c)*e*log(c*x^8 + b*x^4 +
a) + 2*(2*c*d - b*e)*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^
2 - 4*a*c)))/(sqrt(-b^2 + 4*a*c)*c)]
```

Sympy [A] time = 24.314, size = 287, normalized size = 3.99

$$\left(\frac{e}{8c} - \frac{\sqrt{-4ac+b^2}\,(be-2cd)}{8c\,(4ac-b^2)}\right)\log\left(x^4+\frac{-16ac\left(\frac{e}{8c}-\frac{\sqrt{-4ac+b^2}(be-2cd)}{8c(4ac-b^2)}\right)+2ae+4b^2\left(\frac{e}{8c}-\frac{\sqrt{-4ac+b^2}(be-2cd)}{8c(4ac-b^2)}\right)-bd}{be-2cd}\right) + \left(\frac{e}{8c} + \frac{\sqrt{-4ac+b^2}\,(be-2cd)}{8c\,(4ac-b^2)}\right)\log\left(x^4+\frac{-16ac\left(\frac{e}{8c}+\frac{\sqrt{-4ac+b^2}(be-2cd)}{8c(4ac-b^2)}\right)+2ae+4b^2\left(\frac{e}{8c}+\frac{\sqrt{-4ac+b^2}(be-2cd)}{8c(4ac-b^2)}\right)-bd}{be-2cd}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] (e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2))
)*log(x**4 + (-16*a*c*(e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d
)/(8*c*(4*a*c - b**2))) + 2*a*e + 4*b**2*(e/(8*c) - sqrt(-4*a*c +
 b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*d))
+ (e/(8*c) + sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2
)))*log(x**4 + (-16*a*c*(e/(8*c) + sqrt(-4*a*c + b**2)*(b*e - 2*c
*d)/(8*c*(4*a*c - b**2))) + 2*a*e + 4*b**2*(e/(8*c) + sqrt(-4*a*c
 + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*d)
```

)

GIAC/XCAS [A] time = 0.275882, size = 95, normalized size = 1.32

$$\frac{e\ln\left(cx^8 + bx^4 + a\right)}{8\,c} + \frac{(2\,cd - be)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2 + 4\,ac}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)*x^3/(c*x^8 + b*x^4 + a),x, algorithm="giac")
```

```
[Out] 1/8*e*ln(c*x^8 + b*x^4 + a)/c + 1/4*(2*c*d - b*e)*arctan((2*c*x^4
 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

3.45 $\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=375

$$\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

```
[Out] ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)
/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b
^2 - 4*a*c])^(1/4)) + ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcT
an[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4
)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ((e - (2*c*d - b*e)/S
qrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*
a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))
- ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)
*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b + Sqr
t[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 0.947886, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)
/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b
^2 - 4*a*c])^(1/4)) + ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcT
an[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4
)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ((e - (2*c*d - b*e)/S
qrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*
a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))
 - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)
*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b + Sqr
t[b^2 - 4*a*c])^(1/4))
```

Rubi in Sympy [A] time = 119.001, size = 379, normalized size = 1.01

$$-\frac{\sqrt[4]{2}\left(be-2cd-e\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{3}{4}}\sqrt[4]{-b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt[4]{2}\left(be-2cd-e\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{3}{4}}\sqrt[4]{-b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt[4]{2}\left(be-2cd+e\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{3}{4}}\sqrt[4]{-b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt[4]{2}\left(be-2cd+e\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{3}{4}}\sqrt[4]{-b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] -2**(1/4)*(b*e - 2*c*d - e*sqrt(-4*a*c + b**2))*atan(2**(1/4)*c**
(1/4)*x/(-b + sqrt(-4*a*c + b**2))**(1/4))/(4*c**(3/4)*(-b + sqrt
(-4*a*c + b**2))**(1/4)*sqrt(-4*a*c + b**2)) + 2**(1/4)*(b*e - 2*
c*d - e*sqrt(-4*a*c + b**2))*atanh(2**(1/4)*c**(1/4)*x/(-b + sqrt
(-4*a*c + b**2))**(1/4))/(4*c**(3/4)*(-b + sqrt(-4*a*c + b**2))**
(1/4)*sqrt(-4*a*c + b**2)) + 2**(1/4)*(b*e - 2*c*d + e*sqrt(-4*a*
c + b**2))*atan(2**(1/4)*c**(1/4)*x/(-b - sqrt(-4*a*c + b**2))**(
```

```
1/4))/(4*c**(3/4)*(-b - sqrt(-4*a*c + b**2))**(1/4)*sqrt(-4*a*c +
 b**2)) - 2**(1/4)*(b*e - 2*c*d + e*sqrt(-4*a*c + b**2))*atanh(2*
*(1/4)*c**(1/4)*x/(-b - sqrt(-4*a*c + b**2))**(1/4))/(4*c**(3/4)*
(-b - sqrt(-4*a*c + b**2))**(1/4)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0641124, size = 59, normalized size = 0.16

$$\frac{1}{4}\text{RootSum}\left[\#1^8c + \#1^4b + a\&, \frac{\#1^4e\log(x - \#1) + d\log(x - \#1)}{2\#1^5c + \#1b}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1
^4)/(b*#1 + 2*c*#1^5) & ]/4
```

Maple [C] time = 0.005, size = 51, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=RootOf\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(\_R^6e + \_R^2d\right)\ln\left(x - \_R\right)}{2\_R^7c + \_R^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*sum((_R^6*e+_R^2*d)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8
*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^4 + d\right)x^2}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)*x^2/(c*x^8 + b*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)*x^2/(c*x^8 + b*x^4 + a), x)
```

Fricas [A] time = 16.7277, size = 19648, normalized size = 52.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4 + d)*x^2/(c*x^8 + b*x^4 + a),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*
e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (
a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*
d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3
 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 -
8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^
2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c
^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*arctan(1/2*sqrt(
1/2)*((b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d^7 - 9*(a*b^4*c^4 - 8
*a^2*b^2*c^5 + 16*a^3*c^6)*d^5*e^2 + 5*(a*b^5*c^3 - 8*a^2*b^3*c^4
 + 16*a^3*b*c^5)*d^4*e^3 - (a*b^6*c^2 - 27*a^2*b^4*c^3 + 168*a^3*
b^2*c^4 - 304*a^4*c^5)*d^3*e^4 - 18*(a^2*b^5*c^2 - 8*a^3*b^3*c^3
+ 16*a^4*b*c^4)*d^2*e^5 + (7*a^2*b^6*c - 59*a^3*b^4*c^2 + 136*a^4
*b^2*c^3 - 48*a^5*c^4)*d*e^6 - (a^2*b^7 - 9*a^3*b^5*c + 24*a^4*b^
3*c^2 - 16*a^5*b*c^3)*e^7 - ((a*b^7*c^5 - 12*a^2*b^5*c^6 + 48*a^3
*b^3*c^7 - 64*a^4*b*c^8)*d^3 - 6*(a^2*b^6*c^5 - 12*a^3*b^4*c^6 +
48*a^4*b^2*c^7 - 64*a^5*c^8)*d^2*e + 3*(a^2*b^7*c^4 - 12*a^3*b^5*
c^5 + 48*a^4*b^3*c^6 - 64*a^5*b*c^7)*d*e^2 - (a^2*b^8*c^3 - 14*a^
3*b^6*c^4 + 72*a^4*b^4*c^5 - 160*a^5*b^2*c^6 + 128*a^6*c^7)*e^3)*
sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c
^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^
2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b
^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 +
48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 -
8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3
 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*
c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^
2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b
^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (
a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c
^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 +
16*a^3*c^5)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^
2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*
b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^
6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 -
 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*
(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)
*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9
)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5))/((c^6*d^10 - 3*b*c^
5*d^9*e + 3*(b^2*c^4 - a*c^5)*d^8*e^2 - (b^3*c^3 - 16*a*b*c^4)*d^
7*e^3 - 14*(2*a*b^2*c^3 + a^2*c^4)*d^6*e^4 + 21*(a*b^3*c^2 + 2*a^
2*b*c^3)*d^5*e^5 - 7*(a*b^4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e^
```

6 + (a*b^5 + 17*a^2*b^3*c + 24*a^3*b*c^2)*d^3*e^7 - 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^2*e^8 + (3*a^3*b^3 + a^4*b*c)*d*e^9 - (a^4*b^2 - a^5*c)*e^10)*x + sqrt(1/2)*(c^6*d^10 - 3*b*c^5*d^9*e + 3*(b^2*c^4 - a*c^5)*d^8*e^2 - (b^3*c^3 - 16*a*b*c^4)*d^7*e^3 - 14*(2*a*b^2*c^3 + a^2*c^4)*d^6*e^4 + 21*(a*b^3*c^2 + 2*a^2*b*c^3)*d^5*e^5 - 7*(a*b^4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e^6 + (a*b^5 + 17*a^2*b^3*c + 24*a^3*b*c^2)*d^3*e^7 - 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^2*e^8 + (3*a^3*b^3 + a^4*b*c)*d*e^9 - (a^4*b^2 - a^5*c)*e^10)*sqrt((2*(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e^8)*x^2 - sqrt(1/2)*((b^3*c^4 - 4*a*b*c^5)*d^6 - 4*(a*b^2*c^4 - 4*a^2*c^5)*d^5*e - 5*(a*b^3*c^3 - 4*a^2*b*c^4)*d^4*e^2 + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^3 - (a*b^5*c + 17*a^2*b^3*c^2 - 84*a^3*b*c^3)*d^2*e^4 + 4*(2*a^2*b^4*c - 9*a^3*b^2*c^2 + 4*a^4*c^3)*d*e^5 - (a^2*b^5 - 5*a^3*b^3*c + 4*a^4*b*c^2)*e^6 - ((a*b^6*c^4 - 12*a^2*b^4*c^5 + 48*a^3*b^2*c^6 - 64*a^4*c^7)*d^2 - (a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6)*e^2)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))/(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e^8)))) + sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*arctan(-1/2*sqrt(1/2)*((b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d^7 - 9*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^5*e^2 + 5*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^4*e^3 - (a*b^6*c^2 - 27*a^2*b^4*c^3 + 168*a^3*b^2*c^4 - 304*a^4*c^5)*d^3*e^4 - 18*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^2*e^5 + (7*a^2*b^6*c - 59*a^3*b^4*c^2 + 136*a^4*b^2*c^3 - 48*a^5*c^4)*d*e^6 - (a^2*b^7 - 9*a^3*b^5*c + 24*a^4*b^3*c^2 - 16*a^5*b*c^3)*e^7 + ((a*b^7*c^5 - 12*a^2*b^5*c^6 + 48*a^3*b^3*c^7 - 64*a^4*b*c^8)*d^3 - 6*(a^2*b^6*c^5 - 12*a^3*b^4*c^6 + 48*a^4*b^2*c^7 - 64*a^5*c^8)*d^2*e + 3*(a^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5*b*c^7)*d*e^2 - (a^2*b^8*c^3 - 14*a^3*b^6*c^4 + 72*a^4*b^4*c^5 - 160*a^5*b^2*c^6 + 128*a^6*c^7)*e^3)*sqrt

```
((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d
^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 -
3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 -
 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a
^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*
c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (
a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)
*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*
c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c
^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*
b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 +
 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a
^3*c^5)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 -
4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*
c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^
2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*
a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2
*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8
)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/
(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5))/((c^6*d^10 - 3*b*c^5*d^
9*e + 3*(b^2*c^4 - a*c^5)*d^8*e^2 - (b^3*c^3 - 16*a*b*c^4)*d^7*e^
3 - 14*(2*a*b^2*c^3 + a^2*c^4)*d^6*e^4 + 21*(a*b^3*c^2 + 2*a^2*b*
c^3)*d^5*e^5 - 7*(a*b^4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e^6 +
(a*b^5 + 17*a^2*b^3*c + 24*a^3*b*c^2)*d^3*e^7 - 3*(a^2*b^4 + 4*a^
3*b^2*c + a^4*c^2)*d^2*e^8 + (3*a^3*b^3 + a^4*b*c)*d*e^9 - (a^4*b
^2 - a^5*c)*e^10)*x + sqrt(1/2)*(c^6*d^10 - 3*b*c^5*d^9*e + 3*(b^
2*c^4 - a*c^5)*d^8*e^2 - (b^3*c^3 - 16*a*b*c^4)*d^7*e^3 - 14*(2*a
*b^2*c^3 + a^2*c^4)*d^6*e^4 + 21*(a*b^3*c^2 + 2*a^2*b*c^3)*d^5*e^
5 - 7*(a*b^4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e^6 + (a*b^5 + 17
*a^2*b^3*c + 24*a^3*b*c^2)*d^3*e^7 - 3*(a^2*b^4 + 4*a^3*b^2*c + a
^4*c^2)*d^2*e^8 + (3*a^3*b^3 + a^4*b*c)*d*e^9 - (a^4*b^2 - a^5*c)
*e^10)*sqrt((2*(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*e^3 + (b
^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 +
 6*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2*c + 4*a^3
*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e
^8)*x^2 - sqrt(1/2)*((b^3*c^4 - 4*a*b*c^5)*d^6 - 4*(a*b^2*c^4 - 4
*a^2*c^5)*d^5*e - 5*(a*b^3*c^3 - 4*a^2*b*c^4)*d^4*e^2 + 4*(a*b^4*
c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^3 - (a*b^5*c + 17*a^2*b^3
*c^2 - 84*a^3*b*c^3)*d^2*e^4 + 4*(2*a^2*b^4*c - 9*a^3*b^2*c^2 + 4
*a^4*c^3)*d*e^5 - (a^2*b^5 - 5*a^3*b^3*c + 4*a^4*b*c^2)*e^6 + ((a
*b^6*c^4 - 12*a^2*b^4*c^5 + 48*a^3*b^2*c^6 - 64*a^4*c^7)*d^2 - (a
^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6)*e^2)*s
qrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^
3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2
 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^
4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 4
8*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e +
6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^
2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d
^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5
- 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c
^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*
b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*
c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))/(c
```

^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a*c^4)
*d^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a
^2*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^6 + 2
*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e^8)))) - 1/4*sqrt
(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 -
 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4
*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e
^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19
*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^
2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^
8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))
/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*log(1/2*sqrt(1/2)*((b
^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d^7 - 9*(a*b^4*c^4 - 8*a^2*b^2
*c^5 + 16*a^3*c^6)*d^5*e^2 + 5*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^
3*b*c^5)*d^4*e^3 - (a*b^6*c^2 - 27*a^2*b^4*c^3 + 168*a^3*b^2*c^4
- 304*a^4*c^5)*d^3*e^4 - 18*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4
*b*c^4)*d^2*e^5 + (7*a^2*b^6*c - 59*a^3*b^4*c^2 + 136*a^4*b^2*c^3
 - 48*a^5*c^4)*d*e^6 - (a^2*b^7 - 9*a^3*b^5*c + 24*a^4*b^3*c^2 -
16*a^5*b*c^3)*e^7 - ((a*b^7*c^5 - 12*a^2*b^5*c^6 + 48*a^3*b^3*c^7
 - 64*a^4*b*c^8)*d^3 - 6*(a^2*b^6*c^5 - 12*a^3*b^4*c^6 + 48*a^4*b
^2*c^7 - 64*a^5*c^8)*d^2*e + 3*(a^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48
*a^4*b^3*c^6 - 64*a^5*b*c^7)*d*e^2 - (a^2*b^8*c^3 - 14*a^3*b^6*c^
4 + 72*a^4*b^4*c^5 - 160*a^5*b^2*c^6 + 128*a^6*c^7)*e^3)*sqrt((c^
6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e
^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^
3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a
^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b
^2*c^8 - 64*a^5*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*
d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^
3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqr
t((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*
d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 -
 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4
- 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*
a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c
^5)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a
*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3
- 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 +
8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*
c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3
*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a
^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b
^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)) + (c^6*d^10 - 3*b*c^5*d^9*e
 + 3*(b^2*c^4 - a*c^5)*d^8*e^2 - (b^3*c^3 - 16*a*b*c^4)*d^7*e^3 -
 14*(2*a*b^2*c^3 + a^2*c^4)*d^6*e^4 + 21*(a*b^3*c^2 + 2*a^2*b*c^3
)*d^5*e^5 - 7*(a*b^4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e^6 + (a*
b^5 + 17*a^2*b^3*c + 24*a^3*b*c^2)*d^3*e^7 - 3*(a^2*b^4 + 4*a^3*b
^2*c + a^4*c^2)*d^2*e^8 + (3*a^3*b^3 + a^4*b*c)*d*e^9 - (a^4*b^2
- a^5*c)*e^10)*x) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3
*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b
^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sq
rt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3
*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2
- 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4

```
 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48
*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*
c^5)))*log(-1/2*sqrt(1/2)*((b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d
^7 - 9*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^5*e^2 + 5*(a*b^
5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^4*e^3 - (a*b^6*c^2 - 27*a
^2*b^4*c^3 + 168*a^3*b^2*c^4 - 304*a^4*c^5)*d^3*e^4 - 18*(a^2*b^5
*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^2*e^5 + (7*a^2*b^6*c - 59*
a^3*b^4*c^2 + 136*a^4*b^2*c^3 - 48*a^5*c^4)*d*e^6 - (a^2*b^7 - 9*
a^3*b^5*c + 24*a^4*b^3*c^2 - 16*a^5*b*c^3)*e^7 - ((a*b^7*c^5 - 12
*a^2*b^5*c^6 + 48*a^3*b^3*c^7 - 64*a^4*b*c^8)*d^3 - 6*(a^2*b^6*c^
5 - 12*a^3*b^4*c^6 + 48*a^4*b^2*c^7 - 64*a^5*c^8)*d^2*e + 3*(a^2*
b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5*b*c^7)*d*e^2 -
 (a^2*b^8*c^3 - 14*a^3*b^6*c^4 + 72*a^4*b^4*c^5 - 160*a^5*b^2*c^6
 + 128*a^6*c^7)*e^3)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4
*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*
e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*
b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^
6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(sqrt(1/2
)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2
*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*
a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*
b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)
*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c -
 a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b
^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c
^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3
*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 -
 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((
c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3
*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*
a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2
*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4
*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)
) + (c^6*d^10 - 3*b*c^5*d^9*e + 3*(b^2*c^4 - a*c^5)*d^8*e^2 - (b^
3*c^3 - 16*a*b*c^4)*d^7*e^3 - 14*(2*a*b^2*c^3 + a^2*c^4)*d^6*e^4
+ 21*(a*b^3*c^2 + 2*a^2*b*c^3)*d^5*e^5 - 7*(a*b^4*c + 6*a^2*b^2*c
^2 + 2*a^3*c^3)*d^4*e^6 + (a*b^5 + 17*a^2*b^3*c + 24*a^3*b*c^2)*d
^3*e^7 - 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^2*e^8 + (3*a^3*b^3
 + a^4*b*c)*d*e^9 - (a^4*b^2 - a^5*c)*e^10)*x) - 1/4*sqrt(sqrt(1/
2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^
2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^3 - 8
*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a
*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4
)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c
- a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*
b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*
c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*log(1/2*sqrt(1/2)*((b^4*c^5 -
 8*a*b^2*c^6 + 16*a^2*c^7)*d^7 - 9*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 1
6*a^3*c^6)*d^5*e^2 + 5*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)
*d^4*e^3 - (a*b^6*c^2 - 27*a^2*b^4*c^3 + 168*a^3*b^2*c^4 - 304*a^
4*c^5)*d^3*e^4 - 18*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*
d^2*e^5 + (7*a^2*b^6*c - 59*a^3*b^4*c^2 + 136*a^4*b^2*c^3 - 48*a^
5*c^4)*d*e^6 - (a^2*b^7 - 9*a^3*b^5*c + 24*a^4*b^3*c^2 - 16*a^5*b
*c^3)*e^7 + ((a*b^7*c^5 - 12*a^2*b^5*c^6 + 48*a^3*b^3*c^7 - 64*a^
```

```
4*b*c^8)*d^3 - 6*(a^2*b^6*c^5 - 12*a^3*b^4*c^6 + 48*a^4*b^2*c^7 -
 64*a^5*c^8)*d^2*e + 3*(a^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4*b^3
*c^6 - 64*a^5*b*c^7)*d*e^2 - (a^2*b^8*c^3 - 14*a^3*b^6*c^4 + 72*a
^4*b^4*c^5 - 160*a^5*b^2*c^6 + 128*a^6*c^7)*e^3)*sqrt((c^6*d^8 -
12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(
a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d
^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c
 + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 -
 64*a^5*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e +
6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^
2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d
^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5
- 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c
^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*
b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*
c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*sq
rt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c -
 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*
b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^
4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4
*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3
*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c
^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 -
 8*a^2*b^2*c^4 + 16*a^3*c^5)) + (c^6*d^10 - 3*b*c^5*d^9*e + 3*(b^
2*c^4 - a*c^5)*d^8*e^2 - (b^3*c^3 - 16*a*b*c^4)*d^7*e^3 - 14*(2*a
*b^2*c^3 + a^2*c^4)*d^6*e^4 + 21*(a*b^3*c^2 + 2*a^2*b*c^3)*d^5*e^
5 - 7*(a*b^4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e^6 + (a*b^5 + 17
*a^2*b^3*c + 24*a^3*b*c^2)*d^3*e^7 - 3*(a^2*b^4 + 4*a^3*b^2*c + a
^4*c^2)*d^2*e^8 + (3*a^3*b^3 + a^4*b*c)*d*e^9 - (a^4*b^2 - a^5*c)
*e^10)*x) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e +
 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a
^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*
d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5
 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*
c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3
*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2
*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*l
og(-1/2*sqrt(1/2)*((b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d^7 - 9*(
a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^5*e^2 + 5*(a*b^5*c^3 -
8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^4*e^3 - (a*b^6*c^2 - 27*a^2*b^4*c
^3 + 168*a^3*b^2*c^4 - 304*a^4*c^5)*d^3*e^4 - 18*(a^2*b^5*c^2 - 8
*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^2*e^5 + (7*a^2*b^6*c - 59*a^3*b^4*
c^2 + 136*a^4*b^2*c^3 - 48*a^5*c^4)*d*e^6 - (a^2*b^7 - 9*a^3*b^5*
c + 24*a^4*b^3*c^2 - 16*a^5*b*c^3)*e^7 + ((a*b^7*c^5 - 12*a^2*b^5
*c^6 + 48*a^3*b^3*c^7 - 64*a^4*b*c^8)*d^3 - 6*(a^2*b^6*c^5 - 12*a
^3*b^4*c^6 + 48*a^4*b^2*c^7 - 64*a^5*c^8)*d^2*e + 3*(a^2*b^7*c^4
- 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5*b*c^7)*d*e^2 - (a^2*b^
8*c^3 - 14*a^3*b^6*c^4 + 72*a^4*b^4*c^5 - 160*a^5*b^2*c^6 + 128*a
^6*c^7)*e^3)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3
 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*
(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d
*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a
^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(sqrt(1/2)*sqrt(-
(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a
```

```
^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*
c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^
5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4
 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c
^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 -
 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a
^2*b^2*c^4 + 16*a^3*c^5)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a
*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b
*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8
- 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2
*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)
*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2
*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8
 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)) + (c^6
*d^10 - 3*b*c^5*d^9*e + 3*(b^2*c^4 - a*c^5)*d^8*e^2 - (b^3*c^3 -
16*a*b*c^4)*d^7*e^3 - 14*(2*a*b^2*c^3 + a^2*c^4)*d^6*e^4 + 21*(a*
b^3*c^2 + 2*a^2*b*c^3)*d^5*e^5 - 7*(a*b^4*c + 6*a^2*b^2*c^2 + 2*a
^3*c^3)*d^4*e^6 + (a*b^5 + 17*a^2*b^3*c + 24*a^3*b*c^2)*d^3*e^7 -
 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^2*e^8 + (3*a^3*b^3 + a^4*b
*c)*d*e^9 - (a^4*b^2 - a^5*c)*e^10)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(e*x**4+d)/(c*x**8+b*x**4+a),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^4 + d)\,x^2}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)*x^2/(c*x^8 + b*x^4 + a),x, algorithm="giac")

[Out] integrate((e*x^4 + d)*x^2/(c*x^8 + b*x^4 + a), x)
```

3.46 $\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=184

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^
2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt
[b^2 - 4*a*c]]) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqr
t[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.420859, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```


```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^
2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt
[b^2 - 4*a*c]]) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqr
t[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 54.6371, size = 189, normalized size = 1.03

$$\frac{\sqrt{2}\left(be-2cd+e\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{4\sqrt{c}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt{2}\left(be-2cd-e\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{4\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] sqrt(2)*(b*e - 2*c*d + e*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c
)*x**2/sqrt(b + sqrt(-4*a*c + b**2)))/(4*sqrt(c)*sqrt(b + sqrt(-4
*a*c + b**2))*sqrt(-4*a*c + b**2)) - sqrt(2)*(b*e - 2*c*d - e*sqr
t(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x**2/sqrt(b - sqrt(-4*a*c
+ b**2)))/(4*sqrt(c)*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c +
b**2))
```

Mathematica [A] time = 0.285521, size = 179, normalized size = 0.97

$$\frac{\frac{\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] (((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^
2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((
-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqr
t[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [B] time = 0.023, size = 340, normalized size = 1.9

$$\frac{\sqrt{2}e}{4}\arctan\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}} + \frac{b\sqrt{2}e}{4}\arctan\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}} - \frac{c\sqrt{2}d}{2}\arctan\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}} - \frac{\sqrt{2}e}{4}Artanh\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}} + \frac{b\sqrt{2}e}{4}Artanh\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}} - \frac{c\sqrt{2}d}{2}Artanh\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e+1/4/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*b*e-1/2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*d-1/4*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e+1/
4/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e-1/2*c/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^4 + d)x}{cx^8 + bx^4 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)*x/(c*x^8 + b*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)*x/(c*x^8 + b*x^4 + a), x)
```

Fricas [A] time = 0.398143, size = 2072, normalized size = 11.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)*x/(c*x^8 + b*x^4 + a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4
*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 -
 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-(c^2*d^4 - b*c*d^3*e +
a*b*d*e^3 - a^2*e^4)*x^2 + 1/2*sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 -
 (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^
2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2
*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a
*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2
*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/4*sqrt(1/2)*s
qrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt(
(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(
a*b^2*c - 4*a^2*c^2))*log(-(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2
*e^4)*x^2 - 1/2*sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2
*c)*d*e^2 - ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2
)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^
3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*
c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^
3*c^3)))/(a*b^2*c - 4*a^2*c^2))) + 1/4*sqrt(1/2)*sqrt(-(b*c*d^2 -
 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*
c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2
*c^2))*log(-(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x^2 + 1/2
*sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 + ((a
*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*
d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-
(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*
d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2
*c - 4*a^2*c^2))) - 1/4*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*
b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2
```

```
*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-(c^
2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x^2 - 1/2*sqrt(1/2)*((b^
2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 + ((a*b^3*c - 4*a^2*
b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2
*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*
c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2
*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2)
))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.997629, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)*x/(c*x^8 + b*x^4 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.47 $\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=375

$$-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[Out] -((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b - Sqrt[
b^2 - 4*a*c])^(3/4)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/
4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - ((e - (2*c*d - b*e)/
Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4
*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))
 - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4
)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b + Sq
rt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 0.803978, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^4)/(a + b*x^4 + c*x^8),x]
```

```
[Out] -((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b - Sqrt[
b^2 - 4*a*c])^(3/4)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/
4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - ((e - (2*c*d - b*e)/
Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4
*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))
 - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4
)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b + Sq
rt[b^2 - 4*a*c])^(3/4))
```

Rubi in Sympy [A] time = 113.172, size = 379, normalized size = 1.01

$$\frac{2^{\frac{3}{4}}\left(be-2cd-e\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{4\sqrt[4]{c}\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}+\frac{2^{\frac{3}{4}}\left(be-2cd-e\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{4\sqrt[4]{c}\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}-\frac{2^{\frac{3}{4}}\left(be-2cd+e\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{4\sqrt[4]{c}\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}-\frac{2^{\frac{3}{4}}\left(be-2cd+e\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{4\sqrt[4]{c}\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] 2**(3/4)*(b*e - 2*c*d - e*sqrt(-4*a*c + b**2))*atan(2**(1/4)*c**(
1/4)*x/(-b + sqrt(-4*a*c + b**2))**(1/4))/(4*c**(1/4)*(-b + sqrt(
-4*a*c + b**2))**(3/4)*sqrt(-4*a*c + b**2)) + 2**(3/4)*(b*e - 2*c
*d - e*sqrt(-4*a*c + b**2))*atanh(2**(1/4)*c**(1/4)*x/(-b + sqrt(
-4*a*c + b**2))**(1/4))/(4*c**(1/4)*(-b + sqrt(-4*a*c + b**2))**(
3/4)*sqrt(-4*a*c + b**2)) - 2**(3/4)*(b*e - 2*c*d + e*sqrt(-4*a*c
```

```
 + b**2))*atan(2**(1/4)*c**(1/4)*x/(-b - sqrt(-4*a*c + b**2))**(1
/4))/(4*c**(1/4)*(-b - sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4*a*c +
b**2)) - 2**(3/4)*(b*e - 2*c*d + e*sqrt(-4*a*c + b**2))*atanh(2**
(1/4)*c**(1/4)*x/(-b - sqrt(-4*a*c + b**2))**(1/4))/(4*c**(1/4)*(
-b - sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0659693, size = 61, normalized size = 0.16

$$\frac{1}{4}\text{RootSum}\left[\#1^8 c + \#1^4 b + a\&, \frac{\#1^4 e \log(x - \#1) + d\log(x - \#1)}{2\#1^7 c + \#1^3 b}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^4)/(a + b*x^4 + c*x^8),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1
^4)/(b*#1^3 + 2*c*#1^7) & ]/4
```

Maple [C] time = 0.009, size = 47, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R = RootOf\left(c\_Z^8 + \_Z^4 b + a\right)} \frac{\left(\_R^4 e + d\right)\ln\left(x - \_R\right)}{2\_R^7 c + \_R^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^4+d)/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*sum((_R^4*e+d)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z
^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^4 + d}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)/(c*x^8 + b*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)/(c*x^8 + b*x^4 + a), x)
```

Fricas [A] time = 1.84214, size = 12872, normalized size = 34.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)/(c*x^8 + b*x^4 + a),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*
e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (
a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e
^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 -
2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*
(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*
d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*
c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))*arctan(1/2*((b^
4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^5 - 4*(a*b^3*c - 4*a^2*b*c^2)*d^
4*e + 6*(a^2*b^2*c - 4*a^3*c^2)*d^3*e^2 - (a^3*b^2 - 4*a^4*c)*d*e
^4 - ((a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d - 2*(a^4*b^4*c
 - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8
*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^
2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2
*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^
4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))
*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*
e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (
a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e
^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 -
2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*
(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*
d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*
c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/((10*a^2*b*c*d^
3*e^3 - 5*a^3*c*d^2*e^4 - a^3*b*d*e^5 + a^4*e^6 - (b^2*c^2 - a*c^
3)*d^6 + (b^3*c + 3*a*b*c^2)*d^5*e - 5*(a*b^2*c + a^2*c^2)*d^4*e^
2)*x + sqrt(1/2)*(10*a^2*b*c*d^3*e^3 - 5*a^3*c*d^2*e^4 - a^3*b*d*
e^5 + a^4*e^6 - (b^2*c^2 - a*c^3)*d^6 + (b^3*c + 3*a*b*c^2)*d^5*e
 - 5*(a*b^2*c + a^2*c^2)*d^4*e^2)*sqrt((2*(14*a^3*b*c*d^3*e^5 - 2
*a^4*b*d*e^7 + a^5*e^8 - (b^2*c^3 - a*c^4)*d^8 + 2*(b^3*c^2 + a*b
*c^3)*d^7*e - (b^4*c + 9*a*b^2*c^2 + 4*a^2*c^3)*d^6*e^2 + 6*(a*b^
3*c + 3*a^2*b*c^2)*d^5*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^4*e^4
+ (a^3*b^2 - 4*a^4*c)*d^2*e^6)*x^2 - sqrt(1/2)*((b^6*c - 7*a*b^4*
c^2 + 14*a^2*b^2*c^3 - 8*a^3*c^4)*d^6 - 2*(3*a*b^5*c - 17*a^2*b^3
*c^2 + 20*a^3*b*c^3)*d^5*e + 2*(8*a^2*b^4*c - 39*a^3*b^2*c^2 + 28
*a^4*c^3)*d^4*e^2 - 20*(a^3*b^3*c - 4*a^4*b*c^2)*d^3*e^3 - (a^3*b
^4 - 18*a^4*b^2*c + 56*a^5*c^2)*d^2*e^4 + 2*(a^4*b^3 - 4*a^5*b*c)
*d*e^5 - 2*(a^5*b^2 - 4*a^6*c)*e^6 - ((a^3*b^7*c - 12*a^4*b^5*c^2
 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^2 - 2*(a^4*b^6*c - 12*a^5*b^4
*c^2 + 48*a^6*b^2*c^3 - 64*a^7*c^4)*d*e)*sqrt(-(48*a^3*b*c^2*d^5*
```

e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(14*a^3*b*c*d^3*e^5 - 2*a^4*b*d*e^7 + a^5*e^8 - (b^2*c^3 - a*c^4)*d^8 + 2*(b^3*c^2 + a*b*c^3)*d^7*e - (b^4*c + 9*a*b^2*c^2 + 4*a^2*c^3)*d^6*e^2 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^5*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^4*e^4 + (a^3*b^2 - 4*a^4*c)*d^2*e^6)))) + sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))*arctan(-1/2*((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^5 - 4*(a*b^3*c - 4*a^2*b*c^2)*d^4*e + 6*(a^2*b^2*c - 4*a^3*c^2)*d^3*e^2 - (a^3*b^2 - 4*a^4*c)*d*e^4 + ((a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/((10*a^2*b*c*d^3*e^3 - 5*a^3*c*d^2*e^4 - a^3*b*d*e^5 + a^4*e^6 - (b^2*c^2 - a*c^3)*d^6 + (b^3*c + 3*a*b*c^2)*d^5*e - 5*(a*b^2*c + a^2*c^2)*d^4*e^2)*x + sqrt(1/2)*(10*a^2*b*c*d^3*e^3 - 5*a^3*c*d^2*e^4 - a^3*b*d*e^5 + a^4*e^6 - (b^2*c^2 - a*c^3)*d^6 + (b^3*c + 3*a*b*c^2)*d^5*e - 5*(a*b^2*c + a^2*c^2)*d^4*e^2)*sqrt((2*(14*a^3*b*c*d^3*e^5 - 2*a^4*b*d*e^7 + a^5*e^8 - (b^2*c^3 - a*c^4)*d^8 + 2*(b^3*c^2 + a*b*c^3)*d^7*e - (b^4*c + 9*a*b^2*c^2 + 4*a^2*c^3)*d^6*e^2 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^5*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^4*e^4 + (a^3*b^2 - 4*a^4*c)*d^2*e^6)*x^2 - sqrt(1/2)*((b^6*c - 7*a*b^4*c^2 + 14*a^2*b^2*c^3 - 8*a^3*c^4)*d^6 - 2*(3*a*b^5*c - 17*a^2*b^3*c^2 + 20*a^3*b*c^3)*d^5*e + 2*(8*a^2*b^4*c - 39*a^3*b^2*c^2 + 28*a^4*c^3)*d^4*e^2 - 20*(a^3*b^3*c - 4*a^4*b*c^2)*d^3*e^3 - (a^3*b^4 - 18*a^4*b^2*c + 56*a^5*c^2)*d^2*e^4 + 2*(a^4*b^3 - 4*a^5*b*c)*d*e^5 - 2*(a^5*b^2 - 4*a^6*c)*e^6 + ((a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*

c^4)*d^2 - 2*(a^4*b^6*c - 12*a^5*b^4*c^2 + 48*a^6*b^2*c^3 - 64*a^
7*c^4)*d*e)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*
a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 +
 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*
d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a
^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(-(6*a^2*b*c*d^2*
e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*
b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^
3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2
*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3
*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 +
 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^
3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 1
6*a^5*c^3)))/(14*a^3*b*c*d^3*e^5 - 2*a^4*b*d*e^7 + a^5*e^8 - (b^2
*c^3 - a*c^4)*d^8 + 2*(b^3*c^2 + a*b*c^3)*d^7*e - (b^4*c + 9*a*b^
2*c^2 + 4*a^2*c^3)*d^6*e^2 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^5*e^3 -
5*(3*a^2*b^2*c + 2*a^3*c^2)*d^4*e^4 + (a^3*b^2 - 4*a^4*c)*d^2*e^6
)))) + 1/4*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^
3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)
*d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b
*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (
b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*
d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19
*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4
 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))*log((
10*a^2*b*c*d^3*e^3 - 5*a^3*c*d^2*e^4 - a^3*b*d*e^5 + a^4*e^6 - (b
^2*c^2 - a*c^3)*d^6 + (b^3*c + 3*a*b*c^2)*d^5*e - 5*(a*b^2*c + a^
2*c^2)*d^4*e^2)*x + 1/2*((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^5 -
4*(a*b^3*c - 4*a^2*b*c^2)*d^4*e + 6*(a^2*b^2*c - 4*a^3*c^2)*d^3*e
^2 - (a^3*b^2 - 4*a^4*c)*d*e^4 - ((a^3*b^5*c - 8*a^4*b^3*c^2 + 16
*a^5*b*c^3)*d - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e)*sqr
t(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 -
 a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 -
 a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^
3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48
*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*
e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*
b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^
3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2
*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3
*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 +
 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^
3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 1
6*a^5*c^3)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a
^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2
*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-
(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^
6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^
2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b
^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^
8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3
)))*log((10*a^2*b*c*d^3*e^3 - 5*a^3*c*d^2*e^4 - a^3*b*d*e^5 + a^4
*e^6 - (b^2*c^2 - a*c^3)*d^6 + (b^3*c + 3*a*b*c^2)*d^5*e - 5*(a*b
^2*c + a^2*c^2)*d^4*e^2)*x - 1/2*((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^

```
3)*d^5 - 4*(a*b^3*c - 4*a^2*b*c^2)*d^4*e + 6*(a^2*b^2*c - 4*a^3*c
^2)*d^3*e^2 - (a^3*b^2 - 4*a^4*c)*d*e^4 - ((a^3*b^5*c - 8*a^4*b^3
*c^2 + 16*a^5*b*c^3)*d - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^
3)*e)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*
d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*
b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^
2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4
*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(sqrt(1/2)*sqrt(-(6*a^2
*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^
4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 +
16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*
a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 +
 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*
d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a
^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^
2*c^2 + 16*a^5*c^3)))) + 1/4*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*
e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*
b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^
3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2
*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3
*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 +
 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^
3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 1
6*a^5*c^3)))*log((10*a^2*b*c*d^3*e^3 - 5*a^3*c*d^2*e^4 - a^3*b*d*
e^5 + a^4*e^6 - (b^2*c^2 - a*c^3)*d^6 + (b^3*c + 3*a*b*c^2)*d^5*e
 - 5*(a*b^2*c + a^2*c^2)*d^4*e^2)*x + 1/2*((b^4*c - 5*a*b^2*c^2 +
 4*a^2*c^3)*d^5 - 4*(a*b^3*c - 4*a^2*b*c^2)*d^4*e + 6*(a^2*b^2*c
- 4*a^3*c^2)*d^3*e^2 - (a^3*b^2 - 4*a^4*c)*d*e^4 + ((a^3*b^5*c -
8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 +
16*a^6*c^3)*e)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 +
12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^
8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^
3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 1
2*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(sqrt(1/2)*sqr
t(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*
b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b
^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*
e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c
^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*
a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c
^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c -
 8*a^4*b^2*c^2 + 16*a^5*c^3)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-(6*a^2
*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^
4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b^2*c^2 +
16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*
a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 +
 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*
d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a
^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^
2*c^2 + 16*a^5*c^3)))*log((10*a^2*b*c*d^3*e^3 - 5*a^3*c*d^2*e^4 -
 a^3*b*d*e^5 + a^4*e^6 - (b^2*c^2 - a*c^3)*d^6 + (b^3*c + 3*a*b*c
^2)*d^5*e - 5*(a*b^2*c + a^2*c^2)*d^4*e^2)*x - 1/2*((b^4*c - 5*a*
b^2*c^2 + 4*a^2*c^3)*d^5 - 4*(a*b^3*c - 4*a^2*b*c^2)*d^4*e + 6*(a
^2*b^2*c - 4*a^3*c^2)*d^3*e^2 - (a^3*b^2 - 4*a^4*c)*d*e^4 + ((a^3
```

```
*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d - 2*(a^4*b^4*c - 8*a^5*b
^2*c^2 + 16*a^6*c^3)*e)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d
^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^
2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 -
 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^
6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(sqrt
(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3
*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*c
- 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4
*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^
3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2
*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(
a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^
3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3))))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^4 + d}{cx^8 + bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)/(c*x^8 + b*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^4 + d)/(c*x^8 + b*x^4 + a), x)
```

3.48 $\int \frac{d+ex^4}{x(a+bx^4+cx^8)} dx$

Optimal. Leaf size=78

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bx^4+cx^8\right)}{8a}+\frac{d\log(x)}{a}$$

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*a*Sqr
t[b^2 - 4*a*c]) + (d*Log[x])/a - (d*Log[a + b*x^4 + c*x^8])/(8*a)
```

Rubi [A] time = 0.277635, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bx^4+cx^8\right)}{8a}+\frac{d\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^4)/(x*(a + b*x^4 + c*x^8)),x]
```

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*a*Sqr
t[b^2 - 4*a*c]) + (d*Log[x])/a - (d*Log[a + b*x^4 + c*x^8])/(8*a)
```

Rubi in Sympy [A] time = 36.7427, size = 73, normalized size = 0.94

$$\frac{d\log\left(x^4\right)}{4a}-\frac{d\log\left(a+bx^4+cx^8\right)}{8a}-\frac{(2ae-bd)\operatorname{atanh}\left(\frac{b+2cx^4}{\sqrt{-4ac+b^2}}\right)}{4a\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**4+d)/x/(c*x**8+b*x**4+a),x)
```

```
[Out] d*log(x**4)/(4*a) - d*log(a + b*x**4 + c*x**8)/(8*a) - (2*a*e - b
*d)*atanh((b + 2*c*x**4)/sqrt(-4*a*c + b**2))/(4*a*sqrt(-4*a*c +
b**2))
```

Mathematica [C] time = 0.0567099, size = 80, normalized size = 1.03

$$\frac{d\log(x)}{a}-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{2\#1^4c+b}\&\right]}{4a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^4)/(x*(a + b*x^4 + c*x^8)),x]
```

```
[Out] (d*Log[x])/a - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] -
 a*e*Log[x - #1] + c*d*Log[x - #1]*#1^4)/(b + 2*c*#1^4) & ]/(4*a)
```

Maple [A] time = 0.009, size = 106, normalized size = 1.4

$$-\frac{d\ln\left(cx^8+bx^4+a\right)}{8\,a}+\frac{e}{2}\arctan\left((2\,cx^4+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}-\frac{bd}{4\,a}\arctan\left((2\,cx^4+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{d\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^4+d)/x/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/8*d*ln(c*x^8+b*x^4+a)/a+1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+
b)/(4*a*c-b^2)^(1/2))*e-1/4/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b
)/(4*a*c-b^2)^(1/2))*b*d+d*ln(x)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)/((c*x^8 + b*x^4 + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.861789, size = 1, normalized size = 0.01

$$\left[-\frac{(bd-2\,ae)\log\left(-\frac{2\,(b^2c-4\,ac^2)x^4+b^3-4\,abc-(2\,c^2x^8+2\,bcx^4+b^2-2\,ac)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)+\sqrt{b^2-4\,ac}\left(d\log\left(cx^8+bx^4+a\right)-8\,d\log\left(x\right)\right)}{8\,\sqrt{b^2-4\,ac}a},\; -\frac{2\,(bd-2\,ae)\arctan\left(-\frac{(2\,cx^4+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+\sqrt{-b^2+4\,ac}\left(d\log\left(cx^8+bx^4+a\right)-8\,d\log\left(x\right)\right)}{8\,\sqrt{-b^2+4\,ac}a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)/((c*x^8 + b*x^4 + a)*x),x, algorithm="fricas")
```

```
[Out] [-1/8*((b*d - 2*a*e)*log(-(2*(b^2*c - 4*a*c^2)*x^4 + b^3 - 4*a*b*
c - (2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x
^8 + b*x^4 + a)) + sqrt(b^2 - 4*a*c)*(d*log(c*x^8 + b*x^4 + a) -
8*d*log(x)))/(sqrt(b^2 - 4*a*c)*a), -1/8*(2*(b*d - 2*a*e)*arctan(
-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + sqrt(-b^2 + 4*
a*c)*(d*log(c*x^8 + b*x^4 + a) - 8*d*log(x)))/(sqrt(-b^2 + 4*a*c)
*a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**4+d)/x/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.282423, size = 105, normalized size = 1.35

$$-\frac{d\ln\left(cx^8+bx^4+a\right)}{8\,a}+\frac{d\ln\left(x^4\right)}{4\,a}-\frac{(bd-2\,ae)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)/((c*x^8 + b*x^4 + a)*x),x, algorithm="giac")
```

```
[Out] -1/8*d*ln(c*x^8 + b*x^4 + a)/a + 1/4*d*ln(x^4)/a - 1/4*(b*d - 2*a
*e)*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*
a)
```

3.49 $\int \frac{d+ex^4}{x^2(a+bx^4+cx^8)}\,dx$

Optimal. Leaf size=392

$$-\frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{d}{ax}$$

```
[Out] -(d/(a*x)) - (c^(1/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)
*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(d + (b*d - 2*a*e)/
Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*
a*c])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(
1/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/
4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sqrt[b^
2 - 4*a*c])^(1/4)) + (c^(1/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c
])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(
2*2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 1.45766, antiderivative size = 392, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{d}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -(d/(a*x)) - (c^(1/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)
*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(d + (b*d - 2*a*e)/
Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*
a*c])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(
1/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/
4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sqrt[b^
2 - 4*a*c])^(1/4)) + (c^(1/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c
])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(
2*2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi in Sympy [A] time = 160.037, size = 391, normalized size = 1.

$$\frac{\sqrt[4]{2}\sqrt[4]{c}\left(2ae-bd-d\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{4a\sqrt[4]{-b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}} - \frac{\sqrt[4]{2}\sqrt[4]{c}\left(2ae-bd-d\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{4a\sqrt[4]{-b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}} - \frac{\sqrt[4]{2}\sqrt[4]{c}\left(2ae-bd+d\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{4a\sqrt[4]{-b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}} + \frac{\sqrt[4]{2}\sqrt[4]{c}\left(2ae-bd+d\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{4a\sqrt[4]{-b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}} - \frac{d}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**4+d)/x**2/(c*x**8+b*x**4+a),x)
```

```
[Out] 2**(1/4)*c**(1/4)*(2*a*e - b*d - d*sqrt(-4*a*c + b**2))*atan(2**(
1/4)*c**(1/4)*x/(-b + sqrt(-4*a*c + b**2))**(1/4))/(4*a*(-b + sqr
t(-4*a*c + b**2))**(1/4)*sqrt(-4*a*c + b**2)) - 2**(1/4)*c**(1/4)
*(2*a*e - b*d - d*sqrt(-4*a*c + b**2))*atanh(2**(1/4)*c**(1/4)*x/
(-b + sqrt(-4*a*c + b**2))**(1/4))/(4*a*(-b + sqrt(-4*a*c + b**2)
)**(1/4)*sqrt(-4*a*c + b**2)) - 2**(1/4)*c**(1/4)*(2*a*e - b*d +
d*sqrt(-4*a*c + b**2))*atan(2**(1/4)*c**(1/4)*x/(-b - sqrt(-4*a*c
```

```
 + b**2))**(1/4))/(4*a*(-b - sqrt(-4*a*c + b**2))**(1/4)*sqrt(-4*
a*c + b**2)) + 2**(1/4)*c**(1/4)*(2*a*e - b*d + d*sqrt(-4*a*c + b
**2))*atanh(2**(1/4)*c**(1/4)*x/(-b - sqrt(-4*a*c + b**2))**(1/4)
)/(4*a*(-b - sqrt(-4*a*c + b**2))**(1/4)*sqrt(-4*a*c + b**2)) - d
/(a*x)
```

Mathematica [C] time = 0.0889316, size = 85, normalized size = 0.22

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{2\#1^5c+\#1b}\&\right]}{4a}-\frac{d}{ax}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -(d/(a*x)) - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a
*e*Log[x - #1] + c*d*Log[x - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(4*a
)
```

Maple [C] time = 0.01, size = 72, normalized size = 0.2

$$-\frac{1}{4a}\sum_{\_R=RootOf\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(\_R^6cd+(-ae+bd)\_R^2\right)\ln(x-\_R)}{2\_R^7c+\_R^3b}-\frac{d}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/4/a*sum((_R^6*c*d+(-a*e+b*d)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x-_R),
_R=RootOf(_Z^8*c+_Z^4*b+a))-d/a/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{cdx^6+(bd-ae)x^2}{cx^8+bx^4+a}\,dx}{a}-\frac{d}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4 + d)/((c*x^8 + b*x^4 + a)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((c*d*x^6 + (b*d - a*e)*x^2)/(c*x^8 + b*x^4 + a), x)/a
- d/(a*x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4 + d)/((c*x^8 + b*x^4 + a)*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/x**2/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^4 + d}{(cx^8 + bx^4 + a)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4 + d)/((c*x^8 + b*x^4 + a)*x^2),x, algorithm="giac")
```

```
[Out] integrate((e*x^4 + d)/((c*x^8 + b*x^4 + a)*x^2), x)
```

3.50 $\int \frac{d+ex^4}{x^3(a+bx^4+cx^8)}\,dx$

Optimal. Leaf size=199

$$-\frac{\sqrt{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{2ax^2}$$

```
[Out] -d/(2*a*x^2) - (Sqrt[c]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2
]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(d - (b*d - 2*a*e)/Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.696665, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16

$$-\frac{\sqrt{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -d/(2*a*x^2) - (Sqrt[c]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2
]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(d - (b*d - 2*a*e)/Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 84.805, size = 201, normalized size = 1.01

$$-\frac{\sqrt{2}\sqrt{c}\left(2ae-bd+d\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{4a\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\sqrt{c}\left(2ae-bd-d\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{4a\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{d}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**4+d)/x**3/(c*x**8+b*x**4+a),x)
```

```
[Out] -sqrt(2)*sqrt(c)*(2*a*e - b*d + d*sqrt(-4*a*c + b**2))*atan(sqrt(
2)*sqrt(c)*x**2/sqrt(b + sqrt(-4*a*c + b**2)))/(4*a*sqrt(b + sqrt
(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) + sqrt(2)*sqrt(c)*(2*a*e -
b*d - d*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x**2/sqrt(b - s
qrt(-4*a*c + b**2)))/(4*a*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a
*c + b**2)) - d/(2*a*x**2)
```

Mathematica [C] time = 0.0741951, size = 89, normalized size = 0.45

$$-\frac{\text{RootSum}\left[\#1^8 c+\#1^4 b+a\&,\frac{\#1^4 c d \log (x-\#1)-a e \log (x-\#1)+b d \log (x-\#1)}{2 \#1^6 c+\#1^2 b}\&\right]}{4 a}-\frac{d}{2 a x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -d/(2*a*x^2) - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] -
 a*e*Log[x - #1] + c*d*Log[x - #1]*#1^4)/(b*#1^2 + 2*c*#1^6) & ]/
(4*a)
```

Maple [B] time = 0.025, size = 365, normalized size = 1.8

$$-\frac{c\sqrt{2}d}{4a}\arctan\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$-\frac{c\sqrt{2}e}{2}\arctan\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$+\frac{c\sqrt{2}bd}{4a}\arctan\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$+\frac{c\sqrt{2}d}{4a}Artanh\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$
$$-\frac{c\sqrt{2}e}{2}Artanh\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$
$$+\frac{c\sqrt{2}bd}{4a}Artanh\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{d}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/4*c/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d-1/2*c/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e+1/4*c/a/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*b*d+1/4*c/a*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*d-1/2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*e+1/4*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*b*d-1/2*d/a/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{(cdx^4+bd-ae)x}{cx^8+bx^4+a}\,dx}{a} - \frac{d}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4 + d)/((c*x^8 + b*x^4 + a)*x^3),x, algorithm="maxima")
```

```
[Out] -integrate((c*d*x^4 + b*d - a*e)*x/(c*x^8 + b*x^4 + a), x)/a - 1/
2*d/(a*x^2)
```

Fricas [A] time = 0.834555, size = 3742, normalized size = 18.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4 + d)/((c*x^8 + b*x^4 + a)*x^3),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(1/2)*a*x^2*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(
a*b^2 - 2*a^2*c)*d*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 -
 a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*
d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3
*b^2 - 4*a^4*c))*log((3*a*b^2*c*d^2*e^2 - 3*a^2*b*c*d*e^3 + a^3*c
*e^4 + (b^2*c^2 - a*c^3)*d^4 - (b^3*c + a*b*c^2)*d^3*e)*x^2 + 1/2
*sqrt(1/2)*((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 13*a
^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 4*a^3*b*c)*d*e^2 - (a^
3*b^2 - 4*a^4*c)*e^3 - ((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*d - (
a^4*b^3 - 4*a^5*b*c)*e)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2
*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^
2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(a^2*b*e^2 + (b^3
 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + (a^3*b^2 - 4*a^4*c)*s
qrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 +
 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*
b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^2*sqrt(-(a
^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + (a^3*b
^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c +
 a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)
*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((3*a*b^2
*c*d^2*e^2 - 3*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^2 - a*c^3)*d^4
- (b^3*c + a*b*c^2)*d^3*e)*x^2 - 1/2*sqrt(1/2)*((b^5 - 5*a*b^3*c
+ 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 13*a^2*b^2*c + 4*a^3*c^2)*d^2*e +
 3*(a^2*b^3 - 4*a^3*b*c)*d*e^2 - (a^3*b^2 - 4*a^4*c)*e^3 - ((a^3*
b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5*b*c)*e)*sqrt(
-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(
a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2
```

```
- 4*a^7*c)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 -
2*a^2*c)*d*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4
 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e -
2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4
*a^4*c))) + sqrt(1/2)*a*x^2*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^
2 - 2*(a*b^2 - 2*a^2*c)*d*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*
d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^
2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)
))/(a^3*b^2 - 4*a^4*c))*log((3*a*b^2*c*d^2*e^2 - 3*a^2*b*c*d*e^3
+ a^3*c*e^4 + (b^2*c^2 - a*c^3)*d^4 - (b^3*c + a*b*c^2)*d^3*e)*x^
2 + 1/2*sqrt(1/2)*((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^3 - (3*a*b^4
 - 13*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 4*a^3*b*c)*d*e^
2 - (a^3*b^2 - 4*a^4*c)*e^3 + ((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2
)*d - (a^4*b^3 - 4*a^5*b*c)*e)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (
b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3
*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(a^2*b*e^2
 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - (a^3*b^2 - 4*a
^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2
)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2
)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^2*s
qrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e -
 (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*
b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 -
 a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((
3*a*b^2*c*d^2*e^2 - 3*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^2 - a*c^
3)*d^4 - (b^3*c + a*b*c^2)*d^3*e)*x^2 - 1/2*sqrt(1/2)*((b^5 - 5*a
*b^3*c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 13*a^2*b^2*c + 4*a^3*c^2)*
d^2*e + 3*(a^2*b^3 - 4*a^3*b*c)*d*e^2 - (a^3*b^2 - 4*a^4*c)*e^3 +
 ((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5*b*c)*e
)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^
4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a
^6*b^2 - 4*a^7*c)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a
*b^2 - 2*a^2*c)*d*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 -
a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d
^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*
b^2 - 4*a^4*c))) - 2*d)/(a*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**4+d)/x**3/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.495389, size = 1, normalized size = 0.01

$$Done$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)/((c*x^8 + b*x^4 + a)*x^3),x, algorithm="giac")
```

```
[Out] Done
```

3.51 $\int \frac{d+ex^4}{x^4(a+bx^4+cx^8)}\,dx$

Optimal. Leaf size=394

$$\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$
$$+\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$
$$+\frac{c^{3/4}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{d}{3ax^3}$$

```
[Out] -d/(3*a*x^3) + (c^(3/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/
4)*a*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(d + (b*d - 2*a*e
)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 -
4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c
^(3/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(
1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b - Sqrt[
b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a
*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])
/(2*2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 1.38581, antiderivative size = 394, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{d}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -d/(3*a*x^3) + (c^(3/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/
4)*a*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(d + (b*d - 2*a*e
)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 -
4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c
^(3/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(
1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b - Sqrt[
b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a
*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])
/(2*2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi in Sympy [A] time = 153.716, size = 394, normalized size = 1.

$$-\frac{2^{\frac{3}{4}}c^{\frac{3}{4}}\left(2ae-bd-d\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{4a\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}-\frac{2^{\frac{3}{4}}c^{\frac{3}{4}}\left(2ae-bd-d\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{4a\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}+\frac{2^{\frac{3}{4}}c^{\frac{3}{4}}\left(2ae-bd+d\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{4a\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}+\frac{2^{\frac{3}{4}}c^{\frac{3}{4}}\left(2ae-bd+d\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{4a\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}-\frac{d}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((e*x**4+d)/x**4/(c*x**8+b*x**4+a),x)
```

```
[Out] -2**(3/4)*c**(3/4)*(2*a*e - b*d - d*sqrt(-4*a*c + b**2))*atan(2**
(1/4)*c**(1/4)*x/(-b + sqrt(-4*a*c + b**2))**(1/4))/(4*a*(-b + sq
rt(-4*a*c + b**2))**(3/4)*sqrt(-4*a*c + b**2)) - 2**(3/4)*c**(3/4
)*(2*a*e - b*d - d*sqrt(-4*a*c + b**2))*atanh(2**(1/4)*c**(1/4)*x
/(-b + sqrt(-4*a*c + b**2))**(1/4))/(4*a*(-b + sqrt(-4*a*c + b**2
))**(3/4)*sqrt(-4*a*c + b**2)) + 2**(3/4)*c**(3/4)*(2*a*e - b*d +
 d*sqrt(-4*a*c + b**2))*atan(2**(1/4)*c**(1/4)*x/(-b - sqrt(-4*a*
c + b**2))**(1/4))/(4*a*(-b - sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4
*a*c + b**2)) + 2**(3/4)*c**(3/4)*(2*a*e - b*d + d*sqrt(-4*a*c +
b**2))*atanh(2**(1/4)*c**(1/4)*x/(-b - sqrt(-4*a*c + b**2))**(1/4
))/(4*a*(-b - sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4*a*c + b**2)) -
d/(3*a*x**3)
```

Mathematica [C] time = 0.0993131, size = 86, normalized size = 0.22

$$-\frac{3\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{2\#1^7c+\#1^3b}\&\right]+\frac{4d}{x^3}}{12a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -((4*d)/x^3 + 3*RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1]
- a*e*Log[x - #1] + c*d*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]
)/(12*a)
```

Maple [C] time = 0.009, size = 68, normalized size = 0.2

$$-\frac{d}{3\,ax^{3}}+\frac{1}{4\,a}\sum_{\_R=RootOf\left(c\_Z^{8}+\_Z^{4}b+a\right)}\frac{\left(-cd\_R^{4}+ae-bd\right)\ln\left(x-\_R\right)}{2\_R^{7}c+\_R^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/3*d/a/x^3+1/4/a*sum((-_R^4*c*d+a*e-b*d)/(2*_R^7*c+_R^3*b)*ln(x
-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{cdx^{4}+bd-ae}{cx^{8}+bx^{4}+a}\,dx}{a}-\frac{d}{3\,ax^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)/((c*x^8 + b*x^4 + a)*x^4),x, algorithm="maxima")
```

```
[Out] -integrate((c*d*x^4 + b*d - a*e)/(c*x^8 + b*x^4 + a), x)/a - 1/3*
d/(a*x^3)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)/((c*x^8 + b*x^4 + a)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x**4+d)/x**4/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^4 + d}{(cx^8 + bx^4 + a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^4 + d)/((c*x^8 + b*x^4 + a)*x^4),x, algorithm="giac")
```

```
[Out] integrate((e*x^4 + d)/((c*x^8 + b*x^4 + a)*x^4), x)
```

3.52 $\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx$

Optimal. Leaf size=278

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-x-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

```
[Out] -x - ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[
6]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt
[6]) + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqr
t[6]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sq
rt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 +
 Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 - Sqrt[2 + Sqrt[3
]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*S
qrt[6])
```

Rubi [A] time = 0.662404, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-x-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^4*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -x - ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[
6]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt
[6]) + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqr
t[6]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sq
```

```
rt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 +
 Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 - Sqrt[2 + Sqrt[3
]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*S
qrt[6])
```

Rubi in Sympy [A] time = 80.625, size = 530, normalized size = 1.91

$$-x+\frac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}\right)\log\left(x^2-x\sqrt{-\sqrt{3}+2}+1\right)}{12\sqrt{-\sqrt{3}+2}}-\frac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}\right)\log\left(x^2+x\sqrt{-\sqrt{3}+2}+1\right)}{12\sqrt{-\sqrt{3}+2}}$$
$$-\frac{\sqrt{3}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)\log\left(x^2-x\sqrt{\sqrt{3}+2}+1\right)}{12\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)\log\left(x^2+x\sqrt{\sqrt{3}+2}+1\right)}{12\sqrt{\sqrt{3}+2}}$$
$$+\frac{\sqrt{3}\left(-\frac{\left(1+\sqrt{3}\right)\sqrt{\sqrt{3}+2}}{2}+\sqrt{3}\sqrt{\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}$$
$$+\frac{\sqrt{3}\left(-\frac{\left(1+\sqrt{3}\right)\sqrt{\sqrt{3}+2}}{2}+\sqrt{3}\sqrt{\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}$$
$$+\frac{\sqrt{3}\left(\frac{\left(-\sqrt{3}+1\right)\sqrt{-\sqrt{3}+2}}{2}+\sqrt{3}\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}$$
$$+\frac{\sqrt{3}\left(\frac{\left(-\sqrt{3}+1\right)\sqrt{-\sqrt{3}+2}}{2}+\sqrt{3}\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -x + sqrt(3)*(-sqrt(3)/2 + 1/2)*log(x**2 - x*sqrt(-sqrt(3) + 2) +
 1)/(12*sqrt(-sqrt(3) + 2)) - sqrt(3)*(-sqrt(3)/2 + 1/2)*log(x**2
 + x*sqrt(-sqrt(3) + 2) + 1)/(12*sqrt(-sqrt(3) + 2)) - sqrt(3)*(1
/2 + sqrt(3)/2)*log(x**2 - x*sqrt(sqrt(3) + 2) + 1)/(12*sqrt(sqrt
(3) + 2)) + sqrt(3)*(1/2 + sqrt(3)/2)*log(x**2 + x*sqrt(sqrt(3) +
 2) + 1)/(12*sqrt(sqrt(3) + 2)) + sqrt(3)*(-(1 + sqrt(3))*sqrt(sq
rt(3) + 2)/2 + sqrt(3)*sqrt(sqrt(3) + 2))*atan((2*x - sqrt(sqrt(3
) + 2))/sqrt(-sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) +
2)) + sqrt(3)*(-(1 + sqrt(3))*sqrt(sqrt(3) + 2)/2 + sqrt(3)*sqrt(
sqrt(3) + 2))*atan((2*x + sqrt(sqrt(3) + 2))/sqrt(-sqrt(3) + 2))/
(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2)) + sqrt(3)*((-sqrt(3) + 1
)*sqrt(-sqrt(3) + 2)/2 + sqrt(3)*sqrt(-sqrt(3) + 2))*atan((2*x -
sqrt(-sqrt(3) + 2))/sqrt(sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt
```

```
(sqrt(3) + 2)) + sqrt(3)*((-sqrt(3) + 1)*sqrt(-sqrt(3) + 2)/2 + s
qrt(3)*sqrt(-sqrt(3) + 2))*atan((2*x + sqrt(-sqrt(3) + 2))/sqrt(s
qrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2))
```

Mathematica [C] time = 0.0254188, size = 46, normalized size = 0.17

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\log(x-\#1)}{2\#1^7-\#1^3}\&\right]-x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^4*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -x + RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) & ]
/4
```

Maple [C] time = 0.01, size = 34, normalized size = 0.1

$$-x+\frac{\sum_{\_R=RootOf(9\_Z^4+1)}\_R\ln\left(3\_R^2+3\_Rx+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(-x^4+1)/(x^8-x^4+1),x)
```

```
[Out] -x+1/4*sum(_R*ln(3*_R^2+3*_R*x+x^2),_R=RootOf(9*_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-x+\int\frac{1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)*x^4/(x^8 - x^4 + 1),x, algorithm="maxima")
```

```
[Out] -x + integrate(1/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 0.263153, size = 261, normalized size = 0.94

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(\frac{\sqrt{3}\sqrt{2}x+2}{\sqrt{3}\sqrt{2}x+2\,x^2+2\,\sqrt{x^4+\sqrt{3}\sqrt{2}(x^3+x)+3\,x^2+1}}\right)-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}x-2}{\sqrt{3}\sqrt{2}x-2\,x^2-2\,\sqrt{x^4-\sqrt{3}\sqrt{2}(x^3+x)+3\,x^2+1}}\right)+\frac{1}{24}\sqrt{3}\sqrt{2}\log\left(x^4+\sqrt{3}\sqrt{2}(x^3+x)+3\,x^2+1\right)-\frac{1}{24}\sqrt{3}\sqrt{2}\log\left(x^4-\sqrt{3}\sqrt{2}(x^3+x)+3\,x^2+1\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)*x^4/(x^8 - x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*sqrt(2)*arctan((sqrt(3)*sqrt(2)*x + 2)/(sqrt(3)*sqrt
(2)*x + 2*x^2 + 2*sqrt(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 +
1))) - 1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*x - 2)/(sqrt(
3)*sqrt(2)*x - 2*x^2 - 2*sqrt(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3
*x^2 + 1))) + 1/24*sqrt(3)*sqrt(2)*log(x^4 + sqrt(3)*sqrt(2)*(x^3
 + x) + 3*x^2 + 1) - 1/24*sqrt(3)*sqrt(2)*log(x^4 - sqrt(3)*sqrt(
2)*(x^3 + x) + 3*x^2 + 1) - x
```

Sympy [A] time = 0.651772, size = 170, normalized size = 0.61

$$-x-\frac{\sqrt{6}\left(-2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)-2\operatorname{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24}-\frac{\sqrt{6}\left(-2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)-2\operatorname{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24}-\frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24}+\frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -x - sqrt(6)*(-2*atan(sqrt(6)*x/3 - 1/3) - 2*atan(sqrt(6)*x**3 -
4*x**2 + 2*sqrt(6)*x - 3))/24 - sqrt(6)*(-2*atan(sqrt(6)*x/3 + 1/
3) - 2*atan(sqrt(6)*x**3 + 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6
)*log(x**4 - sqrt(6)*x**3 + 3*x**2 - sqrt(6)*x + 1)/24 + sqrt(6)*
log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x + 1)/24
```

GIAC/XCAS [A] time = 0.278603, size = 281, normalized size = 1.01

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\sqrt{6}\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)*x^4/(x^8 - x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))
) + 1/12*sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/12*sqrt(6)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) -
sqrt(2))) + 1/12*sqrt(6)*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6
) - sqrt(2))) + 1/24*sqrt(6)*ln(x^2 + 1/2*x*(sqrt(6) + sqrt(2)) +
 1) - 1/24*sqrt(6)*ln(x^2 - 1/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/24
*sqrt(6)*ln(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*sqrt(6)*l
n(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - x
```

3.53 $\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8-x^4+1\right)$$

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x^4 + x^8]/8
```

Rubi [A] time = 0.0923199, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^3*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x^4 + x^8]/8
```

Rubi in Sympy [A] time = 14.0539, size = 34, normalized size = 0.87

$$-\frac{\log\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^4}{3}-\frac{1}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(sqrt(3)*(2*x**4/3 - 1/3))/
12
```

Mathematica [A] time = 0.0199397, size = 39, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{2x^4-1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^3*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x^4 + x^8]/8
```

Maple [A] time = 0.005, size = 33, normalized size = 0.9

$$-\frac{\ln\left(x^{8}-x^{4}+1\right)}{8}+\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2x^{4}-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(-x^4+1)/(x^8-x^4+1),x)
```

```
[Out] -1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Maxima [A] time = 0.825658, size = 43, normalized size = 1.1

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{4}-1\right)\right)-\frac{1}{8}\log\left(x^{8}-x^{4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)*x^3/(x^8 - x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4
+ 1)
```

Fricas [A] time = 0.253155, size = 49, normalized size = 1.26

$$-\frac{1}{24}\sqrt{3}\left(\sqrt{3}\log\left(x^{8}-x^{4}+1\right)-2\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{4}-1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)*x^3/(x^8 - x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(3)*(sqrt(3)*log(x^8 - x^4 + 1) - 2*arctan(1/3*sqrt(3)*
(2*x^4 - 1)))
```

Sympy [A] time = 0.329236, size = 37, normalized size = 0.95

$$-\frac{\log\left(x^8 - x^4 + 1\right)}{8} + \frac{\sqrt{3}\,\text{atan}\left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)
/3)/12
```

GIAC/XCAS [A] time = 0.273112, size = 43, normalized size = 1.1

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4 - 1\right)\right) - \frac{1}{8}\ln\left(x^8 - x^4 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)*x^3/(x^8 - x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*ln(x^8 - x^4 +
 1)
```

3.54 $\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx$

Optimal. Leaf size=355

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

```
[Out] ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
- Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]
]/(4*Sqrt[3*(2 + Sqrt[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sq
rt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqr
t[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])]) + (Sqrt
[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[(
2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[(2
+ Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 +
Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rubi [A] time = 0.716673, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
- Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]
]/(4*Sqrt[3*(2 + Sqrt[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sq
rt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqr
```

```
t[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])]) + (Sqrt
[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[(
2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[(2
+ Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 +
Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rubi in Sympy [A] time = 102.214, size = 481, normalized size = 1.35

$$\frac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}+1\right)\log\left(x^{2}-x\sqrt{-\sqrt{3}+2}+1\right)}{12\sqrt{-\sqrt{3}+2}}-\frac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}+1\right)\log\left(x^{2}+x\sqrt{-\sqrt{3}+2}+1\right)}{12\sqrt{-\sqrt{3}+2}}-\frac{\sqrt{3}\left(\frac{\sqrt{3}}{2}+1\right)\log\left(x^{2}-x\sqrt{\sqrt{3}+2}+1\right)}{12\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(\frac{\sqrt{3}}{2}+1\right)\log\left(x^{2}+x\sqrt{\sqrt{3}+2}+1\right)}{12\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(-\frac{\left(\sqrt{3}+2\right)^{\frac{3}{2}}}{2}+2\sqrt{\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(-\frac{\left(\sqrt{3}+2\right)^{\frac{3}{2}}}{2}+2\sqrt{\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}-\frac{\sqrt{3}\left(-\frac{\left(-\sqrt{3}+2\right)^{\frac{3}{2}}}{2}+2\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}-\frac{\sqrt{3}\left(-\frac{\left(-\sqrt{3}+2\right)^{\frac{3}{2}}}{2}+2\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] sqrt(3)*(-sqrt(3)/2 + 1)*log(x**2 - x*sqrt(-sqrt(3) + 2) + 1)/(12
*sqrt(-sqrt(3) + 2)) - sqrt(3)*(-sqrt(3)/2 + 1)*log(x**2 + x*sqrt
(-sqrt(3) + 2) + 1)/(12*sqrt(-sqrt(3) + 2)) - sqrt(3)*(sqrt(3)/2
+ 1)*log(x**2 - x*sqrt(sqrt(3) + 2) + 1)/(12*sqrt(sqrt(3) + 2)) +
 sqrt(3)*(sqrt(3)/2 + 1)*log(x**2 + x*sqrt(sqrt(3) + 2) + 1)/(12*
sqrt(sqrt(3) + 2)) + sqrt(3)*(-(sqrt(3) + 2)**(3/2)/2 + 2*sqrt(sq
rt(3) + 2))*atan((2*x - sqrt(sqrt(3) + 2))/sqrt(-sqrt(3) + 2))/(6
*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2)) + sqrt(3)*(-(sqrt(3) + 2)*
*(3/2)/2 + 2*sqrt(sqrt(3) + 2))*atan((2*x + sqrt(sqrt(3) + 2))/sq
rt(-sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2)) - sqrt
(3)*(-(-sqrt(3) + 2)**(3/2)/2 + 2*sqrt(-sqrt(3) + 2))*atan((2*x -
 sqrt(-sqrt(3) + 2))/sqrt(sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)*sqr
t(sqrt(3) + 2)) - sqrt(3)*(-(-sqrt(3) + 2)**(3/2)/2 + 2*sqrt(-sqr
t(3) + 2))*atan((2*x + sqrt(-sqrt(3) + 2))/sqrt(sqrt(3) + 2))/(6*
```

```
sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2))
```

Mathematica [C] time = 0.0240451, size = 55, normalized size = 0.15

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^5-\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-
#1 + 2*#1^5) & ]/4
```

Maple [C] time = 0.01, size = 46, normalized size = 0.1

$$-\frac{1}{4}\sum_{\_R=RootOf(\_Z^8-\_Z^4+1)}\frac{(\_R^6-\_R^2)\ln(x-\_R)}{2\_R^7-\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(-x^4+1)/(x^8-x^4+1),x)
```

```
[Out] -1/4*sum((_R^6-_R^2)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{(x^4-1)x^2}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)*x^2/(x^8 - x^4 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)*x^2/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 0.282052, size = 1223, normalized size = 3.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)*x^2/(x^8 - x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/24*(4*(7*sqrt(3) + 12)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*arc
tan(1/(2*(sqrt(3) - 2)*sqrt(-(97*x^2 - 56*sqrt(3)*(x^2 + 1) + (20
9*sqrt(3)*x - 362*x)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 97)/(5
6*sqrt(3) - 97))*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2*(sqrt(3)
*x - 2*x)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) - sqrt(3) + 2)) + 4
*(7*sqrt(3) + 12)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*arctan(1/(2
*(sqrt(3) - 2)*sqrt(-(97*x^2 - 56*sqrt(3)*(x^2 + 1) - (209*sqrt(3
)*x - 362*x)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 97)/(56*sqrt(3
) - 97))*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2*(sqrt(3)*x - 2*x
)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + sqrt(3) - 2)) - (2*sqrt(3
) + 3)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7))*log(194*x^2 + 112*sqrt
(3)*(x^2 + 1) + 2*(209*sqrt(3)*x + 362*x)*sqrt((sqrt(3) + 2)/(4*s
qrt(3) + 7)) + 194) + (2*sqrt(3) + 3)*sqrt((sqrt(3) - 2)/(4*sqrt(
3) - 7))*log(194*x^2 + 112*sqrt(3)*(x^2 + 1) - 2*(209*sqrt(3)*x +
 362*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 194) + (2*sqrt(3) +
 3)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*log(-194*x^2 + 112*sqrt(3
)*(x^2 + 1) + 2*(209*sqrt(3)*x - 362*x)*sqrt((sqrt(3) - 2)/(4*sqr
t(3) - 7)) - 194) - (2*sqrt(3) + 3)*sqrt((sqrt(3) + 2)/(4*sqrt(3)
 + 7))*log(-194*x^2 + 112*sqrt(3)*(x^2 + 1) - 2*(209*sqrt(3)*x -
362*x)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) - 194) + 4*sqrt(3)*sqr
t((sqrt(3) - 2)/(4*sqrt(3) - 7))*arctan(1/(2*(sqrt(3) + 2)*sqrt((
97*x^2 + 56*sqrt(3)*(x^2 + 1) + (209*sqrt(3)*x + 362*x)*sqrt((sqr
t(3) + 2)/(4*sqrt(3) + 7)) + 97)/(56*sqrt(3) + 97))*sqrt((sqrt(3)
 + 2)/(4*sqrt(3) + 7)) + 2*(sqrt(3)*x + 2*x)*sqrt((sqrt(3) + 2)/(
4*sqrt(3) + 7)) + sqrt(3) + 2)) + 4*sqrt(3)*sqrt((sqrt(3) - 2)/(4
*sqrt(3) - 7))*arctan(1/(2*(sqrt(3) + 2)*sqrt((97*x^2 + 56*sqrt(3
)*(x^2 + 1) - (209*sqrt(3)*x + 362*x)*sqrt((sqrt(3) + 2)/(4*sqrt(
3) + 7)) + 97)/(56*sqrt(3) + 97))*sqrt((sqrt(3) + 2)/(4*sqrt(3) +
 7)) + 2*(sqrt(3)*x + 2*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) -
sqrt(3) - 2)))/((sqrt(3) + 2)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))
*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)))
```

Sympy [A] time = 4.61781, size = 27, normalized size = 0.08

$$-\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(442368t^7 - 384t^3 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(442368
*_t**7 - 384*_t**3 + x)))
```

GIAC/XCAS [A] time = 0.283308, size = 342, normalized size = 0.96

$$-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)*x^2/(x^8 - x^4 + 1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqr
t(6) + sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(
6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*a
rctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1/24*(sqrt
(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt
(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6) + sqrt
(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*ln(x^2 - 1/2*x*(sqrt(6) +
sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6
) - sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*ln(x^2 - 1/2*x*(sq
rt(6) - sqrt(2)) + 1)
```

3.55 $\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx$

Optimal. Leaf size=50

$$\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{4\sqrt{3}}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{4\sqrt{3}}$$

```
[Out] -Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x
^4]/(4*Sqrt[3])
```

Rubi [A] time = 0.0802674, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{4\sqrt{3}}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x
^4]/(4*Sqrt[3])
```

Rubi in Sympy [A] time = 20.2581, size = 42, normalized size = 0.84

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{12}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 + sqrt(3)*log(x**4 + sqr
t(3)*x**2 + 1)/12
```

Mathematica [A] time = 0.0259039, size = 44, normalized size = 0.88

$$\frac{\log\left(x^4+\sqrt{3}x^2+1\right)-\log\left(-x^4+\sqrt{3}x^2-1\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] (-Log[-1 + Sqrt[3]*x^2 - x^4] + Log[1 + Sqrt[3]*x^2 + x^4])/(4*Sq
rt[3])
```

Maple [A] time = 0.015, size = 39, normalized size = 0.8

$$-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(-x^4+1)/(x^8-x^4+1),x)
```

```
[Out] -1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)+1/12*ln(1+x^4+x^2*3^(1/2))*3^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(x^4-1)\,x}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)*x/(x^8 - x^4 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 0.24828, size = 59, normalized size = 1.18

$$\frac{1}{12}\sqrt{3}\log\left(\frac{6\,x^6+6\,x^2+\sqrt{3}\left(x^8+5\,x^4+1\right)}{x^8-x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)*x/(x^8 - x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log((6*x^6 + 6*x^2 + sqrt(3)*(x^8 + 5*x^4 + 1))/(x^8
 - x^4 + 1))
```

Sympy [A] time = 0.288936, size = 42, normalized size = 0.84

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{12}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 + sqrt(3)*log(x**4 + sqr
t(3)*x**2 + 1)/12
```

GIAC/XCAS [A] time = 0.286236, size = 42, normalized size = 0.84

$$-\frac{1}{12}\sqrt{3}\ln\left(\frac{x^2-\sqrt{3}+\frac{1}{x^2}}{x^2+\sqrt{3}+\frac{1}{x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)*x/(x^8 - x^4 + 1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*ln((x^2 - sqrt(3) + 1/x^2)/(x^2 + sqrt(3) + 1/x^2))
```

3.56 $\int \frac{1-x^4}{1-x^4+x^8}\,dx$

Optimal. Leaf size=355

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
 - Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]
]]/(4*Sqrt[3*(2 + Sqrt[3])]) + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/S
qrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[(Sqrt[2 + Sq
rt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])]) + (Sqr
t[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[
(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[(2
 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 +
 Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rubi [A] time = 0.506497, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
 - Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]
]]/(4*Sqrt[3*(2 + Sqrt[3])]) + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/S
qrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[(Sqrt[2 + Sq
```

```
rt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])]) + (Sqr
t[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[
(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[(2
 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 +
 Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rubi in Sympy [A] time = 78.6325, size = 495, normalized size = 1.39

$$\frac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}+1\right)\log\left(x^2-x\sqrt{-\sqrt{3}+2}+1\right)}{12\sqrt{-\sqrt{3}+2}}-\frac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}+1\right)\log\left(x^2+x\sqrt{-\sqrt{3}+2}+1\right)}{12\sqrt{-\sqrt{3}+2}}-\frac{\sqrt{3}\left(\frac{\sqrt{3}}{2}+1\right)\log\left(x^2-x\sqrt{\sqrt{3}+2}+1\right)}{12\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(\frac{\sqrt{3}}{2}+1\right)\log\left(x^2+x\sqrt{\sqrt{3}+2}+1\right)}{12\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(-\frac{\left(\sqrt{3}+2\right)^{\frac{3}{2}}}{2}+\sqrt{3}\sqrt{\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(-\frac{\left(\sqrt{3}+2\right)^{\frac{3}{2}}}{2}+\sqrt{3}\sqrt{\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(\frac{\left(-\sqrt{3}+2\right)^{\frac{3}{2}}}{2}+\sqrt{3}\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(\frac{\left(-\sqrt{3}+2\right)^{\frac{3}{2}}}{2}+\sqrt{3}\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] sqrt(3)*(-sqrt(3)/2 + 1)*log(x**2 - x*sqrt(-sqrt(3) + 2) + 1)/(12
*sqrt(-sqrt(3) + 2)) - sqrt(3)*(-sqrt(3)/2 + 1)*log(x**2 + x*sqrt
(-sqrt(3) + 2) + 1)/(12*sqrt(-sqrt(3) + 2)) - sqrt(3)*(sqrt(3)/2
+ 1)*log(x**2 - x*sqrt(sqrt(3) + 2) + 1)/(12*sqrt(sqrt(3) + 2)) +
 sqrt(3)*(sqrt(3)/2 + 1)*log(x**2 + x*sqrt(sqrt(3) + 2) + 1)/(12*
sqrt(sqrt(3) + 2)) + sqrt(3)*(-(sqrt(3) + 2)**(3/2)/2 + sqrt(3)*s
qrt(sqrt(3) + 2))*atan((2*x - sqrt(sqrt(3) + 2))/sqrt(-sqrt(3) +
2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2)) + sqrt(3)*(-(sqrt(3)
 + 2)**(3/2)/2 + sqrt(3)*sqrt(sqrt(3) + 2))*atan((2*x + sqrt(sqrt
(3) + 2))/sqrt(-sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3)
+ 2)) + sqrt(3)*((-sqrt(3) + 2)**(3/2)/2 + sqrt(3)*sqrt(-sqrt(3)
+ 2))*atan((2*x - sqrt(-sqrt(3) + 2))/sqrt(sqrt(3) + 2))/(6*sqrt(
-sqrt(3) + 2)*sqrt(sqrt(3) + 2)) + sqrt(3)*((-sqrt(3) + 2)**(3/2)
/2 + sqrt(3)*sqrt(-sqrt(3) + 2))*atan((2*x + sqrt(-sqrt(3) + 2))/
sqrt(sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2))
```

Mathematica [C] time = 0.0213857, size = 57, normalized size = 0.16

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^7-\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-
#1^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.001, size = 44, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=RootOf(\_Z^8-\_Z^4+1)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+1)/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum((-_R^4+1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^4-1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 - x^4 + 1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 0.279543, size = 1223, normalized size = 3.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 - x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/24*(4*(7*sqrt(3) + 12)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*arct
an(1/(2*(sqrt(3) - 2)*sqrt(-(97*x^2 - 56*sqrt(3)*(x^2 + 1) + (209
*sqrt(3)*x - 362*x)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 97)/(56
*sqrt(3) - 97))*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2*(sqrt(3)*
x - 2*x)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) - sqrt(3) + 2)) + 4*
(7*sqrt(3) + 12)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*arctan(1/(2*
(sqrt(3) - 2)*sqrt(-(97*x^2 - 56*sqrt(3)*(x^2 + 1) - (209*sqrt(3)
*x - 362*x)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 97)/(56*sqrt(3)
 - 97))*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2*(sqrt(3)*x - 2*x)
*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + sqrt(3) - 2)) + (2*sqrt(3)
 + 3)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7))*log(194*x^2 + 112*sqrt(
3)*(x^2 + 1) + 2*(209*sqrt(3)*x + 362*x)*sqrt((sqrt(3) + 2)/(4*sq
rt(3) + 7)) + 194) - (2*sqrt(3) + 3)*sqrt((sqrt(3) - 2)/(4*sqrt(3
) - 7))*log(194*x^2 + 112*sqrt(3)*(x^2 + 1) - 2*(209*sqrt(3)*x +
362*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 194) - (2*sqrt(3) +
3)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*log(-194*x^2 + 112*sqrt(3)
*(x^2 + 1) + 2*(209*sqrt(3)*x - 362*x)*sqrt((sqrt(3) - 2)/(4*sqrt
(3) - 7)) - 194) + (2*sqrt(3) + 3)*sqrt((sqrt(3) + 2)/(4*sqrt(3)
+ 7))*log(-194*x^2 + 112*sqrt(3)*(x^2 + 1) - 2*(209*sqrt(3)*x - 3
62*x)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) - 194) + 4*sqrt(3)*sqrt
((sqrt(3) - 2)/(4*sqrt(3) - 7))*arctan(1/(2*(sqrt(3) + 2)*sqrt((9
7*x^2 + 56*sqrt(3)*(x^2 + 1) + (209*sqrt(3)*x + 362*x)*sqrt((sqrt
(3) + 2)/(4*sqrt(3) + 7)) + 97)/(56*sqrt(3) + 97))*sqrt((sqrt(3)
+ 2)/(4*sqrt(3) + 7)) + 2*(sqrt(3)*x + 2*x)*sqrt((sqrt(3) + 2)/(4
*sqrt(3) + 7)) + sqrt(3) + 2)) + 4*sqrt(3)*sqrt((sqrt(3) - 2)/(4*
sqrt(3) - 7))*arctan(1/(2*(sqrt(3) + 2)*sqrt((97*x^2 + 56*sqrt(3)
*(x^2 + 1) - (209*sqrt(3)*x + 362*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3
) + 7)) + 97)/(56*sqrt(3) + 97))*sqrt((sqrt(3) + 2)/(4*sqrt(3) +
7)) + 2*(sqrt(3)*x + 2*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) - s
qrt(3) - 2)))/((sqrt(3) + 2)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*
sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)))
```

Sympy [A] time = 4.64108, size = 26, normalized size = 0.07

$$-\operatorname{RootSum}\left(5308416t^{8}-2304t^{4}+1,\left(t\mapsto t\log\left(9216t^{5}-8t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(9216*_
t**5 - 8*_t + x)))
```

GIAC/XCAS [A] time = 0.305732, size = 342, normalized size = 0.96

$$\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)$$
$$+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)$$
$$+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)$$
$$+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/(x^8 - x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt
(6) + sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6
) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*ar
ctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(
6) - 3*sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(
2))) + 1/48*(sqrt(6) + 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6) + sqrt(
2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*ln(x^2 - 1/2*x*(sqrt(6) + s
qrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6)
 - sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*ln(x^2 - 1/2*x*(sqr
t(6) - sqrt(2)) + 1)
```

3.57 $\int \frac{1-x^4}{x(1-x^4+x^8)}\,dx$

Optimal. Leaf size=41

$$\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log(x)$$

```
[Out] ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 - x^4 +
x^8]/8
```

Rubi [A] time = 0.117419, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261

$$\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^4)/(x*(1 - x^4 + x^8)),x]
```

```
[Out] ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 - x^4 +
x^8]/8
```

Rubi in Sympy [A] time = 16.4536, size = 41, normalized size = 1.

$$\frac{\log\left(x^4\right)}{4}-\frac{\log\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^4}{3}-\frac{1}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+1)/x/(x**8-x**4+1),x)
```

```
[Out] log(x**4)/4 - log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(sqrt(3)*(2*x*
*4/3 - 1/3))/12
```

Mathematica [C] time = 0.0206338, size = 44, normalized size = 1.07

$$\log(x)-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)}{2\#1^4-1}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^4)/(x*(1 - x^4 + x^8)),x]
```

```
[Out] Log[x] - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^4)/(-1 + 2*#
1^4) & ]/4
```

Maple [A] time = 0.009, size = 35, normalized size = 0.9

$$\ln(x) - \frac{\ln(x^8 - x^4 + 1)}{8} - \frac{\sqrt{3}}{12}\arctan\left(\frac{(2x^4 - 1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+1)/x/(x^8-x^4+1),x)
```

```
[Out] ln(x)-1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2)
)
```

Maxima [A] time = 0.822459, size = 51, normalized size = 1.24

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^4 - 1)\right) - \frac{1}{8}\log(x^8 - x^4 + 1) + \frac{1}{4}\log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/((x^8 - x^4 + 1)*x),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4
 + 1) + 1/4*log(x^4)
```

Fricas [A] time = 0.251461, size = 58, normalized size = 1.41

$$-\frac{1}{24}\sqrt{3}\left(\sqrt{3}\log(x^8 - x^4 + 1) - 8\sqrt{3}\log(x) + 2\arctan\left(\frac{1}{3}\sqrt{3}(2x^4 - 1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/((x^8 - x^4 + 1)*x),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(3)*(sqrt(3)*log(x^8 - x^4 + 1) - 8*sqrt(3)*log(x) + 2*
arctan(1/3*sqrt(3)*(2*x^4 - 1)))
```

Sympy [A] time = 0.386296, size = 41, normalized size = 1.

$$\log(x) - \frac{\log(x^8 - x^4 + 1)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+1)/x/(x**8-x**4+1),x)
```

```
[Out] log(x) - log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 -
 sqrt(3)/3)/12
```

GIAC/XCAS [A] time = 0.302827, size = 51, normalized size = 1.24

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4 - 1\right)\right) - \frac{1}{8}\ln\left(x^8 - x^4 + 1\right) + \frac{1}{4}\ln\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/((x^8 - x^4 + 1)*x),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*ln(x^8 - x^4
+ 1) + 1/4*ln(x^4)
```

3.58 $\int \frac{1-x^4}{x^2(1-x^4+x^8)}\,dx$

Optimal. Leaf size=280

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

```
[Out] -x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*
Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2
*Sqrt[6]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(
2*Sqrt[6]) - ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/
(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Lo
g[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 - Sqrt[2 + S
qrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]
/(4*Sqrt[6])
```

Rubi [A] time = 0.535533, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^4)/(x^2*(1 - x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*
Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2
*Sqrt[6]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(
2*Sqrt[6]) - ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/
(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Lo
```

```
g[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 - Sqrt[2 + S
qrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]
/(4*Sqrt[6])
```

Rubi in Sympy [A] time = 84.4493, size = 512, normalized size = 1.83

$$\frac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}\right)\log\left(x^2-x\sqrt{-\sqrt{3}+2}+1\right)}{12\sqrt{-\sqrt{3}+2}}-\frac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}\right)\log\left(x^2+x\sqrt{-\sqrt{3}+2}+1\right)}{12\sqrt{-\sqrt{3}+2}}-\frac{\sqrt{3}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)\log\left(x^2-x\sqrt{\sqrt{3}+2}+1\right)}{12\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)\log\left(x^2+x\sqrt{\sqrt{3}+2}+1\right)}{12\sqrt{\sqrt{3}+2}}-\frac{\sqrt{3}\left(-\sqrt{\sqrt{3}+2}+\frac{\left(1+\sqrt{3}\right)\sqrt{\sqrt{3}+2}}{2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}-\frac{\sqrt{3}\left(-\sqrt{\sqrt{3}+2}+\frac{\left(1+\sqrt{3}\right)\sqrt{\sqrt{3}+2}}{2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}-\frac{\sqrt{3}\left(-\frac{\left(-\sqrt{3}+1\right)\sqrt{-\sqrt{3}+2}}{2}+\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x-\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}-\frac{\sqrt{3}\left(-\frac{\left(-\sqrt{3}+1\right)\sqrt{-\sqrt{3}+2}}{2}+\sqrt{-\sqrt{3}+2}\right)\operatorname{atan}\left(\frac{2x+\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}\sqrt{\sqrt{3}+2}}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+1)/x**2/(x**8-x**4+1),x)
```

```
[Out] sqrt(3)*(-sqrt(3)/2 + 1/2)*log(x**2 - x*sqrt(-sqrt(3) + 2) + 1)/(
12*sqrt(-sqrt(3) + 2)) - sqrt(3)*(-sqrt(3)/2 + 1/2)*log(x**2 + x*
sqrt(-sqrt(3) + 2) + 1)/(12*sqrt(-sqrt(3) + 2)) - sqrt(3)*(1/2 +
sqrt(3)/2)*log(x**2 - x*sqrt(sqrt(3) + 2) + 1)/(12*sqrt(sqrt(3) +
 2)) + sqrt(3)*(1/2 + sqrt(3)/2)*log(x**2 + x*sqrt(sqrt(3) + 2) +
 1)/(12*sqrt(sqrt(3) + 2)) - sqrt(3)*(-sqrt(sqrt(3) + 2) + (1 + s
qrt(3))*sqrt(sqrt(3) + 2)/2)*atan((2*x - sqrt(sqrt(3) + 2))/sqrt(
-sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2)) - sqrt(3)
*(-sqrt(sqrt(3) + 2) + (1 + sqrt(3))*sqrt(sqrt(3) + 2)/2)*atan((2
*x + sqrt(sqrt(3) + 2))/sqrt(-sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)
*sqrt(sqrt(3) + 2)) - sqrt(3)*(-(-sqrt(3) + 1)*sqrt(-sqrt(3) + 2)
/2 + sqrt(-sqrt(3) + 2))*atan((2*x - sqrt(-sqrt(3) + 2))/sqrt(sqr
t(3) + 2))/(6*sqrt(-sqrt(3) + 2)*sqrt(sqrt(3) + 2)) - sqrt(3)*(-(
-sqrt(3) + 1)*sqrt(-sqrt(3) + 2)/2 + sqrt(-sqrt(3) + 2))*atan((2*
```

```
x + sqrt(-sqrt(3) + 2))/sqrt(sqrt(3) + 2))/(6*sqrt(-sqrt(3) + 2)*
sqrt(sqrt(3) + 2)) - 1/x
```

Mathematica [C] time = 0.0233242, size = 47, normalized size = 0.17

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^4)/(x^2*(1 - x^4 + x^8)),x]
```

```
[Out] -x^(-1) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 + 2*
#1^4) & ]/4
```

Maple [C] time = 0.013, size = 38, normalized size = 0.1

$$-x^{-1}-\frac{\sum_{\_R=RootOf(9\_Z^4+1)}\_R\ln\left(9\_R^3x-3\_R^2+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+1)/x^2/(x^8-x^4+1),x)
```

```
[Out] -1/x-1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{x}-\int\frac{x^6}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/((x^8 - x^4 + 1)*x^2),x, algorithm="maxima")
```

```
[Out] -1/x - integrate(x^6/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 0.274038, size = 284, normalized size = 1.01

$$\frac{4\sqrt{3}\sqrt{2}x\arctan\left(\frac{\sqrt{3}\sqrt{2}+3x}{\sqrt{3}\sqrt{2}x^2+\sqrt{3}\sqrt{2}\sqrt{x^4+\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1}+3x}\right)-4\sqrt{3}\sqrt{2}x\arctan\left(\frac{\sqrt{3}\sqrt{2}-3x}{\sqrt{3}\sqrt{2}x^2+\sqrt{3}\sqrt{2}\sqrt{x^4-\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1}-3x}\right)+\sqrt{3}\sqrt{2}x}{24x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/((x^8 - x^4 + 1)*x^2),x, algorithm="fricas")
```

```
[Out] 1/24*(4*sqrt(3)*sqrt(2)*x*arctan((sqrt(3)*sqrt(2) + 3*x)/(sqrt(3)
*sqrt(2)*x^2 + sqrt(3)*sqrt(2)*sqrt(x^4 + sqrt(3)*sqrt(2)*(x^3 +
x) + 3*x^2 + 1) + 3*x)) - 4*sqrt(3)*sqrt(2)*x*arctan((sqrt(3)*sqr
t(2) - 3*x)/(sqrt(3)*sqrt(2)*x^2 + sqrt(3)*sqrt(2)*sqrt(x^4 - sqr
t(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1) - 3*x)) + sqrt(3)*sqrt(2)*x*l
og(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1) - sqrt(3)*sqrt(2)
*x*log(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1) - 24)/x
```

Sympy [A] time = 0.696164, size = 168, normalized size = 0.6

$$-\frac{\sqrt{6}\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24} - \frac{\sqrt{6}\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24} - \frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24} + \frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+1)/x**2/(x**8-x**4+1),x)
```

```
[Out] -sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**
2 + 2*sqrt(6)*x - 3))/24 - sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2
*atan(sqrt(6)*x**3 + 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(
x**4 - sqrt(6)*x**3 + 3*x**2 - sqrt(6)*x + 1)/24 + sqrt(6)*log(x*
*4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x + 1)/24 - 1/x
```

GIAC/XCAS [A] time = 0.312053, size = 284, normalized size = 1.01

$$-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\sqrt{6}\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/((x^8 - x^4 + 1)*x^2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2)
)) - 1/12*sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/12*sqrt(6)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) -
 sqrt(2))) - 1/12*sqrt(6)*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(
6) - sqrt(2))) + 1/24*sqrt(6)*ln(x^2 + 1/2*x*(sqrt(6) + sqrt(2))
+ 1) - 1/24*sqrt(6)*ln(x^2 - 1/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/2
4*sqrt(6)*ln(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*sqrt(6)*
ln(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/x
```

3.59 $\int \frac{1-x^4}{x^3(1-x^4+x^8)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2x^2}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}$$

```
[Out] -1/(2*x^2) + ArcTan[Sqrt[3] - 2*x^2]/4 - ArcTan[Sqrt[3] + 2*x^2]/
4 - Log[1 - Sqrt[3]*x^2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2
+ x^4]/(8*Sqrt[3])
```

Rubi [A] time = 0.188991, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348

$$-\frac{1}{2x^2}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^4)/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) + ArcTan[Sqrt[3] - 2*x^2]/4 - ArcTan[Sqrt[3] + 2*x^2]/
4 - Log[1 - Sqrt[3]*x^2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2
+ x^4]/(8*Sqrt[3])
```

Rubi in Sympy [A] time = 40.0168, size = 76, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24}-\frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}-\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+1)/x**3/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqr
t(3)*x**2 + 1)/24 - atan(2*x**2 - sqrt(3))/4 - atan(2*x**2 + sqrt
(3))/4 - 1/(2*x**2)
```

Mathematica [C] time = 0.0242982, size = 49, normalized size = 0.55

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^2\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^4)/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^2)/(-1 +
 2*#1^4) & ]/4
```

Maple [A] time = 0.007, size = 70, normalized size = 0.8

$$-\frac{1}{2\,x^2}-\frac{\arctan\left(2\,x^2-\sqrt{3}\right)}{4}-\frac{\arctan\left(2\,x^2+\sqrt{3}\right)}{4}-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{24}+\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+1)/x^3/(x^8-x^4+1),x)
```

```
[Out] -1/2/x^2-1/4*arctan(2*x^2-3^(1/2))-1/4*arctan(2*x^2+3^(1/2))-1/24
*ln(1+x^4-x^2*3^(1/2))*3^(1/2)+1/24*ln(1+x^4+x^2*3^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2\,x^2}-\int\frac{x^5}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/((x^8 - x^4 + 1)*x^3),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate(x^5/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 0.267569, size = 188, normalized size = 2.11

$$\frac{\sqrt{3}\left(4\sqrt{3}x^2\arctan\left(\frac{\sqrt{3}}{2\sqrt{3}x^2+2\sqrt{3}\sqrt{x^4+\sqrt{3}x^2+1}+3}\right)+4\sqrt{3}x^2\arctan\left(\frac{\sqrt{3}}{2\sqrt{3}x^2+2\sqrt{3}\sqrt{x^4-\sqrt{3}x^2+1}-3}\right)+x^2\log\left(x^4+\sqrt{3}x^2+1\right)-x^2\log\left(x\right.\right.}{24\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/((x^8 - x^4 + 1)*x^3),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(3)*(4*sqrt(3)*x^2*arctan(sqrt(3)/(2*sqrt(3)*x^2 + 2*sqr
t(3)*sqrt(x^4 + sqrt(3)*x^2 + 1) + 3)) + 4*sqrt(3)*x^2*arctan(sqr
t(3)/(2*sqrt(3)*x^2 + 2*sqrt(3)*sqrt(x^4 - sqrt(3)*x^2 + 1) - 3))
 + x^2*log(x^4 + sqrt(3)*x^2 + 1) - x^2*log(x^4 - sqrt(3)*x^2 + 1
) - 4*sqrt(3))/x^2
```

Sympy [A] time = 0.698227, size = 76, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24}-\frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}-\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+1)/x**3/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqr
t(3)*x**2 + 1)/24 - atan(2*x**2 - sqrt(3))/4 - atan(2*x**2 + sqrt
(3))/4 - 1/(2*x**2)
```

GIAC/XCAS [A] time = 0.348744, size = 348, normalized size = 3.91

$$-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)$$
$$-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)$$
$$-\frac{1}{96}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{96}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)$$
$$-\frac{1}{96}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)+\frac{1}{96}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/((x^8 - x^4 + 1)*x^3),x, algorithm="giac")
```

```
[Out] -1/48*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqr
t(6) + sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(
```

```
6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/48*(sqrt(6) + 3*sqrt(2))*a
rctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt
(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt
(2))) - 1/96*(sqrt(6) - 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6) + sqrt
(2)) + 1) + 1/96*(sqrt(6) - 3*sqrt(2))*ln(x^2 - 1/2*x*(sqrt(6) +
sqrt(2)) + 1) - 1/96*(sqrt(6) + 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6
) - sqrt(2)) + 1) + 1/96*(sqrt(6) + 3*sqrt(2))*ln(x^2 - 1/2*x*(sq
rt(6) - sqrt(2)) + 1) - 1/2/x^2
```

3.60 $\int \frac{1-x^4}{x^4(1-x^4+x^8)}\,dx$

Optimal. Leaf size=370

$$-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

```
[Out] -1/(3*x^3) - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2
*x)/Sqrt[2 + Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2
 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]
*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]])/4 - (Sqrt[(
2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]
]])/4 + (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]
)/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/
8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8
+ (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rubi [A] time = 0.612242, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391

$$-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^4)/(x^4*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2
*x)/Sqrt[2 + Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2
```

```
 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]
*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]])/4 - (Sqrt[(
2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]
]])/4 + (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]
)/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/
8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8
+ (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rubi in Sympy [A] time = 77.1026, size = 318, normalized size = 0.86

$$\frac{\sqrt{3}\log\left(x^2 - x\sqrt{-\sqrt{3}+2}+1\right)}{24\sqrt{-\sqrt{3}+2}} - \frac{\sqrt{3}\log\left(x^2 + x\sqrt{-\sqrt{3}+2}+1\right)}{24\sqrt{-\sqrt{3}+2}} - \frac{\sqrt{3}\log\left(x^2 - x\sqrt{\sqrt{3}+2}+1\right)}{24\sqrt{\sqrt{3}+2}} + \frac{\sqrt{3}\log\left(x^2 + x\sqrt{\sqrt{3}+2}+1\right)}{24\sqrt{\sqrt{3}+2}} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2x-\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{12\sqrt{-\sqrt{3}+2}} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2x+\sqrt{\sqrt{3}+2}}{\sqrt{-\sqrt{3}+2}}\right)}{12\sqrt{-\sqrt{3}+2}} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2x-\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{12\sqrt{\sqrt{3}+2}} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2x+\sqrt{-\sqrt{3}+2}}{\sqrt{\sqrt{3}+2}}\right)}{12\sqrt{\sqrt{3}+2}} - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4+1)/x**4/(x**8-x**4+1),x)
```

```
[Out] sqrt(3)*log(x**2 - x*sqrt(-sqrt(3) + 2) + 1)/(24*sqrt(-sqrt(3) +
2)) - sqrt(3)*log(x**2 + x*sqrt(-sqrt(3) + 2) + 1)/(24*sqrt(-sqrt
(3) + 2)) - sqrt(3)*log(x**2 - x*sqrt(sqrt(3) + 2) + 1)/(24*sqrt(
sqrt(3) + 2)) + sqrt(3)*log(x**2 + x*sqrt(sqrt(3) + 2) + 1)/(24*s
qrt(sqrt(3) + 2)) - sqrt(3)*atan((2*x - sqrt(sqrt(3) + 2))/sqrt(-
sqrt(3) + 2))/(12*sqrt(-sqrt(3) + 2)) - sqrt(3)*atan((2*x + sqrt(
sqrt(3) + 2))/sqrt(-sqrt(3) + 2))/(12*sqrt(-sqrt(3) + 2)) + sqrt(
3)*atan((2*x - sqrt(-sqrt(3) + 2))/sqrt(sqrt(3) + 2))/(12*sqrt(sq
rt(3) + 2)) + sqrt(3)*atan((2*x + sqrt(-sqrt(3) + 2))/sqrt(sqrt(3
) + 2))/(12*sqrt(sqrt(3) + 2)) - 1/(3*x**3)
```

Mathematica [C] time = 0.022147, size = 47, normalized size = 0.13

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^4)/(x^4*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1 + 2
*#1^4) & ]/4
```

Maple [C] time = 0.014, size = 46, normalized size = 0.1

$$-\frac{1}{3\,x^3}-\frac{1}{4}\sum_{\_R\,=RootOf(\_Z^8-\_Z^4+1)}\frac{\_R^4\ln(x-\_R)}{2\,\_R^7-\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4+1)/x^4/(x^8-x^4+1),x)
```

```
[Out] -1/3/x^3-1/4*sum(_R^4/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+
1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3\,x^3}-\int\frac{x^4}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/((x^8 - x^4 + 1)*x^4),x, algorithm="maxima")
```

```
[Out] -1/3/x^3 - integrate(x^4/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 0.29883, size = 1197, normalized size = 3.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/((x^8 - x^4 + 1)*x^4),x, algorithm="fricas")
```

```
[Out] -1/24*(4*sqrt(3)*x^3*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7))*arctan((
sqrt(3)*sqrt(2) + 2*sqrt(2))/(2*sqrt(2*x^2 + 2*x*sqrt((sqrt(3) +
2)/(4*sqrt(3) + 7)) + 2)*(sqrt(3) + 2)*sqrt((sqrt(3) + 2)/(4*sqrt
(3) + 7)) + 2*(sqrt(3)*sqrt(2)*x + 2*sqrt(2)*x)*sqrt((sqrt(3) + 2
)/(4*sqrt(3) + 7)) + sqrt(2))) + 4*sqrt(3)*x^3*sqrt((sqrt(3) - 2)
```

```
/(4*sqrt(3) - 7))*arctan((sqrt(3)*sqrt(2) + 2*sqrt(2))/(2*sqrt(2*
x^2 - 2*x*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 2)*(sqrt(3) + 2)*
sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 2*(sqrt(3)*sqrt(2)*x + 2*sq
rt(2)*x)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) - sqrt(2))) - 4*(7*s
qrt(3)*x^3 + 12*x^3)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*arctan((
sqrt(3)*sqrt(2) - 2*sqrt(2))/(2*sqrt(2*x^2 + 2*x*sqrt((sqrt(3) -
2)/(4*sqrt(3) - 7)) + 2)*(sqrt(3) - 2)*sqrt((sqrt(3) - 2)/(4*sqrt
(3) - 7)) + 2*(sqrt(3)*sqrt(2)*x - 2*sqrt(2)*x)*sqrt((sqrt(3) - 2
)/(4*sqrt(3) - 7)) - sqrt(2))) - 4*(7*sqrt(3)*x^3 + 12*x^3)*sqrt(
(sqrt(3) + 2)/(4*sqrt(3) + 7))*arctan((sqrt(3)*sqrt(2) - 2*sqrt(2
))/(2*sqrt(2*x^2 - 2*x*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2)*(
sqrt(3) - 2)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2*(sqrt(3)*sqr
t(2)*x - 2*sqrt(2)*x)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)) + sqrt(
2))) + (2*sqrt(3)*x^3 + 3*x^3)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)
)*log(2*x^2 + 2*x*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 2) - (2*s
qrt(3)*x^3 + 3*x^3)*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7))*log(2*x^2
 - 2*x*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7)) + 2) - (2*sqrt(3)*x^3
+ 3*x^3)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*log(2*x^2 + 2*x*sqrt
((sqrt(3) - 2)/(4*sqrt(3) - 7)) + 2) + (2*sqrt(3)*x^3 + 3*x^3)*sq
rt((sqrt(3) + 2)/(4*sqrt(3) + 7))*log(2*x^2 - 2*x*sqrt((sqrt(3) -
 2)/(4*sqrt(3) - 7)) + 2) + 8*(sqrt(3) + 2)*sqrt((sqrt(3) + 2)/(4
*sqrt(3) + 7))*sqrt((sqrt(3) - 2)/(4*sqrt(3) - 7)))/((sqrt(3)*x^3
 + 2*x^3)*sqrt((sqrt(3) + 2)/(4*sqrt(3) + 7))*sqrt((sqrt(3) - 2)/
(4*sqrt(3) - 7)))
```

Sympy [A] time = 4.68959, size = 32, normalized size = 0.09

$$-\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(-18432t^5 + 4t + x\right)\right)\right) - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4+1)/x**4/(x**8-x**4+1),x)
```

```
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-18432
*_t**5 + 4*_t + x))) - 1/(3*x**3)
```

GIAC/XCAS [A] time = 0.294462, size = 348, normalized size = 0.94

$$-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\ln\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^4 - 1)/((x^8 - x^4 + 1)*x^4),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqr
t(6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(
6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*a
rctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1/24*(sqrt
(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt
(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6) + sqrt
(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*ln(x^2 - 1/2*x*(sqrt(6) +
sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*ln(x^2 + 1/2*x*(sqrt(6
) - sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*ln(x^2 - 1/2*x*(sq
rt(6) - sqrt(2)) + 1) - 1/3/x^3
```

3.61 $\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal. Leaf size=280

$$-\frac{x^2(ad+be)}{2a^2e^2}+\frac{\left(a^2c^2d-3ab^2cd+2abc^2e+b^4d-b^3ce\right)\log\left(ax^2+bx+c\right)}{2a^4\left(ad^2-e(bd-ce)\right)}+\frac{\left(5a^2bc^2d-2a^2c^3e-5ab^3cd+4ab^2c^2e+b^5d-b^4ce\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{x\left(a^2d^2+ae(bd-ce)+b^2e^2\right)}{a^3e^3}-\frac{d^5\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)}+\frac{x^3}{3ae}$$

```
[Out] ((a^2*d^2 + b^2*e^2 + a*e*(b*d - c*e))*x)/(a^3*e^3) - ((a*d + b*e
)*x^2)/(2*a^2*e^2) + x^3/(3*a*e) + ((b^5*d - 5*a*b^3*c*d + 5*a^2*
b*c^2*d - b^4*c*e + 4*a*b^2*c^2*e - 2*a^2*c^3*e)*ArcTanh[(b + 2*a
*x)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d -
c*e))) - (d^5*Log[d + e*x])/(e^4*(a*d^2 - e*(b*d - c*e))) + ((b^4
*d - 3*a*b^2*c*d + a^2*c^2*d - b^3*c*e + 2*a*b*c^2*e)*Log[c + b*x
 + a*x^2])/(2*a^4*(a*d^2 - e*(b*d - c*e)))
```

Rubi [A] time = 1.20744, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24

$$-\frac{x^2(ad+be)}{2a^2e^2}+\frac{\left(a^2c^2d-3ab^2cd+2abc^2e+b^4d-b^3ce\right)\log\left(ax^2+bx+c\right)}{2a^4\left(ad^2-e(bd-ce)\right)}+\frac{\left(5a^2bc^2d-2a^2c^3e-5ab^3cd+4ab^2c^2e+b^5d-b^4ce\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{x\left(a^2d^2+ae(bd-ce)+b^2e^2\right)}{a^3e^3}-\frac{d^5\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)}+\frac{x^3}{3ae}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((a + c/x^2 + b/x)*(d + e*x)),x]
```


```
[Out] ((a^2*d^2 + b^2*e^2 + a*e*(b*d - c*e))*x)/(a^3*e^3) - ((a*d + b*e
)*x^2)/(2*a^2*e^2) + x^3/(3*a*e) + ((b^5*d - 5*a*b^3*c*d + 5*a^2*
b*c^2*d - b^4*c*e + 4*a*b^2*c^2*e - 2*a^2*c^3*e)*ArcTanh[(b + 2*a
*x)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d -
c*e))) - (d^5*Log[d + e*x])/(e^4*(a*d^2 - e*(b*d - c*e))) + ((b^4
*d - 3*a*b^2*c*d + a^2*c^2*d - b^3*c*e + 2*a*b*c^2*e)*Log[c + b*x
 + a*x^2])/(2*a^4*(a*d^2 - e*(b*d - c*e)))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.404622, size = 283, normalized size = 1.01

$$-\frac{x^2(ad+be)}{2a^2e^2}+\frac{\left(a^2c^2d-3ab^2cd+2abc^2e+b^4d-b^3ce\right)\log\left(ax^2+bx+c\right)}{2a^4\left(ad^2-bde+ce^2\right)}+\frac{\left(5a^2bc^2d-2a^2c^3e-5ab^3cd+4ab^2c^2e+b^5d-b^4ce\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a^4\sqrt{4ac-b^2}\left(-ad^2+bde-ce^2\right)}+\frac{x\left(a^2d^2+abde-ace^2+b^2e^2\right)}{a^3e^3}-\frac{d^5\log(d+ex)}{e^4\left(ad^2-bde+ce^2\right)}+\frac{x^3}{3ae}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] ((a^2*d^2 + a*b*d*e + b^2*e^2 - a*c*e^2)*x)/(a^3*e^3) - ((a*d + b
*e)*x^2)/(2*a^2*e^2) + x^3/(3*a*e) + ((b^5*d - 5*a*b^3*c*d + 5*a^
2*b*c^2*d - b^4*c*e + 4*a*b^2*c^2*e - 2*a^2*c^3*e)*ArcTan[(b + 2*
a*x)/Sqrt[-b^2 + 4*a*c]])/(a^4*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + b*d
*e - c*e^2)) - (d^5*Log[d + e*x])/(e^4*(a*d^2 - b*d*e + c*e^2)) +
 ((b^4*d - 3*a*b^2*c*d + a^2*c^2*d - b^3*c*e + 2*a*b*c^2*e)*Log[c
 + b*x + a*x^2])/(2*a^4*(a*d^2 - b*d*e + c*e^2))
```

Maple [B] time = 0.018, size = 662, normalized size = 2.4

$$\frac{x^3}{3\,ae}-\frac{x^2d}{2\,ae^2}-\frac{bx^2}{2\,a^2e}+\frac{d^2x}{e^3a}+\frac{bdx}{a^2e^2}-\frac{cx}{a^2e}+\frac{b^2x}{a^3e}-\frac{d^5\ln(ex+d)}{e^4\,(ad^2-bde+e^2c)}+\frac{\ln\left(ax^2+bx+c\right)c^2d}{(2\,ad^2-2\,bde+2\,e^2c)\,a^2}$$
$$-\frac{3\,\ln\left(ax^2+bx+c\right)b^2cd}{(2\,ad^2-2\,bde+2\,e^2c)\,a^3}+\frac{\ln\left(ax^2+bx+c\right)bc^2e}{(ad^2-bde+e^2c)\,a^3}+\frac{\ln\left(ax^2+bx+c\right)b^4d}{(2\,ad^2-2\,bde+2\,e^2c)\,a^4}$$
$$-\frac{\ln\left(ax^2+bx+c\right)b^3ce}{(2\,ad^2-2\,bde+2\,e^2c)\,a^4}-5\,\frac{c^2db}{(ad^2-bde+e^2c)\,a^2\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right)$$
$$+2\,\frac{c^3e}{(ad^2-bde+e^2c)\,a^2\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right)$$
$$+5\,\frac{b^3cd}{(ad^2-bde+e^2c)\,a^3\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right)$$
$$-4\,\frac{b^2c^2e}{(ad^2-bde+e^2c)\,a^3\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right)$$
$$-\frac{b^5d}{(ad^2-bde+e^2c)\,a^4}\arctan\left((2\,ax+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$
$$+\frac{b^4ce}{(ad^2-bde+e^2c)\,a^4}\arctan\left((2\,ax+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^3/(a+c/x^2+b/x)/(e*x+d),x)
```

```
[Out] 1/3*x^3/a/e-1/2/a/e^2*x^2*d-1/2/a^2/e*x^2*b+1/a/e^3*d^2*x+1/a^2/e
^2*b*d*x-1/a^2/e*c*x+1/a^3/e*b^2*x-1/e^4*d^5/(a*d^2-b*d*e+c*e^2)*
ln(e*x+d)+1/2/(a*d^2-b*d*e+c*e^2)/a^2*ln(a*x^2+b*x+c)*c^2*d-3/2/(
a*d^2-b*d*e+c*e^2)/a^3*ln(a*x^2+b*x+c)*b^2*c*d+1/(a*d^2-b*d*e+c*e
^2)/a^3*ln(a*x^2+b*x+c)*b*c^2*e+1/2/(a*d^2-b*d*e+c*e^2)/a^4*ln(a*
x^2+b*x+c)*b^4*d-1/2/(a*d^2-b*d*e+c*e^2)/a^4*ln(a*x^2+b*x+c)*b^3*
c*e-5/(a*d^2-b*d*e+c*e^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/
(4*a*c-b^2)^(1/2))*b*c^2*d+2/(a*d^2-b*d*e+c*e^2)/a^2/(4*a*c-b^2)^
(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c^3*e+5/(a*d^2-b*d*e+c*
e^2)/a^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^
3*c*d-4/(a*d^2-b*d*e+c*e^2)/a^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b
)/(4*a*c-b^2)^(1/2))*b^2*c^2*e-1/(a*d^2-b*d*e+c*e^2)/a^4/(4*a*c-b
^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^5*d+1/(a*d^2-b*d*
e+c*e^2)/a^4/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)
)*b^4*c*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((e*x + d)*(a + b/x + c/x^2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 28.9781, size = 1, normalized size = 0.

$$\left[-\frac{3\left(\left(b^5-5\,ab^3c+5\,a^2bc^2\right)de^4-\left(b^4c-4\,ab^2c^2+2\,a^2c^3\right)e^5\right)\log\left(-\frac{b^3-4\,abc+2\,\left(ab^2-4\,a^2c\right)x-\left(2\,a^2x^2+2\,abx+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{ax^2+bx+c}\right)+(}{}\right.$$

$$-\frac{6\left(\left(b^5-5\,ab^3c+5\,a^2bc^2\right)de^4-\left(b^4c-4\,ab^2c^2+2\,a^2c^3\right)e^5\right)\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,ax+b)}{b^2-4\,ac}\right)+\left(6\,a^4d^5\log\left(ex+d\right)-2\,\left(a^4d^2e^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((e*x + d)*(a + b/x + c/x^2)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d*e^4 - (b^4*c - 4*a*b^
2*c^2 + 2*a^2*c^3)*e^5)*log(-(b^3 - 4*a*b*c + 2*(a*b^2 - 4*a^2*c)
*x - (2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(a*x^
2 + b*x + c)) + (6*a^4*d^5*log(e*x + d) - 2*(a^4*d^2*e^3 - a^3*b*
d*e^4 + a^3*c*e^5)*x^3 + 3*(a^4*d^3*e^2 + a^2*b*c*e^5 - (a^2*b^2
- a^3*c)*d*e^4)*x^2 - 6*(a^4*d^4*e - (a*b^3 - 2*a^2*b*c)*d*e^4 +
(a*b^2*c - a^2*c^2)*e^5)*x - 3*((b^4 - 3*a*b^2*c + a^2*c^2)*d*e^4
 - (b^3*c - 2*a*b*c^2)*e^5)*log(a*x^2 + b*x + c))*sqrt(b^2 - 4*a*
c))/((a^5*d^2*e^4 - a^4*b*d*e^5 + a^4*c*e^6)*sqrt(b^2 - 4*a*c)),
-1/6*(6*((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d*e^4 - (b^4*c - 4*a*b^2
*c^2 + 2*a^2*c^3)*e^5)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^
2 - 4*a*c)) + (6*a^4*d^5*log(e*x + d) - 2*(a^4*d^2*e^3 - a^3*b*d*
e^4 + a^3*c*e^5)*x^3 + 3*(a^4*d^3*e^2 + a^2*b*c*e^5 - (a^2*b^2 -
a^3*c)*d*e^4)*x^2 - 6*(a^4*d^4*e - (a*b^3 - 2*a^2*b*c)*d*e^4 + (a
*b^2*c - a^2*c^2)*e^5)*x - 3*((b^4 - 3*a*b^2*c + a^2*c^2)*d*e^4 -
 (b^3*c - 2*a*b*c^2)*e^5)*log(a*x^2 + b*x + c))*sqrt(-b^2 + 4*a*c
))/((a^5*d^2*e^4 - a^4*b*d*e^5 + a^4*c*e^6)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.291317, size = 398, normalized size = 1.42

$$-\frac{d^5\ln\left(|xe+d|\right)}{ad^2e^4-bde^5+ce^6}+\frac{\left(b^4d-3\,ab^2cd+a^2c^2d-b^3ce+2\,abc^2e\right)\ln\left(ax^2+bx+c\right)}{2\,(a^5d^2-a^4bde+a^4ce^2)}-\frac{\left(b^5d-5\,ab^3cd+5\,a^2bc^2d-b^4ce+4\,ab^2c^2e-2\,a^2c^3e\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^5d^2-a^4bde+a^4ce^2)\sqrt{-b^2+4\,ac}}+\frac{\left(2\,a^2x^3e^2-3\,a^2dx^2e+6\,a^2d^2x-3\,abx^2e^2+6\,abdxe+6\,b^2xe^2-6\,acxe^2\right)e^{(-3)}}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((e*x + d)*(a + b/x + c/x^2)),x, algorithm="giac")
```

```
[Out] -d^5*ln(abs(x*e + d))/(a*d^2*e^4 - b*d*e^5 + c*e^6) + 1/2*(b^4*d
- 3*a*b^2*c*d + a^2*c^2*d - b^3*c*e + 2*a*b*c^2*e)*ln(a*x^2 + b*x
 + c)/(a^5*d^2 - a^4*b*d*e + a^4*c*e^2) - (b^5*d - 5*a*b^3*c*d +
5*a^2*b*c^2*d - b^4*c*e + 4*a*b^2*c^2*e - 2*a^2*c^3*e)*arctan((2*
a*x + b)/sqrt(-b^2 + 4*a*c))/((a^5*d^2 - a^4*b*d*e + a^4*c*e^2)*s
qrt(-b^2 + 4*a*c)) + 1/6*(2*a^2*x^3*e^2 - 3*a^2*d*x^2*e + 6*a^2*d
^2*x - 3*a*b*x^2*e^2 + 6*a*b*d*x*e + 6*b^2*x*e^2 - 6*a*c*x*e^2)*e
^(-3)/a^3
```

3.62 $\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal. Leaf size=218

$$-\frac{\left(-2abcd+ac^2e+b^3d-b^2ce\right)\log\left(ax^2+bx+c\right)}{2a^3\left(ad^2-e(bd-ce)\right)}-\frac{x(ad+be)}{a^2e^2}-\frac{\left(2a^2c^2d-4ab^2cd+3abc^2e+b^4d-b^3ce\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{d^4\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)}+\frac{x^2}{2ae}$$

```
[Out] -(((a*d + b*e)*x)/(a^2*e^2)) + x^2/(2*a*e) - ((b^4*d - 4*a*b^2*c*
d + 2*a^2*c^2*d - b^3*c*e + 3*a*b*c^2*e)*ArcTanh[(b + 2*a*x)/Sqrt
[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) +
 (d^4*Log[d + e*x])/(e^3*(a*d^2 - e*(b*d - c*e))) - ((b^3*d - 2*a
*b*c*d - b^2*c*e + a*c^2*e)*Log[c + b*x + a*x^2])/(2*a^3*(a*d^2 -
 e*(b*d - c*e)))
```

Rubi [A] time = 0.793686, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24

$$-\frac{\left(-2abcd+ac^2e+b^3d-b^2ce\right)\log\left(ax^2+bx+c\right)}{2a^3\left(ad^2-e(bd-ce)\right)}-\frac{x(ad+be)}{a^2e^2}-\frac{\left(2a^2c^2d-4ab^2cd+3abc^2e+b^4d-b^3ce\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{d^4\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)}+\frac{x^2}{2ae}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -(((a*d + b*e)*x)/(a^2*e^2)) + x^2/(2*a*e) - ((b^4*d - 4*a*b^2*c*
d + 2*a^2*c^2*d - b^3*c*e + 3*a*b*c^2*e)*ArcTanh[(b + 2*a*x)/Sqrt
[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) +
 (d^4*Log[d + e*x])/(e^3*(a*d^2 - e*(b*d - c*e))) - ((b^3*d - 2*a
*b*c*d - b^2*c*e + a*c^2*e)*Log[c + b*x + a*x^2])/(2*a^3*(a*d^2 -
 e*(b*d - c*e)))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.308177, size = 218, normalized size = 1.

$$\frac{\left(2abcd-ac^2e+b^3(-d)+b^2ce\right)\log(x(ax+b)+c)}{2a^3\left(ad^2+e(ce-bd)\right)}-\frac{x(ad+be)}{a^2e^2}+\frac{\left(2a^2c^2d-4ab^2cd+3abc^2e+b^4d-b^3ce\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a^3\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)}+\frac{d^4\log(d+ex)}{e^3\left(ad^2+e(ce-bd)\right)}+\frac{x^2}{2ae}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -(((a*d + b*e)*x)/(a^2*e^2)) + x^2/(2*a*e) + ((b^4*d - 4*a*b^2*c*
d + 2*a^2*c^2*d - b^3*c*e + 3*a*b*c^2*e)*ArcTan[(b + 2*a*x)/Sqrt[
-b^2 + 4*a*c]])/(a^3*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e)
)) + (d^4*Log[d + e*x])/(e^3*(a*d^2 + e*(-(b*d) + c*e))) + ((-(b^
3*d) + 2*a*b*c*d + b^2*c*e - a*c^2*e)*Log[c + x*(b + a*x)])/(2*a^
3*(a*d^2 + e*(-(b*d) + c*e)))
```

Maple [B] time = 0.012, size = 512, normalized size = 2.4

$$\frac{x^2}{2\,ae}-\frac{dx}{ae^2}-\frac{bx}{a^2e}+\frac{d^4\ln(ex+d)}{e^3\left(ad^2-bde+e^2c\right)}+\frac{\ln\left(ax^2+bx+c\right)bcd}{\left(ad^2-bde+e^2c\right)a^2}-\frac{\ln\left(ax^2+bx+c\right)c^2e}{\left(2\,ad^2-2\,bde+2\,e^2c\right)a^2}-\frac{\ln\left(ax^2+bx+c\right)b^3d}{\left(2\,ad^2-2\,bde+2\,e^2c\right)a^3}+\frac{\ln\left(ax^2+bx+c\right)b^2ce}{\left(2\,ad^2-2\,bde+2\,e^2c\right)a^3}+2\,\frac{c^2d}{\left(ad^2-bde+e^2c\right)a\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right)-4\,\frac{b^2cd}{\left(ad^2-bde+e^2c\right)a^2\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right)+3\,\frac{bc^2e}{\left(ad^2-bde+e^2c\right)a^2\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right)+\frac{b^4d}{\left(ad^2-bde+e^2c\right)a^3}\arctan\left((2\,ax+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}-\frac{b^3ce}{\left(ad^2-bde+e^2c\right)a^3}\arctan\left((2\,ax+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(a+c/x^2+b/x)/(e*x+d),x)
```

```
[Out] 1/2*x^2/a/e-1/a/e^2*d*x-1/a^2/e*b*x+1/e^3*d^4/(a*d^2-b*d*e+c*e^2)
*ln(e*x+d)+1/(a*d^2-b*d*e+c*e^2)/a^2*ln(a*x^2+b*x+c)*b*c*d-1/2/(a
*d^2-b*d*e+c*e^2)/a^2*ln(a*x^2+b*x+c)*c^2*e-1/2/(a*d^2-b*d*e+c*e^
2)/a^3*ln(a*x^2+b*x+c)*b^3*d+1/2/(a*d^2-b*d*e+c*e^2)/a^3*ln(a*x^2
+b*x+c)*b^2*c*e+2/(a*d^2-b*d*e+c*e^2)/a/(4*a*c-b^2)^(1/2)*arctan(
(2*a*x+b)/(4*a*c-b^2)^(1/2))*c^2*d-4/(a*d^2-b*d*e+c*e^2)/a^2/(4*a
*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*c*d+3/(a*d^
2-b*d*e+c*e^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)
^(1/2))*b*c^2*e+1/(a*d^2-b*d*e+c*e^2)/a^3/(4*a*c-b^2)^(1/2)*arcta
n((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^4*d-1/(a*d^2-b*d*e+c*e^2)/a^3/(4
*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^3*c*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((e*x + d)*(a + b/x + c/x^2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 17.9436, size = 1, normalized size = 0.

$$\left[\frac{\left(\left(b^4 - 4\,ab^2c + 2\,a^2c^2\right)de^3 - \left(b^3c - 3\,abc^2\right)e^4\right)\log\left(-\frac{b^3 - 4\,abc + 2\,\left(ab^2 - 4\,a^2c\right)x - \left(2\,a^2x^2 + 2\,abx + b^2 - 2\,ac\right)\sqrt{b^2 - 4\,ac}}{ax^2 + bx + c}\right) + \left(2\,a^3d^4\log\left(ex\right.\right.}{2\left(a^4d^2e\right.} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((e*x + d)*(a + b/x + c/x^2)),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d*e^3 - (b^3*c - 3*a*b*c^2)*
e^4)*log(-(b^3 - 4*a*b*c + 2*(a*b^2 - 4*a^2*c)*x - (2*a^2*x^2 + 2
*a*b*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(a*x^2 + b*x + c)) + (2*
a^3*d^4*log(e*x + d) + (a^3*d^2*e^2 - a^2*b*d*e^3 + a^2*c*e^4)*x^
2 - 2*(a^3*d^3*e + a*b*c*e^4 - (a*b^2 - a^2*c)*d*e^3)*x - ((b^3 -
 2*a*b*c)*d*e^3 - (b^2*c - a*c^2)*e^4)*log(a*x^2 + b*x + c))*sqrt
(b^2 - 4*a*c))/((a^4*d^2*e^3 - a^3*b*d*e^4 + a^3*c*e^5)*sqrt(b^2
- 4*a*c)), 1/2*(2*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d*e^3 - (b^3*c -
 3*a*b*c^2)*e^4)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*
```

```
a*c)) + (2*a^3*d^4*log(e*x + d) + (a^3*d^2*e^2 - a^2*b*d*e^3 + a^
2*c*e^4)*x^2 - 2*(a^3*d^3*e + a*b*c*e^4 - (a*b^2 - a^2*c)*d*e^3)*
x - ((b^3 - 2*a*b*c)*d*e^3 - (b^2*c - a*c^2)*e^4)*log(a*x^2 + b*x
 + c))*sqrt(-b^2 + 4*a*c))/((a^4*d^2*e^3 - a^3*b*d*e^4 + a^3*c*e^
5)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.296006, size = 302, normalized size = 1.39

$$\frac{d^4 \ln\left(|xe+d|\right)}{ad^2e^3 - bde^4 + ce^5} - \frac{\left(b^3d - 2\,abcd - b^2ce + ac^2e\right)\ln\left(ax^2+bx+c\right)}{2\left(a^4d^2 - a^3bde + a^3ce^2\right)} + \frac{\left(b^4d - 4\,ab^2cd + 2\,a^2c^2d - b^3ce + 3\,abc^2e\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^4d^2 - a^3bde + a^3ce^2\right)\sqrt{-b^2+4\,ac}} + \frac{\left(ax^2e - 2\,adx - 2\,bxe\right)e^{(-2)}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((e*x + d)*(a + b/x + c/x^2)),x, algorithm="giac")
```

```
[Out] d^4*ln(abs(x*e + d))/(a*d^2*e^3 - b*d*e^4 + c*e^5) - 1/2*(b^3*d -
 2*a*b*c*d - b^2*c*e + a*c^2*e)*ln(a*x^2 + b*x + c)/(a^4*d^2 - a^
3*b*d*e + a^3*c*e^2) + (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - b^3*c
*e + 3*a*b*c^2*e)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a^4*d^
2 - a^3*b*d*e + a^3*c*e^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(a*x^2*e - 2
*a*d*x - 2*b*x*e)*e^(-2)/a^2
```

3.63 $\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal. Leaf size=176

$$\frac{\left(-acd+b^2d-bce\right)\log\left(ax^2+bx+c\right)}{2a^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(-3abcd+2ac^2e+b^3d-b^2ce\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{x}{ae}$$

```
[Out] x/(a*e) + ((b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*ArcTanh[(b +
 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*
d - c*e))) - (d^3*Log[d + e*x])/(e^2*(a*d^2 - e*(b*d - c*e))) + (
(b^2*d - a*c*d - b*c*e)*Log[c + b*x + a*x^2])/(2*a^2*(a*d^2 - e*(
b*d - c*e)))
```

Rubi [A] time = 0.534576, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261

$$\frac{\left(-acd+b^2d-bce\right)\log\left(ax^2+bx+c\right)}{2a^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(-3abcd+2ac^2e+b^3d-b^2ce\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{x}{ae}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] x/(a*e) + ((b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*ArcTanh[(b +
 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*
d - c*e))) - (d^3*Log[d + e*x])/(e^2*(a*d^2 - e*(b*d - c*e))) + (
(b^2*d - a*c*d - b*c*e)*Log[c + b*x + a*x^2])/(2*a^2*(a*d^2 - e*(
b*d - c*e)))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{d^3\log\left(d+ex\right)}{e^2\left(ad^2-bde+ce^2\right)}+\frac{\int\frac{1}{a}\,dx}{e}+\frac{\left(-acd+b^2d-bce\right)\log\left(ax^2+bx+c\right)}{2a^2\left(ad^2-bde+ce^2\right)}+\frac{\left(-3abcd+2ac^2e+b^3d-b^2ce\right)\operatorname{atanh}\left(\frac{2ax+b}{\sqrt{-4ac+b^2}}\right)}{a^2\sqrt{-4ac+b^2}\left(ad^2-bde+ce^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] -d**3*log(d + e*x)/(e**2*(a*d**2 - b*d*e + c*e**2)) + Integral(1/
a, x)/e + (-a*c*d + b**2*d - b*c*e)*log(a*x**2 + b*x + c)/(2*a**2
*(a*d**2 - b*d*e + c*e**2)) + (-3*a*b*c*d + 2*a*c**2*e + b**3*d -
 b**2*c*e)*atanh((2*a*x + b)/sqrt(-4*a*c + b**2))/(a**2*sqrt(-4*a
*c + b**2)*(a*d**2 - b*d*e + c*e**2))
```

Mathematica [A] time = 0.320269, size = 178, normalized size = 1.01

$$\frac{\left(-acd+b^2d-bce\right)\log\left(ax^2+bx+c\right)}{2a^2\left(ad^2-bde+ce^2\right)}+\frac{\left(-3abcd+2ac^2e+b^3d-b^2ce\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a^2\sqrt{4ac-b^2}\left(-ad^2+bde-ce^2\right)}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-bde+ce^2\right)}+\frac{x}{ae}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] x/(a*e) + ((b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*ArcTan[(b +
2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^2*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + b
*d*e - c*e^2)) - (d^3*Log[d + e*x])/(e^2*(a*d^2 - b*d*e + c*e^2))
 + ((b^2*d - a*c*d - b*c*e)*Log[c + b*x + a*x^2])/(2*a^2*(a*d^2 -
 b*d*e + c*e^2))
```

Maple [B] time = 0.011, size = 388, normalized size = 2.2

$$\frac{x}{ae}-\frac{d^3\ln\left(ex+d\right)}{e^2\left(ad^2-bde+e^2c\right)}-\frac{\ln\left(ax^2+bx+c\right)cd}{\left(2\,ad^2-2\,bde+2\,e^2c\right)a}+\frac{\ln\left(ax^2+bx+c\right)b^2d}{\left(2\,ad^2-2\,bde+2\,e^2c\right)a^2}-\frac{\ln\left(ax^2+bx+c\right)bce}{\left(2\,ad^2-2\,bde+2\,e^2c\right)a^2}+3\,\frac{bcd}{\left(ad^2-bde+e^2c\right)a\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right)-2\,\frac{c^2e}{\left(ad^2-bde+e^2c\right)a\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right)-\frac{b^3d}{\left(ad^2-bde+e^2c\right)a^2}\arctan\left((2\,ax+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{b^2ce}{\left(ad^2-bde+e^2c\right)a^2}\arctan\left((2\,ax+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a+c/x^2+b/x)/(e*x+d),x)
```

```
[Out] x/a/e-1/e^2*d^3/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)-1/2/(a*d^2-b*d*e+c*
e^2)/a*ln(a*x^2+b*x+c)*c*d+1/2/(a*d^2-b*d*e+c*e^2)/a^2*ln(a*x^2+b
*x+c)*b^2*d-1/2/(a*d^2-b*d*e+c*e^2)/a^2*ln(a*x^2+b*x+c)*b*c*e+3/(
a*d^2-b*d*e+c*e^2)/a/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^
2)^(1/2))*b*c*d-2/(a*d^2-b*d*e+c*e^2)/a/(4*a*c-b^2)^(1/2)*arctan(
(2*a*x+b)/(4*a*c-b^2)^(1/2))*c^2*e-1/(a*d^2-b*d*e+c*e^2)/a^2/(4*a
*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^3*d+1/(a*d^2-
b*d*e+c*e^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(
1/2))*b^2*c*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((e*x + d)*(a + b/x + c/x^2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.41146, size = 1, normalized size = 0.01

$$\left[\frac{\left(\left(b^3-3\,abc\right)de^2-\left(b^2c-2\,ac^2\right)e^3\right)\log\left(\frac{b^3-4\,abc+2\left(ab^2-4\,a^2c\right)x+\left(2\,a^2x^2+2\,abx+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{ax^2+bx+c}\right)-\left(2\,a^2d^3\log\left(ex+d\right)-2\left(a^2\right.\right.}{2\left(a^3d^2e^2-a^2bde^3+a^2ce^4\right)\sqrt{b^2-4\,ac}}\right.$$

$$-\frac{2\left(\left(b^3-3\,abc\right)de^2-\left(b^2c-2\,ac^2\right)e^3\right)\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\left(2\,ax+b\right)}{b^2-4\,ac}\right)+\left(2\,a^2d^3\log\left(ex+d\right)-2\left(a^2d^2e-abde^2+ace^3\right)x+\left(b\right.\right.}{2\left(a^3d^2e^2-a^2bde^3+a^2ce^4\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((e*x + d)*(a + b/x + c/x^2)),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^3 - 3*a*b*c)*d*e^2 - (b^2*c - 2*a*c^2)*e^3)*log((b^3 -
4*a*b*c + 2*(a*b^2 - 4*a^2*c)*x + (2*a^2*x^2 + 2*a*b*x + b^2 - 2*
a*c)*sqrt(b^2 - 4*a*c))/(a*x^2 + b*x + c)) - (2*a^2*d^3*log(e*x +
 d) - 2*(a^2*d^2*e - a*b*d*e^2 + a*c*e^3)*x + (b*c*e^3 - (b^2 - a
*c)*d*e^2)*log(a*x^2 + b*x + c))*sqrt(b^2 - 4*a*c))/((a^3*d^2*e^2
 - a^2*b*d*e^3 + a^2*c*e^4)*sqrt(b^2 - 4*a*c)), -1/2*(2*((b^3 - 3
```

```
*a*b*c)*d*e^2 - (b^2*c - 2*a*c^2)*e^3)*arctan(-sqrt(-b^2 + 4*a*c)
*(2*a*x + b)/(b^2 - 4*a*c)) + (2*a^2*d^3*log(e*x + d) - 2*(a^2*d^
2*e - a*b*d*e^2 + a*c*e^3)*x + (b*c*e^3 - (b^2 - a*c)*d*e^2)*log(
a*x^2 + b*x + c))*sqrt(-b^2 + 4*a*c))/((a^3*d^2*e^2 - a^2*b*d*e^3
 + a^2*c*e^4)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.31354, size = 250, normalized size = 1.42

$$-\frac{d^{3}\ln\left(|xe+d|\right)}{ad^{2}e^{2}-bde^{3}+ce^{4}}+\frac{xe^{(-1)}}{a}+\frac{\left(b^{2}d-acd-bce\right)\ln\left(ax^{2}+bx+c\right)}{2\left(a^{3}d^{2}-a^{2}bde+a^{2}ce^{2}\right)}-\frac{\left(b^{3}d-3\,abcd-b^{2}ce+2\,ac^{2}e\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(a^{3}d^{2}-a^{2}bde+a^{2}ce^{2}\right)\sqrt{-b^{2}+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((e*x + d)*(a + b/x + c/x^2)),x, algorithm="giac")
```

```
[Out] -d^3*ln(abs(x*e + d))/(a*d^2*e^2 - b*d*e^3 + c*e^4) + x*e^(-1)/a
+ 1/2*(b^2*d - a*c*d - b*c*e)*ln(a*x^2 + b*x + c)/(a^3*d^2 - a^2*
b*d*e + a^2*c*e^2) - (b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*ar
ctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a^3*d^2 - a^2*b*d*e + a^2*
c*e^2)*sqrt(-b^2 + 4*a*c))
```

3.64 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal. Leaf size=149

$$-\frac{\left(-2acd+b^2d-bce\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)}-\frac{(bd-ce)\log\left(ax^2+bx+c\right)}{2a\left(ad^2-e(bd-ce)\right)}$$

```
[Out] -(((b^2*d - 2*a*c*d - b*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c
]])/(a*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e)))) + (d^2*Log[d +
 e*x])/(e*(a*d^2 - b*d*e + c*e^2)) - ((b*d - c*e)*Log[c + b*x + a
*x^2])/(2*a*(a*d^2 - e*(b*d - c*e)))
```

Rubi [A] time = 0.399242, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-\frac{\left(-2acd+b^2d-bce\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)}-\frac{(bd-ce)\log\left(ax^2+bx+c\right)}{2a\left(ad^2-e(bd-ce)\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -(((b^2*d - 2*a*c*d - b*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c
]])/(a*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e)))) + (d^2*Log[d +
 e*x])/(e*(a*d^2 - b*d*e + c*e^2)) - ((b*d - c*e)*Log[c + b*x + a
*x^2])/(2*a*(a*d^2 - e*(b*d - c*e)))
```

Rubi in Sympy [A] time = 80.2854, size = 129, normalized size = 0.87

$$\frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)}-\frac{(bd-ce)\log\left(ax^2+bx+c\right)}{2a\left(ad^2-bde+ce^2\right)}-\frac{\left(-2acd+b^2d-bce\right)\operatorname{atanh}\left(\frac{2ax+b}{\sqrt{-4ac+b^2}}\right)}{a\sqrt{-4ac+b^2}\left(ad^2-bde+ce^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] d**2*log(d + e*x)/(e*(a*d**2 - b*d*e + c*e**2)) - (b*d - c*e)*log
(a*x**2 + b*x + c)/(2*a*(a*d**2 - b*d*e + c*e**2)) - (-2*a*c*d +
```

```
b**2*d - b*c*e)*atanh((2*a*x + b)/sqrt(-4*a*c + b**2))/(a*sqrt(-4
*a*c + b**2)*(a*d**2 - b*d*e + c*e**2))
```

Mathematica [A] time = 0.211338, size = 132, normalized size = 0.89

$$-\frac{\sqrt{4ac-b^2}\left(e(bd-ce)\log(x(ax+b)+c)-2ad^2\log(d+ex)\right)+2e\left(2acd+b^2(-d)+bce\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{2ae\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -(2*e*(-(b^2*d) + 2*a*c*d + b*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 +
 4*a*c]] + Sqrt[-b^2 + 4*a*c]*(-2*a*d^2*Log[d + e*x] + e*(b*d - c
*e)*Log[c + x*(b + a*x)]))/(2*a*Sqrt[-b^2 + 4*a*c]*e*(a*d^2 + e*(
-(b*d) + c*e)))
```

Maple [A] time = 0.009, size = 275, normalized size = 1.9

$$\frac{d^2\ln(ex+d)}{e(ad^2-bde+e^2c)}-\frac{\ln(ax^2+bx+c)\,bd}{(2\,ad^2-2\,bde+2\,e^2c)\,a}+\frac{\ln(ax^2+bx+c)\,ce}{(2\,ad^2-2\,bde+2\,e^2c)\,a}$$
$$-2\,\frac{cd}{(ad^2-bde+e^2c)\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right)$$
$$+\frac{b^2d}{(ad^2-bde+e^2c)\,a}\arctan\left((2\,ax+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$
$$-\frac{bce}{(ad^2-bde+e^2c)\,a}\arctan\left((2\,ax+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+c/x^2+b/x)/(e*x+d),x)
```

```
[Out] d^2*ln(e*x+d)/e/(a*d^2-b*d*e+c*e^2)-1/2/(a*d^2-b*d*e+c*e^2)/a*ln(
a*x^2+b*x+c)*b*d+1/2/(a*d^2-b*d*e+c*e^2)/a*ln(a*x^2+b*x+c)*c*e-2/
(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2
)^(1/2))*c*d+1/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*arctan((2*a*
x+b)/(4*a*c-b^2)^(1/2))*b^2/a*d-1/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)
^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b/a*c*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)*(a + b/x + c/x^2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62713, size = 1, normalized size = 0.01

$$\left[\frac{\left(bce^2-\left(b^2-2\,ac\right)de\right)\log\left(\frac{b^3-4\,abc+2\,\left(ab^2-4\,a^2c\right)x+\left(2\,a^2x^2+2\,abx+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{ax^2+bx+c}\right)+\left(2\,ad^2\log\left(ex+d\right)-\left(bde-ce^2\right)\log\left(ax\right.\right.}{2\left(a^2d^2e-abde^2+ace^3\right)\sqrt{b^2-4\,ac}}\right.$$

$$\left.-\frac{2\left(bce^2-\left(b^2-2\,ac\right)de\right)\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,ax+b)}{b^2-4\,ac}\right)-\left(2\,ad^2\log\left(ex+d\right)-\left(bde-ce^2\right)\log\left(ax^2+bx+c\right)\right)\sqrt{-b^2+4\,ac}}{2\left(a^2d^2e-abde^2+ace^3\right)\sqrt{-b^2+4\,ac}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)*(a + b/x + c/x^2)),x, algorithm="fricas")
```

```
[Out] [1/2*((b*c*e^2 - (b^2 - 2*a*c)*d*e)*log((b^3 - 4*a*b*c + 2*(a*b^2
 - 4*a^2*c)*x + (2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c)*sqrt(b^2 - 4*
a*c))/(a*x^2 + b*x + c)) + (2*a*d^2*log(e*x + d) - (b*d*e - c*e^2
)*log(a*x^2 + b*x + c))*sqrt(b^2 - 4*a*c))/((a^2*d^2*e - a*b*d*e^
2 + a*c*e^3)*sqrt(b^2 - 4*a*c)), -1/2*(2*(b*c*e^2 - (b^2 - 2*a*c)
*d*e)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) - (2*
a*d^2*log(e*x + d) - (b*d*e - c*e^2)*log(a*x^2 + b*x + c))*sqrt(-
b^2 + 4*a*c))/((a^2*d^2*e - a*b*d*e^2 + a*c*e^3)*sqrt(-b^2 + 4*a*
c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.300595, size = 201, normalized size = 1.35

$$\frac{d^2\ln\left(|xe+d|\right)}{ad^2e-bde^2+ce^3}-\frac{(bd-ce)\ln\left(ax^2+bx+c\right)}{2\left(a^2d^2-abde+ace^2\right)}+\frac{\left(b^2d-2\,acd-bce\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^2d^2-abde+ace^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)*(a + b/x + c/x^2)),x, algorithm="giac")
```

```
[Out] d^2*ln(abs(x*e + d))/(a*d^2*e - b*d*e^2 + c*e^3) - 1/2*(b*d - c*e
)*ln(a*x^2 + b*x + c)/(a^2*d^2 - a*b*d*e + a*c*e^2) + (b^2*d - 2*
a*c*d - b*c*e)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*d^2 -
 a*b*d*e + a*c*e^2)*sqrt(-b^2 + 4*a*c))
```

3.65 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx$

Optimal. Leaf size=124

$$\frac{(bd-2ce)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}+\frac{d\log\left(ax^2+bx+c\right)}{2\,(ad^2-e(bd-ce))}-\frac{d\log(d+ex)}{ad^2-e(bd-ce)}$$

```
[Out] ((b*d - 2*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2
- 4*a*c]*(a*d^2 - e*(b*d - c*e))) - (d*Log[d + e*x])/(a*d^2 - e*(
b*d - c*e)) + (d*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e))
)
```

Rubi [A] time = 0.323081, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{(bd-2ce)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}+\frac{d\log\left(ax^2+bx+c\right)}{2\,(ad^2-e(bd-ce))}-\frac{d\log(d+ex)}{ad^2-e(bd-ce)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + c/x^2 + b/x)*x*(d + e*x)),x]
```

```
[Out] ((b*d - 2*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2
- 4*a*c]*(a*d^2 - e*(b*d - c*e))) - (d*Log[d + e*x])/(a*d^2 - e*(
b*d - c*e)) + (d*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e))
)
```

Rubi in Sympy [A] time = 62.2532, size = 112, normalized size = 0.9

$$-\frac{d\log\left(\frac{d}{x}+e\right)}{ad^2-bde+ce^2}+\frac{d\log\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}{2\,(ad^2-bde+ce^2)}-\frac{(bd-2ce)\,\mathrm{atanh}\left(\frac{b+\frac{2c}{x}}{\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}\,(ad^2-bde+ce^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+c/x**2+b/x)/x/(e*x+d),x)
```

```
[Out] -d*log(d/x + e)/(a*d**2 - b*d*e + c*e**2) + d*log(a + b/x + c/x**
2)/(2*(a*d**2 - b*d*e + c*e**2)) - (b*d - 2*c*e)*atanh((b + 2*c/x
```

```
)/sqrt(-4*a*c + b**2))/(sqrt(-4*a*c + b**2)*(a*d**2 - b*d*e + c*e
**2))
```

Mathematica [A] time = 0.128955, size = 107, normalized size = 0.86

$$\frac{d\sqrt{4ac-b^2}(2\log(d+ex)-\log(x(ax+b)+c))+2(bd-2ce)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\left(e(bd-ce)-ad^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + c/x^2 + b/x)*x*(d + e*x)),x]
```

```
[Out] (2*(b*d - 2*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b
^2 + 4*a*c]*d*(2*Log[d + e*x] - Log[c + x*(b + a*x)]))/(2*Sqrt[-b
^2 + 4*a*c]*(-(a*d^2) + e*(b*d - c*e)))
```

Maple [A] time = 0.007, size = 169, normalized size = 1.4

$$-\frac{d\ln(ex+d)}{ad^2-bde+e^2c}+\frac{d\ln\left(ax^2+bx+c\right)}{2\,ad^2-2\,bde+2\,e^2c} -\frac{bd}{ad^2-bde+e^2c}\arctan\left((2\,ax+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}} +2\,\frac{ce}{(ad^2-bde+e^2c)\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+c/x^2+b/x)/x/(e*x+d),x)
```

```
[Out] -d/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)+1/2/(a*d^2-b*d*e+c*e^2)*d*ln(a*x
^2+b*x+c)-1/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b
)/(4*a*c-b^2)^(1/2))*b*d+2/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*
arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)*(a + b/x + c/x^2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.527888, size = 1, normalized size = 0.01

$$\left[-\frac{(bd-2\,ce)\log\left(-\frac{b^3-4\,abc+2\,(ab^2-4\,a^2c)\,x-(2\,a^2x^2+2\,abx+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{ax^2+bx+c}\right)-\sqrt{b^2-4\,ac}\left(d\log\left(ax^2+bx+c\right)-2\,d\log\left(ex+d\right)\right)}{2\,(ad^2-bde+ce^2)\sqrt{b^2-4\,ac}},\ -\frac{2\,(bd-2\,ce)\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,ax+b)}{b^2-4\,ac}\right)-\sqrt{-b^2+4\,ac}\left(d\log\left(ax^2+bx+c\right)-2\,d\log\left(ex+d\right)\right)}{2\,(ad^2-bde+ce^2)\sqrt{-b^2+4\,ac}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)*(a + b/x + c/x^2)*x),x, algorithm="fricas")
```

```
[Out] [-1/2*((b*d - 2*c*e)*log(-(b^3 - 4*a*b*c + 2*(a*b^2 - 4*a^2*c)*x
- (2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(a*x^2 +
 b*x + c)) - sqrt(b^2 - 4*a*c)*(d*log(a*x^2 + b*x + c) - 2*d*log(
e*x + d)))/((a*d^2 - b*d*e + c*e^2)*sqrt(b^2 - 4*a*c)), -1/2*(2*(
b*d - 2*c*e)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)
) - sqrt(-b^2 + 4*a*c)*(d*log(a*x^2 + b*x + c) - 2*d*log(e*x + d)
))/((a*d^2 - b*d*e + c*e^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+c/x**2+b/x)/x/(e*x+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.298772, size = 171, normalized size = 1.38

$$-\frac{d e\ln\left(|xe+d|\right)}{ad^2e-bde^2+ce^3}+\frac{d\ln\left(ax^2+bx+c\right)}{2\,(ad^2-bde+ce^2)}-\frac{(bd-2\,ce)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{(ad^2-bde+ce^2)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)*(a + b/x + c/x^2)*x),x, algorithm="giac")
```

```
[Out] -d*e*ln(abs(x*e + d))/(a*d^2*e - b*d*e^2 + c*e^3) + 1/2*d*ln(a*x^
2 + b*x + c)/(a*d^2 - b*d*e + c*e^2) - (b*d - 2*c*e)*arctan((2*a*
x + b)/sqrt(-b^2 + 4*a*c))/((a*d^2 - b*d*e + c*e^2)*sqrt(-b^2 + 4
*a*c))
```

3.66 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx$

Optimal. Leaf size=123

$$-\frac{(2ad-be)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}-\frac{e\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)}+\frac{e\log(d+ex)}{ad^2-bde+ce^2}$$

```
[Out] -(((2*a*d - b*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^
2 - 4*a*c]*(a*d^2 - e*(b*d - c*e)))) + (e*Log[d + e*x])/(a*d^2 -
b*d*e + c*e^2) - (e*Log[c + b*x + a*x^2])/(2*(a*d^2 - b*d*e + c*e
^2))
```

Rubi [A] time = 0.279885, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28

$$-\frac{(2ad-be)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}-\frac{e\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)}+\frac{e\log(d+ex)}{ad^2-bde+ce^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + c/x^2 + b/x)*x^2*(d + e*x)),x]
```

```
[Out] -(((2*a*d - b*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^
2 - 4*a*c]*(a*d^2 - e*(b*d - c*e)))) + (e*Log[d + e*x])/(a*d^2 -
b*d*e + c*e^2) - (e*Log[c + b*x + a*x^2])/(2*(a*d^2 - b*d*e + c*e
^2))
```

Rubi in Sympy [A] time = 60.69, size = 112, normalized size = 0.91

$$\frac{e\log\left(d+ex\right)}{ad^2-bde+ce^2}-\frac{e\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)}-\frac{(2ad-be)\operatorname{atanh}\left(\frac{2ax+b}{\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}\left(ad^2-bde+ce^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+c/x**2+b/x)/x**2/(e*x+d),x)
```

```
[Out] e*log(d + e*x)/(a*d**2 - b*d*e + c*e**2) - e*log(a*x**2 + b*x + c
)/(2*(a*d**2 - b*d*e + c*e**2)) - (2*a*d - b*e)*atanh((2*a*x + b)
```

```
/sqrt(-4*a*c + b**2))/(sqrt(-4*a*c + b**2)*(a*d**2 - b*d*e + c*e*
*2))
```

Mathematica [A] time = 0.135244, size = 105, normalized size = 0.85

$$\frac{e\sqrt{4ac-b^2}(\log(x(ax+b)+c)-2\log(d+ex))+(2be-4ad)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\,(e(bd-ce)-ad^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + c/x^2 + b/x)*x^2*(d + e*x)),x]
```

```
[Out] ((-4*a*d + 2*b*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-
b^2 + 4*a*c]*e*(-2*Log[d + e*x] + Log[c + x*(b + a*x)]))/(2*Sqrt[
-b^2 + 4*a*c]*(-(a*d^2) + e*(b*d - c*e)))
```

Maple [A] time = 0.007, size = 168, normalized size = 1.4

$$\frac{e\ln(ex+d)}{ad^2-bde+e^2c}-\frac{e\ln\left(ax^2+bx+c\right)}{2\,ad^2-2\,bde+2\,e^2c}+2\,\frac{ad}{(ad^2-bde+e^2c)\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right)$$
$$-\frac{be}{ad^2-bde+e^2c}\arctan\left((2\,ax+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+c/x^2+b/x)/x^2/(e*x+d),x)
```

```
[Out] e*ln(e*x+d)/(a*d^2-b*d*e+c*e^2)-1/2*e*ln(a*x^2+b*x+c)/(a*d^2-b*d*
e+c*e^2)+2/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)
/(4*a*c-b^2)^(1/2))*a*d-1/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*a
rctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)*(a + b/x + c/x^2)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.54644, size = 1, normalized size = 0.01

$$\left[-\frac{(2\,ad-be)\log\left(\frac{b^3-4\,abc+2\,(ab^2-4\,a^2c)\,x+(2\,a^2x^2+2\,abx+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{ax^2+bx+c}\right)+\sqrt{b^2-4\,ac}\left(e\log\left(ax^2+bx+c\right)-2\,e\log\left(ex+d\right)\right)}{2\,(ad^2-bde+ce^2)\sqrt{b^2-4\,ac}},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)*(a + b/x + c/x^2)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((2*a*d - b*e)*log((b^3 - 4*a*b*c + 2*(a*b^2 - 4*a^2*c)*x +
 (2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(a*x^2 +
b*x + c)) + sqrt(b^2 - 4*a*c)*(e*log(a*x^2 + b*x + c) - 2*e*log(e
*x + d)))/((a*d^2 - b*d*e + c*e^2)*sqrt(b^2 - 4*a*c)), 1/2*(2*(2*
a*d - b*e)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c))
- sqrt(-b^2 + 4*a*c)*(e*log(a*x^2 + b*x + c) - 2*e*log(e*x + d)))
/((a*d^2 - b*d*e + c*e^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+c/x**2+b/x)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.296595, size = 170, normalized size = 1.38

$$-\frac{e\ln\left(ax^2+bx+c\right)}{2\,(ad^2-bde+ce^2)}+\frac{e^2\ln\left(|xe+d|\right)}{ad^2e-bde^2+ce^3}+\frac{(2\,ad-be)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{(ad^2-bde+ce^2)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)*(a + b/x + c/x^2)*x^2),x, algorithm="giac")
```

```
[Out] -1/2*e*ln(a*x^2 + b*x + c)/(a*d^2 - b*d*e + c*e^2) + e^2*ln(abs(x
*e + d))/(a*d^2*e - b*d*e^2 + c*e^3) + (2*a*d - b*e)*arctan((2*a*
x + b)/sqrt(-b^2 + 4*a*c))/((a*d^2 - b*d*e + c*e^2)*sqrt(-b^2 + 4
*a*c))
```

3.67 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx$

Optimal. Leaf size=158

$$\frac{\left(abd+2ace+b^2(-e)\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}-\frac{e^2\log(d+ex)}{d\,(ad^2-bde+ce^2)}-\frac{(ad-be)\log\left(ax^2+bx+c\right)}{2c\,(ad^2-e(bd-ce))}+\frac{\log(x)}{cd}$$

```
[Out] ((a*b*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]]
)/(c*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + Log[x]/(c*d) -
(e^2*Log[d + e*x])/(d*(a*d^2 - b*d*e + c*e^2)) - ((a*d - b*e)*Log
[c + b*x + a*x^2])/(2*c*(a*d^2 - e*(b*d - c*e)))
```

Rubi [A] time = 0.552871, antiderivative size = 159, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{\left(abd+2ace+b^2(-e)\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}-\frac{e^2\log(d+ex)}{d\,(ad^2-e(bd-ce))}-\frac{(ad-be)\log\left(ax^2+bx+c\right)}{2c\,(ad^2-e(bd-ce))}+\frac{\log(x)}{cd}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + c/x^2 + b/x)*x^3*(d + e*x)),x]
```

```
[Out] ((a*b*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]]
)/(c*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + Log[x]/(c*d) -
(e^2*Log[d + e*x])/(d*(a*d^2 - e*(b*d - c*e))) - ((a*d - b*e)*Log
[c + b*x + a*x^2])/(2*c*(a*d^2 - e*(b*d - c*e)))
```

Rubi in Sympy [A] time = 99.8035, size = 136, normalized size = 0.86

$$-\frac{e^2\log(d+ex)}{d\,(ad^2-bde+ce^2)}-\frac{(ad-be)\log\left(ax^2+bx+c\right)}{2c\,(ad^2-bde+ce^2)}-\frac{\left(-abd-2ace+b^2e\right)\operatorname{atanh}\left(\frac{2ax+b}{\sqrt{-4ac+b^2}}\right)}{c\sqrt{-4ac+b^2}\,(ad^2-bde+ce^2)}+\frac{\log(x)}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+c/x**2+b/x)/x**3/(e*x+d),x)
```

```
[Out] -e**2*log(d + e*x)/(d*(a*d**2 - b*d*e + c*e**2)) - (a*d - b*e)*lo
g(a*x**2 + b*x + c)/(2*c*(a*d**2 - b*d*e + c*e**2)) - (-a*b*d - 2
```

```
*a*c*e + b**2*e)*atanh((2*a*x + b)/sqrt(-4*a*c + b**2))/(c*sqrt(-
4*a*c + b**2)*(a*d**2 - b*d*e + c*e**2)) + log(x)/(c*d)
```

Mathematica [A] time = 0.34048, size = 152, normalized size = 0.96

$$-\frac{\sqrt{4ac-b^2}\left(-2\log(x)\left(ad^2+e(ce-bd)\right)+d(ad-be)\log(x(ax+b)+c)+2ce^2\log(d+ex)\right)+2d\left(abd+2ace+b^2(-e)\right)\tan}{2cd\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + c/x^2 + b/x)*x^3*(d + e*x)),x]
```

```
[Out] -(2*d*(a*b*d - b^2*e + 2*a*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*
a*c]] + Sqrt[-b^2 + 4*a*c]*(-2*(a*d^2 + e*(-(b*d) + c*e))*Log[x]
+ 2*c*e^2*Log[d + e*x] + d*(a*d - b*e)*Log[c + x*(b + a*x)]))/(2*
c*Sqrt[-b^2 + 4*a*c]*d*(a*d^2 + e*(-(b*d) + c*e)))
```

Maple [A] time = 0.011, size = 285, normalized size = 1.8

$$\frac{\ln(x)}{cd}-\frac{e^2\ln(ex+d)}{d\left(ad^2-bde+e^2c\right)}-\frac{a\ln\left(ax^2+bx+c\right)d}{\left(2\,ad^2-2\,bde+2\,e^2c\right)c}+\frac{\ln\left(ax^2+bx+c\right)be}{\left(2\,ad^2-2\,bde+2\,e^2c\right)c}$$
$$-\frac{abd}{\left(ad^2-bde+e^2c\right)c}\arctan\left((2\,ax+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$
$$-2\,\frac{ae}{\left(ad^2-bde+e^2c\right)\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right)$$
$$+\frac{b^2e}{\left(ad^2-bde+e^2c\right)c}\arctan\left((2\,ax+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+c/x^2+b/x)/x^3/(e*x+d),x)
```

```
[Out] ln(x)/c/d-e^2*ln(e*x+d)/d/(a*d^2-b*d*e+c*e^2)-1/2/(a*d^2-b*d*e+c*
e^2)/c*a*ln(a*x^2+b*x+c)*d+1/2/(a*d^2-b*d*e+c*e^2)/c*ln(a*x^2+b*x
+c)*b*e-1/(a*d^2-b*d*e+c*e^2)/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b
)/(4*a*c-b^2)^(1/2))*a*b*d-2/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2
)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*e+1/(a*d^2-b*d*e+c*e^2)/c
/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)*(a + b/x + c/x^2)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)*(a + b/x + c/x^2)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+c/x**2+b/x)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.293829, size = 221, normalized size = 1.4

$$-\frac{(ad-be)\ln\left(ax^2+bx+c\right)}{2\left(acd^2-bcde+c^2e^2\right)}-\frac{e^3\ln\left(|xe+d|\right)}{ad^3e-bd^2e^2+cde^3}-\frac{\left(abd-b^2e+2\,ace\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(acd^2-bcde+c^2e^2\right)\sqrt{-b^2+4\,ac}}+\frac{\ln\left(|x|\right)}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)*(a + b/x + c/x^2)*x^3),x, algorithm="giac")
```

```
[Out] -1/2*(a*d - b*e)*ln(a*x^2 + b*x + c)/(a*c*d^2 - b*c*d*e + c^2*e^2
) - e^3*ln(abs(x*e + d))/(a*d^3*e - b*d^2*e^2 + c*d*e^3) - (a*b*d
 - b^2*e + 2*a*c*e)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a*c*
d^2 - b*c*d*e + c^2*e^2)*sqrt(-b^2 + 4*a*c)) + ln(abs(x))/(c*d)
```

3.68 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx$

Optimal. Leaf size=193

$$\frac{\left(2a^2cd-ab(bd+3ce)+b^3e\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{\left(abd+ace+b^2(-e)\right)\log\left(ax^2+bx+c\right)}{2c^2\left(ad^2-e(bd-ce)\right)}+\frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)}-\frac{\log(x)(bd+ce)}{c^2d^2}-\frac{1}{cdx}$$

```
[Out] -(1/(c*d*x)) + ((2*a^2*c*d + b^3*e - a*b*(b*d + 3*c*e))*ArcTanh[(
b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*
(b*d - c*e))) - ((b*d + c*e)*Log[x])/(c^2*d^2) + (e^3*Log[d + e*x
])/(d^2*(a*d^2 - e*(b*d - c*e))) + ((a*b*d - b^2*e + a*c*e)*Log[c
 + b*x + a*x^2])/(2*c^2*(a*d^2 - e*(b*d - c*e)))
```

Rubi [A] time = 0.713638, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24

$$\frac{\left(2a^2cd-ab(bd+3ce)+b^3e\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{\left(abd+ace+b^2(-e)\right)\log\left(ax^2+bx+c\right)}{2c^2\left(ad^2-e(bd-ce)\right)}+\frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)}-\frac{\log(x)(bd+ce)}{c^2d^2}-\frac{1}{cdx}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + c/x^2 + b/x)*x^4*(d + e*x)),x]
```

```
[Out] -(1/(c*d*x)) + ((2*a^2*c*d + b^3*e - a*b*(b*d + 3*c*e))*ArcTanh[(
b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*
(b*d - c*e))) - ((b*d + c*e)*Log[x])/(c^2*d^2) + (e^3*Log[d + e*x
])/(d^2*(a*d^2 - e*(b*d - c*e))) + ((a*b*d - b^2*e + a*c*e)*Log[c
 + b*x + a*x^2])/(2*c^2*(a*d^2 - e*(b*d - c*e)))
```

Rubi in Sympy [A] time = 137.446, size = 178, normalized size = 0.92

$$\frac{e^3\log\left(d+ex\right)}{d^2\left(ad^2-bde+ce^2\right)}-\frac{1}{cdx}-\frac{\left(-abd-ace+b^2e\right)\log\left(ax^2+bx+c\right)}{2c^2\left(ad^2-bde+ce^2\right)}+\frac{\left(2a^2cd-ab^2d-3abce+b^3e\right)\operatorname{atanh}\left(\frac{2ax+b}{\sqrt{-4ac+b^2}}\right)}{c^2\sqrt{-4ac+b^2}\left(ad^2-bde+ce^2\right)}-\frac{(bd+ce)\log(x)}{c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+c/x**2+b/x)/x**4/(e*x+d),x)
```

```
[Out] e**3*log(d + e*x)/(d**2*(a*d**2 - b*d*e + c*e**2)) - 1/(c*d*x) -
(-a*b*d - a*c*e + b**2*e)*log(a*x**2 + b*x + c)/(2*c**2*(a*d**2 -
 b*d*e + c*e**2)) + (2*a**2*c*d - a*b**2*d - 3*a*b*c*e + b**3*e)*
atanh((2*a*x + b)/sqrt(-4*a*c + b**2))/(c**2*sqrt(-4*a*c + b**2)*
(a*d**2 - b*d*e + c*e**2)) - (b*d + c*e)*log(x)/(c**2*d**2)
```

Mathematica [A] time = 0.286311, size = 194, normalized size = 1.01

$$\frac{\left(2a^2cd-ab(bd+3ce)+b^3e\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{c^2\sqrt{4ac-b^2}\,(e(bd-ce)-ad^2)}+\frac{\left(abd+ace+b^2(-e)\right)\log(x(ax+b)+c)}{2c^2\,(ad^2+e(ce-bd))}+\frac{e^3\log(d+ex)}{ad^4+d^2e(ce-bd)}-\frac{\log(x)(bd+ce)}{c^2d^2}-\frac{1}{cdx}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + c/x^2 + b/x)*x^4*(d + e*x)),x]
```

```
[Out] -(1/(c*d*x)) + ((2*a^2*c*d + b^3*e - a*b*(b*d + 3*c*e))*ArcTan[(b
 + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]*(-(a*d^2)
+ e*(b*d - c*e))) - ((b*d + c*e)*Log[x])/(c^2*d^2) + (e^3*Log[d +
 e*x])/(a*d^4 + d^2*e*(-(b*d) + c*e)) + ((a*b*d - b^2*e + a*c*e)*
Log[c + x*(b + a*x)])/(2*c^2*(a*d^2 + e*(-(b*d) + c*e)))
```

Maple [B] time = 0.017, size = 412, normalized size = 2.1

$$-\frac{1}{cdx}-\frac{\ln(x)\,b}{c^2d}-\frac{\ln(x)\,e}{cd^2}+\frac{e^3\ln(ex+d)}{d^2\,(ad^2-bde+e^2c)}+\frac{a\ln\left(ax^2+bx+c\right)bd}{(2\,ad^2-2\,bde+2\,e^2c)\,c^2}$$
$$+\frac{a\ln\left(ax^2+bx+c\right)e}{(2\,ad^2-2\,bde+2\,e^2c)\,c}-\frac{\ln\left(ax^2+bx+c\right)b^2e}{(2\,ad^2-2\,bde+2\,e^2c)\,c^2}$$
$$-2\,\frac{a^2d}{(ad^2-bde+e^2c)\,c\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right)$$
$$+\frac{ab^2d}{(ad^2-bde+e^2c)\,c^2}\arctan\left((2\,ax+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$
$$+3\,\frac{abe}{(ad^2-bde+e^2c)\,c\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right)$$
$$-\frac{b^3e}{(ad^2-bde+e^2c)\,c^2}\arctan\left((2\,ax+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+c/x^2+b/x)/x^4/(e*x+d),x)
```

```
[Out] -1/c/d/x-1/c^2/d*ln(x)*b-1/c/d^2*ln(x)*e+e^3/d^2/(a*d^2-b*d*e+c*e
^2)*ln(e*x+d)+1/2/(a*d^2-b*d*e+c*e^2)/c^2*a*ln(a*x^2+b*x+c)*b*d+1
/2/(a*d^2-b*d*e+c*e^2)/c*a*ln(a*x^2+b*x+c)*e-1/2/(a*d^2-b*d*e+c*e
^2)/c^2*ln(a*x^2+b*x+c)*b^2*e-2/(a*d^2-b*d*e+c*e^2)/c/(4*a*c-b^2)
^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*d+1/(a*d^2-b*d*e+c
*e^2)/c^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a
*b^2*d+3/(a*d^2-b*d*e+c*e^2)/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)
/(4*a*c-b^2)^(1/2))*a*b*e-1/(a*d^2-b*d*e+c*e^2)/c^2/(4*a*c-b^2)^(
1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^3*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)*(a + b/x + c/x^2)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)*(a + b/x + c/x^2)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+c/x**2+b/x)/x**4/(e*x+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.30186, size = 284, normalized size = 1.47

$$\frac{\left(abd - b^2e + ace\right)\ln\left(ax^2 + bx + c\right)}{2\left(ac^2d^2 - bc^2de + c^3e^2\right)} + \frac{e^4\ln\left(|xe + d|\right)}{ad^4e - bd^3e^2 + cd^2e^3} + \frac{\left(ab^2d - 2\,a^2cd - b^3e + 3\,abce\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(ac^2d^2 - bc^2de + c^3e^2\right)\sqrt{-b^2 + 4\,ac}} - \frac{(bd + ce)\ln\left(|x|\right)}{c^2d^2} - \frac{1}{cdx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)*(a + b/x + c/x^2)*x^4),x, algorithm="giac")
```

```
[Out] 1/2*(a*b*d - b^2*e + a*c*e)*ln(a*x^2 + b*x + c)/(a*c^2*d^2 - b*c^
2*d*e + c^3*e^2) + e^4*ln(abs(x*e + d))/(a*d^4*e - b*d^3*e^2 + c*
d^2*e^3) + (a*b^2*d - 2*a^2*c*d - b^3*e + 3*a*b*c*e)*arctan((2*a*
x + b)/sqrt(-b^2 + 4*a*c))/((a*c^2*d^2 - b*c^2*d*e + c^3*e^2)*sqr
t(-b^2 + 4*a*c)) - (b*d + c*e)*ln(abs(x))/(c^2*d^2) - 1/(c*d*x)
```

3.69 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx$

Optimal. Leaf size=252

$$\frac{\left(a^2cd-ab(bd+2ce)+b^3e\right)\log\left(ax^2+bx+c\right)}{2c^3\left(ad^2-e(bd-ce)\right)}-\frac{\left(a^2c(3bd+2ce)-ab^2(bd+4ce)+b^4e\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{\log(x)\left(-c\left(ad^2-ce^2\right)+b^2d^2+bcde\right)}{c^3d^3}-\frac{e^4\log(d+ex)}{d^3\left(ad^2-e(bd-ce)\right)}+\frac{bd+ce}{c^2d^2x}-\frac{1}{2cdx^2}$$

```
[Out] -1/(2*c*d*x^2) + (b*d + c*e)/(c^2*d^2*x) - ((b^4*e + a^2*c*(3*b*d
 + 2*c*e) - a*b^2*(b*d + 4*c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4
*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + ((b^2*d
^2 + b*c*d*e - c*(a*d^2 - c*e^2))*Log[x])/(c^3*d^3) - (e^4*Log[d
+ e*x])/(d^3*(a*d^2 - e*(b*d - c*e))) + ((a^2*c*d + b^3*e - a*b*(
b*d + 2*c*e))*Log[c + b*x + a*x^2])/(2*c^3*(a*d^2 - e*(b*d - c*e)
))
```

Rubi [A] time = 0.912672, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24

$$\frac{\left(a^2cd-ab(bd+2ce)+b^3e\right)\log\left(ax^2+bx+c\right)}{2c^3\left(ad^2-e(bd-ce)\right)}-\frac{\left(a^2c(3bd+2ce)-ab^2(bd+4ce)+b^4e\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{\log(x)\left(-c\left(ad^2-ce^2\right)+b^2d^2+bcde\right)}{c^3d^3}-\frac{e^4\log(d+ex)}{d^3\left(ad^2-e(bd-ce)\right)}+\frac{bd+ce}{c^2d^2x}-\frac{1}{2cdx^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + c/x^2 + b/x)*x^5*(d + e*x)),x]
```

```
[Out] -1/(2*c*d*x^2) + (b*d + c*e)/(c^2*d^2*x) - ((b^4*e + a^2*c*(3*b*d
 + 2*c*e) - a*b^2*(b*d + 4*c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4
*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + ((b^2*d
^2 + b*c*d*e - c*(a*d^2 - c*e^2))*Log[x])/(c^3*d^3) - (e^4*Log[d
+ e*x])/(d^3*(a*d^2 - e*(b*d - c*e))) + ((a^2*c*d + b^3*e - a*b*(
b*d + 2*c*e))*Log[c + b*x + a*x^2])/(2*c^3*(a*d^2 - e*(b*d - c*e)
))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+c/x**2+b/x)/x**5/(e*x+d),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.376587, size = 252, normalized size = 1.

$$\frac{\left(a^2cd-ab(bd+2ce)+b^3e\right)\log(x(ax+b)+c)}{2c^3\left(ad^2+e(ce-bd)\right)}-\frac{\left(a^2c(3bd+2ce)-ab^2(bd+4ce)+b^4e\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{c^3\sqrt{4ac-b^2}\left(e(bd-ce)-ad^2\right)}+\frac{\log(x)\left(c\left(ce^2-ad^2\right)+b^2d^2+bcde\right)}{c^3d^3}-\frac{e^4\log(d+ex)}{ad^5+d^3e(ce-bd)}+\frac{bd+ce}{c^2d^2x}-\frac{1}{2cdx^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + c/x^2 + b/x)*x^5*(d + e*x)),x]
```

```
[Out] -1/(2*c*d*x^2) + (b*d + c*e)/(c^2*d^2*x) - ((b^4*e + a^2*c*(3*b*d
 + 2*c*e) - a*b^2*(b*d + 4*c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4
*a*c]])/(c^3*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + e*(b*d - c*e))) + ((b
^2*d^2 + b*c*d*e + c*(-(a*d^2) + c*e^2))*Log[x])/(c^3*d^3) - (e^4
*Log[d + e*x])/(a*d^5 + d^3*e*(-(b*d) + c*e)) + ((a^2*c*d + b^3*e
 - a*b*(b*d + 2*c*e))*Log[c + x*(b + a*x)])/(2*c^3*(a*d^2 + e*(-(
b*d) + c*e)))
```

Maple [B] time = 0.018, size = 562, normalized size = 2.2

$$-\frac{1}{2\,cdx^{2}}+\frac{b}{c^{2}xd}+\frac{e}{cxd^{2}}-\frac{\ln(x)\,a}{c^{2}d}+\frac{\ln(x)\,b^{2}}{dc^{3}}+\frac{\ln(x)\,be}{c^{2}d^{2}}+\frac{\ln(x)\,e^{2}}{cd^{3}}-\frac{e^{4}\ln(ex+d)}{d^{3}\,(ad^{2}-bde+e^{2}c)}$$
$$+\frac{a^{2}\ln\left(ax^{2}+bx+c\right)\,d}{(2\,ad^{2}-2\,bde+2\,e^{2}c)\,c^{2}}-\frac{a\ln\left(ax^{2}+bx+c\right)\,b^{2}d}{(2\,ad^{2}-2\,bde+2\,e^{2}c)\,c^{3}}-\frac{a\ln\left(ax^{2}+bx+c\right)\,be}{(ad^{2}-bde+e^{2}c)\,c^{2}}$$
$$+\frac{\ln\left(ax^{2}+bx+c\right)\,b^{3}e}{(2\,ad^{2}-2\,bde+2\,e^{2}c)\,c^{3}}+3\,\frac{a^{2}bd}{(ad^{2}-bde+e^{2}c)\,c^{2}\sqrt{4\,ac-b^{2}}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^{2}}}\right)$$
$$+2\,\frac{a^{2}e}{(ad^{2}-bde+e^{2}c)\,c\sqrt{4\,ac-b^{2}}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^{2}}}\right)$$
$$-\frac{ab^{3}d}{(ad^{2}-bde+e^{2}c)\,c^{3}}\arctan\left((2\,ax+b)\frac{1}{\sqrt{4\,ac-b^{2}}}\right)\frac{1}{\sqrt{4\,ac-b^{2}}}$$
$$-4\,\frac{ab^{2}e}{(ad^{2}-bde+e^{2}c)\,c^{2}\sqrt{4\,ac-b^{2}}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^{2}}}\right)$$
$$+\frac{b^{4}e}{(ad^{2}-bde+e^{2}c)\,c^{3}}\arctan\left((2\,ax+b)\frac{1}{\sqrt{4\,ac-b^{2}}}\right)\frac{1}{\sqrt{4\,ac-b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+c/x^2+b/x)/x^5/(e*x+d),x)
```

```
[Out] -1/2/c/d/x^2+1/x/c^2/d*b+1/x/c/d^2*e-1/d/c^2*ln(x)*a+1/d/c^3*ln(x
)*b^2+1/d^2/c^2*ln(x)*b*e+1/d^3/c*ln(x)*e^2-e^4/d^3/(a*d^2-b*d*e+
c*e^2)*ln(e*x+d)+1/2/(a*d^2-b*d*e+c*e^2)/c^2*a^2*ln(a*x^2+b*x+c)*
d-1/2/(a*d^2-b*d*e+c*e^2)/c^3*a*ln(a*x^2+b*x+c)*b^2*d-1/(a*d^2-b*
d*e+c*e^2)/c^2*a*ln(a*x^2+b*x+c)*b*e+1/2/(a*d^2-b*d*e+c*e^2)/c^3*
ln(a*x^2+b*x+c)*b^3*e+3/(a*d^2-b*d*e+c*e^2)/c^2/(4*a*c-b^2)^(1/2)
*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*b*d+2/(a*d^2-b*d*e+c*e^2
)/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*e*a^2-1
/(a*d^2-b*d*e+c*e^2)/c^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*
c-b^2)^(1/2))*a*b^3*d-4/(a*d^2-b*d*e+c*e^2)/c^2/(4*a*c-b^2)^(1/2)
*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*e+1/(a*d^2-b*d*e+c*e^2
)/c^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^4*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)*(a + b/x + c/x^2)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)*(a + b/x + c/x^2)*x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+c/x**2+b/x)/x**5/(e*x+d),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.300104, size = 377, normalized size = 1.5

$$-\frac{\left(ab^2d-a^2cd-b^3e+2\,abce\right)\ln\left(ax^2+bx+c\right)}{2\left(ac^3d^2-bc^3de+c^4e^2\right)}-\frac{e^5\ln\left(|xe+d|\right)}{ad^5e-bd^4e^2+cd^3e^3}-\frac{\left(ab^3d-3\,a^2bcd-b^4e+4\,ab^2ce-2\,a^2c^2e\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(ac^3d^2-bc^3de+c^4e^2\right)\sqrt{-b^2+4\,ac}}+\frac{\left(b^2d^2-acd^2+bcde+c^2e^2\right)\ln\left(|x|\right)}{c^3d^3}-\frac{c^2d^2-2\left(bcd^2+c^2de\right)x}{2\,c^3d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)*(a + b/x + c/x^2)*x^5),x, algorithm="giac")
```

```
[Out] -1/2*(a*b^2*d - a^2*c*d - b^3*e + 2*a*b*c*e)*ln(a*x^2 + b*x + c)/
(a*c^3*d^2 - b*c^3*d*e + c^4*e^2) - e^5*ln(abs(x*e + d))/(a*d^5*e
 - b*d^4*e^2 + c*d^3*e^3) - (a*b^3*d - 3*a^2*b*c*d - b^4*e + 4*a*
b^2*c*e - 2*a^2*c^2*e)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a
*c^3*d^2 - b*c^3*d*e + c^4*e^2)*sqrt(-b^2 + 4*a*c)) + (b^2*d^2 -
a*c*d^2 + b*c*d*e + c^2*e^2)*ln(abs(x))/(c^3*d^3) - 1/2*(c^2*d^2
- 2*(b*c*d^2 + c^2*d*e)*x)/(c^3*d^3*x^2)
```

3.70 $\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2} dx$

Optimal. Leaf size=343

$$\frac{\left(-b^2c\left(3ad^2-ce^2\right)+4abc^2de+ac^2\left(ad^2-ce^2\right)+b^4d^2-2b^3cde\right)\log\left(ax^2+bx+c\right)}{2a^3\left(ad^2-e(bd-ce)\right)^2}-\frac{x(2ad+be)}{a^2e^3}$$
$$+\frac{\left(-4a^2c^3de-b^3c\left(5ad^2-ce^2\right)+8ab^2c^2de+abc^2\left(5ad^2-3ce^2\right)+b^5d^2-2b^4cde\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}$$
$$+\frac{d^5}{e^4(d+ex)\left(ad^2-e(bd-ce)\right)}+\frac{d^4\log(d+ex)\left(3ad^2-e(4bd-5ce)\right)}{e^4\left(ad^2-e(bd-ce)\right)^2}+\frac{x^2}{2ae^2}$$

```
[Out] -(((2*a*d + b*e)*x)/(a^2*e^3)) + x^2/(2*a*e^2) + d^5/(e^4*(a*d^2
- e*(b*d - c*e))*(d + e*x)) + ((b^5*d^2 - 2*b^4*c*d*e + 8*a*b^2*c
^2*d*e - 4*a^2*c^3*d*e + a*b*c^2*(5*a*d^2 - 3*c*e^2) - b^3*c*(5*a
*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[
b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + (d^4*(3*a*d^2 - e*(4*b*
d - 5*c*e))*Log[d + e*x])/(e^4*(a*d^2 - e*(b*d - c*e))^2) + ((b^4
*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + a*c^2*(a*d^2 - c*e^2) - b^2*
c*(3*a*d^2 - c*e^2))*Log[c + b*x + a*x^2])/(2*a^3*(a*d^2 - e*(b*d
 - c*e))^2)
```

Rubi [A] time = 1.80418, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{\left(-b^2c\left(3ad^2-ce^2\right)+4abc^2de+ac^2\left(ad^2-ce^2\right)+b^4d^2-2b^3cde\right)\log\left(ax^2+bx+c\right)}{2a^3\left(ad^2-e(bd-ce)\right)^2}-\frac{x(2ad+be)}{a^2e^3}$$
$$+\frac{\left(-4a^2c^3de-b^3c\left(5ad^2-ce^2\right)+8ab^2c^2de+abc^2\left(5ad^2-3ce^2\right)+b^5d^2-2b^4cde\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}$$
$$+\frac{d^5}{e^4(d+ex)\left(ad^2-e(bd-ce)\right)}+\frac{d^4\log(d+ex)\left(3ad^2-e(4bd-5ce)\right)}{e^4\left(ad^2-e(bd-ce)\right)^2}+\frac{x^2}{2ae^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```


```
[Out] -(((2*a*d + b*e)*x)/(a^2*e^3)) + x^2/(2*a*e^2) + d^5/(e^4*(a*d^2
- e*(b*d - c*e))*(d + e*x)) + ((b^5*d^2 - 2*b^4*c*d*e + 8*a*b^2*c
^2*d*e - 4*a^2*c^3*d*e + a*b*c^2*(5*a*d^2 - 3*c*e^2) - b^3*c*(5*a
*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[
b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + (d^4*(3*a*d^2 - e*(4*b*
d - 5*c*e))*Log[d + e*x])/(e^4*(a*d^2 - e*(b*d - c*e))^2) + ((b^4
```

```
*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + a*c^2*(a*d^2 - c*e^2) - b^2*
c*(3*a*d^2 - c*e^2))*Log[c + b*x + a*x^2])/(2*a^3*(a*d^2 - e*(b*d
 - c*e))^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.65833, size = 338, normalized size = 0.99

$$\frac{\left(b^2c\left(ce^2-3ad^2\right)+4abc^2de+ac^2\left(ad^2-ce^2\right)+b^4d^2-2b^3cde\right)\log(x(ax+b)+c)}{2a^3\left(ad^2+e(ce-bd)\right)^2}-\frac{x(2ad+be)}{a^2e^3}-\frac{\left(-4a^2c^3de+b^3c\left(ce^2-5ad^2\right)+8ab^2c^2de+abc^2\left(5ad^2-3ce^2\right)+b^5d^2-2b^4cde\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a^3\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)^2}+\frac{d^5}{e^4(d+ex)\left(ad^2+e(ce-bd)\right)}+\frac{\log(d+ex)\left(3ad^6+d^4e(5ce-4bd)\right)}{e^4\left(ad^2+e(ce-bd)\right)^2}+\frac{x^2}{2ae^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] -((((2*a*d + b*e)*x)/(a^2*e^3)) + x^2/(2*a*e^2) + d^5/(e^4*(a*d^2
+ e*(-(b*d) + c*e))*(d + e*x)) - ((b^5*d^2 - 2*b^4*c*d*e + 8*a*b^
2*c^2*d*e - 4*a^2*c^3*d*e + a*b*c^2*(5*a*d^2 - 3*c*e^2) + b^3*c*(
-5*a*d^2 + c*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^3*S
qrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))^2) + ((3*a*d^6 + d^4
*e*(-4*b*d + 5*c*e))*Log[d + e*x])/(e^4*(a*d^2 + e*(-(b*d) + c*e)
)^2) + ((b^4*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + a*c^2*(a*d^2 - c
*e^2) + b^2*c*(-3*a*d^2 + c*e^2))*Log[c + x*(b + a*x)])/(2*a^3*(a
*d^2 + e*(-(b*d) + c*e))^2)
```

Maple [B] time = 0.02, size = 943, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x)
```

```
[Out] 1/2*x^2/a/e^2-2/a/e^3*d*x-1/a^2/e^2*b*x+3/e^4*d^6/(a*d^2-b*d*e+c*
e^2)^2*ln(e*x+d)*a-4/e^3*d^5/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*b+5/
e^2*d^4/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*c+1/e^4*d^5/(a*d^2-b*d*e+
c*e^2)/(e*x+d)+1/2/(a*d^2-b*d*e+c*e^2)^2/a*ln(a*x^2+b*x+c)*c^2*d^
2-3/2/(a*d^2-b*d*e+c*e^2)^2/a^2*ln(a*x^2+b*x+c)*b^2*c*d^2+2/(a*d^
2-b*d*e+c*e^2)^2/a^2*ln(a*x^2+b*x+c)*b*c^2*d*e-1/2/(a*d^2-b*d*e+c
*e^2)^2/a^2*ln(a*x^2+b*x+c)*c^3*e^2+1/2/(a*d^2-b*d*e+c*e^2)^2/a^3
*ln(a*x^2+b*x+c)*b^4*d^2-1/(a*d^2-b*d*e+c*e^2)^2/a^3*ln(a*x^2+b*x
+c)*b^3*c*d*e+1/2/(a*d^2-b*d*e+c*e^2)^2/a^3*ln(a*x^2+b*x+c)*b^2*c
^2*e^2-5/(a*d^2-b*d*e+c*e^2)^2/a/(4*a*c-b^2)^(1/2)*arctan((2*a*x+
b)/(4*a*c-b^2)^(1/2))*b*c^2*d^2+4/(a*d^2-b*d*e+c*e^2)^2/a/(4*a*c-
b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c^3*d*e+5/(a*d^2-b
*d*e+c*e^2)^2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^
(1/2))*b^3*c*d^2-8/(a*d^2-b*d*e+c*e^2)^2/a^2/(4*a*c-b^2)^(1/2)*ar
ctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*c^2*d*e+3/(a*d^2-b*d*e+c*e^
2)^2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*
c^3*e^2-1/(a*d^2-b*d*e+c*e^2)^2/a^3/(4*a*c-b^2)^(1/2)*arctan((2*a
*x+b)/(4*a*c-b^2)^(1/2))*b^5*d^2+2/(a*d^2-b*d*e+c*e^2)^2/a^3/(4*a
*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^4*c*d*e-1/(a*
d^2-b*d*e+c*e^2)^2/a^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-
b^2)^(1/2))*b^3*c^2*e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((e*x + d)^2*(a + b/x + c/x^2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 118.759, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((e*x + d)^2*(a + b/x + c/x^2)),x, algorithm="fricas")
```

```
[Out] [-1/2*(((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3*e^4 - 2*(b^4*c - 4*a*
b^2*c^2 + 2*a^2*c^3)*d^2*e^5 + (b^3*c^2 - 3*a*b*c^3)*d*e^6 + ((b^
5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^2*e^5 - 2*(b^4*c - 4*a*b^2*c^2 + 2
*a^2*c^3)*d*e^6 + (b^3*c^2 - 3*a*b*c^3)*e^7)*x)*log(-(b^3 - 4*a*b
*c + 2*(a*b^2 - 4*a^2*c)*x - (2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c)*
sqrt(b^2 - 4*a*c))/(a*x^2 + b*x + c)) - (2*a^4*d^7 - 2*a^3*b*d^6*
e + 2*a^3*c*d^5*e^2 + (a^4*d^4*e^3 - 2*a^3*b*d^3*e^4 - 2*a^2*b*c*
d*e^6 + a^2*c^2*e^7 + (a^2*b^2 + 2*a^3*c)*d^2*e^5)*x^3 - (3*a^4*d
^5*e^2 - 4*a^3*b*d^4*e^3 + 2*a*b*c^2*e^7 - (a^2*b^2 - 6*a^3*c)*d^
3*e^4 + 2*(a*b^3 - a^2*b*c)*d^2*e^5 - (4*a*b^2*c - 3*a^2*c^2)*d*e
^6)*x^2 - 2*(2*a^4*d^6*e - 3*a^3*b*d^5*e^2 + 4*a^3*c*d^4*e^3 + a*
b*c^2*d*e^6 + (a*b^3 - 2*a^2*b*c)*d^3*e^4 - 2*(a*b^2*c - a^2*c^2)
*d^2*e^5)*x + ((b^4 - 3*a*b^2*c + a^2*c^2)*d^3*e^4 - 2*(b^3*c - 2
*a*b*c^2)*d^2*e^5 + (b^2*c^2 - a*c^3)*d*e^6 + ((b^4 - 3*a*b^2*c +
 a^2*c^2)*d^2*e^5 - 2*(b^3*c - 2*a*b*c^2)*d*e^6 + (b^2*c^2 - a*c^
3)*e^7)*x)*log(a*x^2 + b*x + c) + 2*(3*a^4*d^7 - 4*a^3*b*d^6*e +
5*a^3*c*d^5*e^2 + (3*a^4*d^6*e - 4*a^3*b*d^5*e^2 + 5*a^3*c*d^4*e^
3)*x)*log(e*x + d))*sqrt(b^2 - 4*a*c))/((a^5*d^5*e^4 - 2*a^4*b*d^
4*e^5 - 2*a^3*b*c*d^2*e^7 + a^3*c^2*d*e^8 + (a^3*b^2 + 2*a^4*c)*d
^3*e^6 + (a^5*d^4*e^5 - 2*a^4*b*d^3*e^6 - 2*a^3*b*c*d*e^8 + a^3*c
^2*e^9 + (a^3*b^2 + 2*a^4*c)*d^2*e^7)*x)*sqrt(b^2 - 4*a*c)), -1/2
*(2*((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3*e^4 - 2*(b^4*c - 4*a*b^2
*c^2 + 2*a^2*c^3)*d^2*e^5 + (b^3*c^2 - 3*a*b*c^3)*d*e^6 + ((b^5 -
 5*a*b^3*c + 5*a^2*b*c^2)*d^2*e^5 - 2*(b^4*c - 4*a*b^2*c^2 + 2*a^
2*c^3)*d*e^6 + (b^3*c^2 - 3*a*b*c^3)*e^7)*x)*arctan(-sqrt(-b^2 +
4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) - (2*a^4*d^7 - 2*a^3*b*d^6*e +
2*a^3*c*d^5*e^2 + (a^4*d^4*e^3 - 2*a^3*b*d^3*e^4 - 2*a^2*b*c*d*e^
6 + a^2*c^2*e^7 + (a^2*b^2 + 2*a^3*c)*d^2*e^5)*x^3 - (3*a^4*d^5*e
^2 - 4*a^3*b*d^4*e^3 + 2*a*b*c^2*e^7 - (a^2*b^2 - 6*a^3*c)*d^3*e^
4 + 2*(a*b^3 - a^2*b*c)*d^2*e^5 - (4*a*b^2*c - 3*a^2*c^2)*d*e^6)*
x^2 - 2*(2*a^4*d^6*e - 3*a^3*b*d^5*e^2 + 4*a^3*c*d^4*e^3 + a*b*c^
2*d*e^6 + (a*b^3 - 2*a^2*b*c)*d^3*e^4 - 2*(a*b^2*c - a^2*c^2)*d^2
*e^5)*x + ((b^4 - 3*a*b^2*c + a^2*c^2)*d^3*e^4 - 2*(b^3*c - 2*a*b
*c^2)*d^2*e^5 + (b^2*c^2 - a*c^3)*d*e^6 + ((b^4 - 3*a*b^2*c + a^2
*c^2)*d^2*e^5 - 2*(b^3*c - 2*a*b*c^2)*d*e^6 + (b^2*c^2 - a*c^3)*e
^7)*x)*log(a*x^2 + b*x + c) + 2*(3*a^4*d^7 - 4*a^3*b*d^6*e + 5*a^
3*c*d^5*e^2 + (3*a^4*d^6*e - 4*a^3*b*d^5*e^2 + 5*a^3*c*d^4*e^3)*x
)*log(e*x + d))*sqrt(-b^2 + 4*a*c))/((a^5*d^5*e^4 - 2*a^4*b*d^4*e
^5 - 2*a^3*b*c*d^2*e^7 + a^3*c^2*d*e^8 + (a^3*b^2 + 2*a^4*c)*d^3*
e^6 + (a^5*d^4*e^5 - 2*a^4*b*d^3*e^6 - 2*a^3*b*c*d*e^8 + a^3*c^2*
e^9 + (a^3*b^2 + 2*a^4*c)*d^2*e^7)*x)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.303156, size = 763, normalized size = 2.22

$$\frac{d^5e^4}{(ad^2e^8 - bde^9 + ce^{10})(xe + d)} + \frac{\left(b^5d^2e^2 - 5\,ab^3cd^2e^2 + 5\,a^2bc^2d^2e^2 - 2\,b^4cde^3 + 8\,ab^2c^2de^3 - 4\,a^2c^3de^3 + b^3c^2e^4 - 3\,abc^3e^4\right)\arctan\left(-\frac{\left(2\,ad - \frac{2\,ad^2}{xe+d} - be + \frac{2\,bde}{xe+d}\right.}{\sqrt{-b^2+4\,ac}}\right.}{(a^5d^4 - 2\,a^4bd^3e + a^3b^2d^2e^2 + 2\,a^4cd^2e^2 - 2\,a^3bcde^3 + a^3c^2e^4)\sqrt{-b^2 + 4\,ac}} + \frac{\left(a^2 - \frac{2\,(3\,a^2de + abe^2)\,e^{(-1)}}{xe+d}\right)(xe + d)^2e^{(-4)}}{2\,a^3} + \frac{\left(b^4d^2 - 3\,ab^2cd^2 + a^2c^2d^2 - 2\,b^3cde + 4\,abc^2de + b^2c^2e^2 - ac^3e^2\right)\ln\left(-a + \frac{2\,ad}{xe+d} - \frac{ad^2}{(xe+d)^2} - \frac{be}{xe+d} + \frac{bde}{(xe+d)^2} - \frac{ce^2}{(xe+d)^2}\right)}{2\,(a^5d^4 - 2\,a^4bd^3e + a^3b^2d^2e^2 + 2\,a^4cd^2e^2 - 2\,a^3bcde^3 + a^3c^2e^4)} - \frac{\left(3\,a^2d^2 + 2\,abde + b^2e^2 - ace^2\right)e^{(-4)}\ln\left(\frac{|xe+d|\,e^{(-1)}}{(xe+d)^2}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/((e*x + d)^2*(a + b/x + c/x^2)),x, algorithm="giac")
```

```
[Out] d^5*e^4/((a*d^2*e^8 - b*d*e^9 + c*e^10)*(x*e + d)) + (b^5*d^2*e^2
 - 5*a*b^3*c*d^2*e^2 + 5*a^2*b*c^2*d^2*e^2 - 2*b^4*c*d*e^3 + 8*a*
b^2*c^2*d*e^3 - 4*a^2*c^3*d*e^3 + b^3*c^2*e^4 - 3*a*b*c^3*e^4)*ar
ctan(-(2*a*d - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*
e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((a^5*d^4 - 2*a^
4*b*d^3*e + a^3*b^2*d^2*e^2 + 2*a^4*c*d^2*e^2 - 2*a^3*b*c*d*e^3 +
 a^3*c^2*e^4)*sqrt(-b^2 + 4*a*c)) + 1/2*(a^2 - 2*(3*a^2*d*e + a*b
*e^2)*e^(-1)/(x*e + d))*(x*e + d)^2*e^(-4)/a^3 + 1/2*(b^4*d^2 - 3
*a*b^2*c*d^2 + a^2*c^2*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + b^2*c^
2*e^2 - a*c^3*e^2)*ln(-a + 2*a*d/(x*e + d) - a*d^2/(x*e + d)^2 -
b*e/(x*e + d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a^5*d^4 -
 2*a^4*b*d^3*e + a^3*b^2*d^2*e^2 + 2*a^4*c*d^2*e^2 - 2*a^3*b*c*d*
e^3 + a^3*c^2*e^4) - (3*a^2*d^2 + 2*a*b*d*e + b^2*e^2 - a*c*e^2)*
e^(-4)*ln(abs(x*e + d)*e^(-1)/(x*e + d)^2)/a^3
```

3.71 $\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2} dx$

Optimal. Leaf size=274

$$-\frac{(bd-ce)\left(-2acd+b^2d-bce\right)\log\left(ax^2+bx+c\right)}{2a^2\left(ad^2-e(bd-ce)\right)^2}-\frac{\left(-b^2c\left(4ad^2-ce^2\right)+6abc^2de+2ac^2\left(ad^2-ce^2\right)+b^4d^2-2b^3cde\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}-\frac{d^4}{e^3(d+ex)\left(ad^2-e(bd-ce)\right)}-\frac{d^3\log(d+ex)\left(2ad^2-e(3bd-4ce)\right)}{e^3\left(ad^2-e(bd-ce)\right)^2}+\frac{x}{ae^2}$$

```
[Out] x/(a*e^2) - d^4/(e^3*(a*d^2 - e*(b*d - c*e))*(d + e*x)) - ((b^4*d
^2 - 2*b^3*c*d*e + 6*a*b*c^2*d*e + 2*a*c^2*(a*d^2 - c*e^2) - b^2*
c*(4*a*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^2
*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) - (d^3*(2*a*d^2 - e
*(3*b*d - 4*c*e))*Log[d + e*x])/(e^3*(a*d^2 - e*(b*d - c*e))^2) -
 ((b*d - c*e)*(b^2*d - 2*a*c*d - b*c*e)*Log[c + b*x + a*x^2])/(2*
a^2*(a*d^2 - e*(b*d - c*e))^2)
```

Rubi [A] time = 1.10399, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24

$$-\frac{(bd-ce)\left(-2acd+b^2d-bce\right)\log\left(ax^2+bx+c\right)}{2a^2\left(ad^2-e(bd-ce)\right)^2}-\frac{\left(-b^2c\left(4ad^2-ce^2\right)+6abc^2de+2ac^2\left(ad^2-ce^2\right)+b^4d^2-2b^3cde\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}-\frac{d^4}{e^3(d+ex)\left(ad^2-e(bd-ce)\right)}-\frac{d^3\log(d+ex)\left(2ad^2-e(3bd-4ce)\right)}{e^3\left(ad^2-e(bd-ce)\right)^2}+\frac{x}{ae^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```


```
[Out] x/(a*e^2) - d^4/(e^3*(a*d^2 - e*(b*d - c*e))*(d + e*x)) - ((b^4*d
^2 - 2*b^3*c*d*e + 6*a*b*c^2*d*e + 2*a*c^2*(a*d^2 - c*e^2) - b^2*
c*(4*a*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^2
*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) - (d^3*(2*a*d^2 - e
*(3*b*d - 4*c*e))*Log[d + e*x])/(e^3*(a*d^2 - e*(b*d - c*e))^2) -
 ((b*d - c*e)*(b^2*d - 2*a*c*d - b*c*e)*Log[c + b*x + a*x^2])/(2*
a^2*(a*d^2 - e*(b*d - c*e))^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.545471, size = 269, normalized size = 0.98

$$\frac{(bd-ce)\left(2acd+b^2(-d)+bce\right)\log(x(ax+b)+c)}{2a^2\left(ad^2+e(ce-bd)\right)^2}+\frac{\left(b^2c\left(ce^2-4ad^2\right)+6abc^2de+2ac^2\left(ad^2-ce^2\right)+b^4d^2-2b^3cde\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a^2\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)^2}-\frac{d^4}{e^3(d+ex)\left(ad^2+e(ce-bd)\right)}-\frac{\log(d+ex)\left(2ad^5+d^3e(4ce-3bd)\right)}{e^3\left(ad^2+e(ce-bd)\right)^2}+\frac{x}{ae^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] x/(a*e^2) - d^4/(e^3*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) + ((b^
4*d^2 - 2*b^3*c*d*e + 6*a*b*c^2*d*e + 2*a*c^2*(a*d^2 - c*e^2) + b
^2*c*(-4*a*d^2 + c*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/
(a^2*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))^2) - ((2*a*d^5
 + d^3*e*(-3*b*d + 4*c*e))*Log[d + e*x])/(e^3*(a*d^2 + e*(-(b*d)
+ c*e))^2) + ((b*d - c*e)*(-(b^2*d) + 2*a*c*d + b*c*e)*Log[c + x*
(b + a*x)])/(2*a^2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

Maple [B] time = 0.016, size = 765, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x)
```

```
[Out] x/a/e^2-1/e^3*d^4/(a*d^2-b*d*e+c*e^2)/(e*x+d)-2/e^3*d^5/(a*d^2-b*
d*e+c*e^2)^2*ln(e*x+d)*a+3/e^2*d^4/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d
)*b-4/e*d^3/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*c+1/(a*d^2-b*d*e+c*e^
2)^2/a*ln(a*x^2+b*x+c)*b*c*d^2-1/(a*d^2-b*d*e+c*e^2)^2/a*ln(a*x^2
+b*x+c)*c^2*d*e-1/2/(a*d^2-b*d*e+c*e^2)^2/a^2*ln(a*x^2+b*x+c)*b^3
*d^2+1/(a*d^2-b*d*e+c*e^2)^2/a^2*ln(a*x^2+b*x+c)*b^2*d*e*c-1/2/(a
*d^2-b*d*e+c*e^2)^2/a^2*ln(a*x^2+b*x+c)*b*c^2*e^2+2/(a*d^2-b*d*e+
c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c^
2*d^2-4/(a*d^2-b*d*e+c*e^2)^2/a/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b
)/(4*a*c-b^2)^(1/2))*b^2*c*d^2+6/(a*d^2-b*d*e+c*e^2)^2/a/(4*a*c-b
^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c^2*d*e-2/(a*d^2-
b*d*e+c*e^2)^2/a/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(
1/2))*c^3*e^2+1/(a*d^2-b*d*e+c*e^2)^2/a^2/(4*a*c-b^2)^(1/2)*arcta
n((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^4*d^2-2/(a*d^2-b*d*e+c*e^2)^2/a^
2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^3*d*e*c
+1/(a*d^2-b*d*e+c*e^2)^2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(
4*a*c-b^2)^(1/2))*b^2*c^2*e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((e*x + d)^2*(a + b/x + c/x^2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 50.4761, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((e*x + d)^2*(a + b/x + c/x^2)),x, algorithm="fricas")
```

```
[Out] [-1/2*(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3*e^3 - 2*(b^3*c - 3*a*b*
c^2)*d^2*e^4 + (b^2*c^2 - 2*a*c^3)*d*e^5 + ((b^4 - 4*a*b^2*c + 2*
a^2*c^2)*d^2*e^4 - 2*(b^3*c - 3*a*b*c^2)*d*e^5 + (b^2*c^2 - 2*a*c
^3)*e^6)*x)*log((b^3 - 4*a*b*c + 2*(a*b^2 - 4*a^2*c)*x + (2*a^2*x
^2 + 2*a*b*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(a*x^2 + b*x + c))
 + (2*a^3*d^6 - 2*a^2*b*d^5*e + 2*a^2*c*d^4*e^2 - 2*(a^3*d^4*e^2
- 2*a^2*b*d^3*e^3 - 2*a*b*c*d*e^5 + a*c^2*e^6 + (a*b^2 + 2*a^2*c)
*d^2*e^4)*x^2 - 2*(a^3*d^5*e - 2*a^2*b*d^4*e^2 - 2*a*b*c*d^2*e^4
+ a*c^2*d*e^5 + (a*b^2 + 2*a^2*c)*d^3*e^3)*x + (b*c^2*d*e^5 + (b^
```

```
3 - 2*a*b*c)*d^3*e^3 - 2*(b^2*c - a*c^2)*d^2*e^4 + (b*c^2*e^6 + (
b^3 - 2*a*b*c)*d^2*e^4 - 2*(b^2*c - a*c^2)*d*e^5)*x)*log(a*x^2 +
b*x + c) + 2*(2*a^3*d^6 - 3*a^2*b*d^5*e + 4*a^2*c*d^4*e^2 + (2*a^
3*d^5*e - 3*a^2*b*d^4*e^2 + 4*a^2*c*d^3*e^3)*x)*log(e*x + d))*sqr
t(b^2 - 4*a*c))/((a^4*d^5*e^3 - 2*a^3*b*d^4*e^4 - 2*a^2*b*c*d^2*e
^6 + a^2*c^2*d*e^7 + (a^2*b^2 + 2*a^3*c)*d^3*e^5 + (a^4*d^4*e^4 -
 2*a^3*b*d^3*e^5 - 2*a^2*b*c*d*e^7 + a^2*c^2*e^8 + (a^2*b^2 + 2*a
^3*c)*d^2*e^6)*x)*sqrt(b^2 - 4*a*c)), 1/2*(2*((b^4 - 4*a*b^2*c +
2*a^2*c^2)*d^3*e^3 - 2*(b^3*c - 3*a*b*c^2)*d^2*e^4 + (b^2*c^2 - 2
*a*c^3)*d*e^5 + ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^2*e^4 - 2*(b^3*c
 - 3*a*b*c^2)*d*e^5 + (b^2*c^2 - 2*a*c^3)*e^6)*x)*arctan(-sqrt(-b
^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) - (2*a^3*d^6 - 2*a^2*b*d^5
*e + 2*a^2*c*d^4*e^2 - 2*(a^3*d^4*e^2 - 2*a^2*b*d^3*e^3 - 2*a*b*c
*d*e^5 + a*c^2*e^6 + (a*b^2 + 2*a^2*c)*d^2*e^4)*x^2 - 2*(a^3*d^5*
e - 2*a^2*b*d^4*e^2 - 2*a*b*c*d^2*e^4 + a*c^2*d*e^5 + (a*b^2 + 2*
a^2*c)*d^3*e^3)*x + (b*c^2*d*e^5 + (b^3 - 2*a*b*c)*d^3*e^3 - 2*(b
^2*c - a*c^2)*d^2*e^4 + (b*c^2*e^6 + (b^3 - 2*a*b*c)*d^2*e^4 - 2*
(b^2*c - a*c^2)*d*e^5)*x)*log(a*x^2 + b*x + c) + 2*(2*a^3*d^6 - 3
*a^2*b*d^5*e + 4*a^2*c*d^4*e^2 + (2*a^3*d^5*e - 3*a^2*b*d^4*e^2 +
 4*a^2*c*d^3*e^3)*x)*log(e*x + d))*sqrt(-b^2 + 4*a*c))/((a^4*d^5*
e^3 - 2*a^3*b*d^4*e^4 - 2*a^2*b*c*d^2*e^6 + a^2*c^2*d*e^7 + (a^2*
b^2 + 2*a^3*c)*d^3*e^5 + (a^4*d^4*e^4 - 2*a^3*b*d^3*e^5 - 2*a^2*b
*c*d*e^7 + a^2*c^2*e^8 + (a^2*b^2 + 2*a^3*c)*d^2*e^6)*x)*sqrt(-b^
2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.299698, size = 643, normalized size = 2.35

$$-\frac{d^4e^3}{(ad^2e^6 - bde^7 + ce^8)(xe + d)} - \frac{\left(b^4d^2e^2 - 4\,ab^2cd^2e^2 + 2\,a^2c^2d^2e^2 - 2\,b^3cde^3 + 6\,abc^2de^3 + b^2c^2e^4 - 2\,ac^3e^4\right)\arctan\left(-\frac{\left(2\,ad - \frac{2\,ad^2}{xe+d} - be + \frac{2\,bde}{xe+d} - \frac{2\,ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}\right)e^{(-2)}}{(a^4d^4 - 2\,a^3bd^3e + a^2b^2d^2e^2 + 2\,a^3cd^2e^2 - 2\,a^2bcde^3 + a^2c^2e^4)\sqrt{-b^2 + 4\,ac}} + \frac{(xe + d)e^{(-3)}}{a} - \frac{\left(b^3d^2 - 2\,abcd^2 - 2\,b^2cde + 2\,ac^2de + bc^2e^2\right)\ln\left(-a + \frac{2\,ad}{xe+d} - \frac{ad^2}{(xe+d)^2} - \frac{be}{xe+d} + \frac{bde}{(xe+d)^2} - \frac{ce^2}{(xe+d)^2}\right)}{2\,(a^4d^4 - 2\,a^3bd^3e + a^2b^2d^2e^2 + 2\,a^3cd^2e^2 - 2\,a^2bcde^3 + a^2c^2e^4)} + \frac{(2\,ad + be)e^{(-3)}\ln\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((e*x + d)^2*(a + b/x + c/x^2)),x, algorithm="giac")
```

```
[Out] -d^4*e^3/((a*d^2*e^6 - b*d*e^7 + c*e^8)*(x*e + d)) - (b^4*d^2*e^2
 - 4*a*b^2*c*d^2*e^2 + 2*a^2*c^2*d^2*e^2 - 2*b^3*c*d*e^3 + 6*a*b*
c^2*d*e^3 + b^2*c^2*e^4 - 2*a*c^3*e^4)*arctan(-(2*a*d - 2*a*d^2/(
x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/sq
rt(-b^2 + 4*a*c))*e^(-2)/((a^4*d^4 - 2*a^3*b*d^3*e + a^2*b^2*d^2*
e^2 + 2*a^3*c*d^2*e^2 - 2*a^2*b*c*d*e^3 + a^2*c^2*e^4)*sqrt(-b^2
+ 4*a*c)) + (x*e + d)*e^(-3)/a - 1/2*(b^3*d^2 - 2*a*b*c*d^2 - 2*b
^2*c*d*e + 2*a*c^2*d*e + b*c^2*e^2)*ln(-a + 2*a*d/(x*e + d) - a*d
^2/(x*e + d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e +
 d)^2)/(a^4*d^4 - 2*a^3*b*d^3*e + a^2*b^2*d^2*e^2 + 2*a^3*c*d^2*e
^2 - 2*a^2*b*c*d*e^3 + a^2*c^2*e^4) + (2*a*d + b*e)*e^(-3)*ln(abs
(x*e + d)*e^(-1)/(x*e + d)^2)/a^2
```

3.72 $\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

Optimal. Leaf size=246

$$\frac{\left(-c\left(ad^2-ce^2\right)+b^2d^2-2bcde\right)\log\left(ax^2+bx+c\right)}{2a\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(-bc\left(3ad^2-ce^2\right)+4ac^2de+b^3d^2-2b^2cde\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{d^2\log(d+ex)\left(ad^2-e(2bd-3ce)\right)}{e^2\left(ad^2-e(bd-ce)\right)^2}+\frac{d^3}{e^2(d+ex)\left(ad^2-e(bd-ce)\right)}$$

```
[Out] d^3/(e^2*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((b^3*d^2 - 2*b^2*c
*d*e + 4*a*c^2*d*e - b*c*(3*a*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/S
qrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2
) + (d^2*(a*d^2 - e*(2*b*d - 3*c*e))*Log[d + e*x])/(e^2*(a*d^2 -
e*(b*d - c*e))^2) + ((b^2*d^2 - 2*b*c*d*e - c*(a*d^2 - c*e^2))*Lo
g[c + b*x + a*x^2])/(2*a*(a*d^2 - e*(b*d - c*e))^2)
```

Rubi [A] time = 0.833699, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261

$$\frac{\left(-c\left(ad^2-ce^2\right)+b^2d^2-2bcde\right)\log\left(ax^2+bx+c\right)}{2a\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(-bc\left(3ad^2-ce^2\right)+4ac^2de+b^3d^2-2b^2cde\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{d^2\log(d+ex)\left(ad^2-e(2bd-3ce)\right)}{e^2\left(ad^2-e(bd-ce)\right)^2}+\frac{d^3}{e^2(d+ex)\left(ad^2-e(bd-ce)\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```


```
[Out] d^3/(e^2*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((b^3*d^2 - 2*b^2*c
*d*e + 4*a*c^2*d*e - b*c*(3*a*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/S
qrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2
) + (d^2*(a*d^2 - e*(2*b*d - 3*c*e))*Log[d + e*x])/(e^2*(a*d^2 -
e*(b*d - c*e))^2) + ((b^2*d^2 - 2*b*c*d*e - c*(a*d^2 - c*e^2))*Lo
g[c + b*x + a*x^2])/(2*a*(a*d^2 - e*(b*d - c*e))^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.406977, size = 207, normalized size = 0.84

$$\frac{\frac{(c(ce^2-ad^2)+b^2d^2-2bcde)\log(x(ax+b)+c)}{a}-\frac{2(bc(ce^2-3ad^2)+4ac^2de+b^3d^2-2b^2cde)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a\sqrt{4ac-b^2}}+\frac{2\log(d+ex)(ad^4+d^2e(3ce-2bd))}{e^2}+\frac{2d^3(ad^2\cdots}{e^2(\cdots}}{2\left(ad^2+e(ce-bd)\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] ((2*d^3*(a*d^2 + e*(-(b*d) + c*e)))/(e^2*(d + e*x)) - (2*(b^3*d^2
 - 2*b^2*c*d*e + 4*a*c^2*d*e + b*c*(-3*a*d^2 + c*e^2))*ArcTan[(b
+ 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a*Sqrt[-b^2 + 4*a*c]) + (2*(a*d^4
+ d^2*e*(-2*b*d + 3*c*e))*Log[d + e*x])/e^2 + ((b^2*d^2 - 2*b*c*d
*e + c*(-(a*d^2) + c*e^2))*Log[c + x*(b + a*x)])/a)/(2*(a*d^2 + e
*(-(b*d) + c*e))^2)
```

Maple [B] time = 0.014, size = 580, normalized size = 2.4

$$\frac{d^4\ln(ex+d)\,a}{e^2(ad^2-bde+e^2c)^2}-2\frac{d^3\ln(ex+d)\,b}{(ad^2-bde+e^2c)^2\,e}+3\frac{d^2\ln(ex+d)\,c}{(ad^2-bde+e^2c)^2}+\frac{d^3}{e^2(ad^2-bde+e^2c)(ex+d)}-\frac{\ln(ax^2+bx+c)\,cd^2}{2(ad^2-bde+e^2c)^2}+\frac{\ln(ax^2+bx+c)\,b^2d^2}{2(ad^2-bde+e^2c)^2\,a}-\frac{\ln(ax^2+bx+c)\,bcde}{(ad^2-bde+e^2c)^2\,a}+\frac{\ln(ax^2+bx+c)\,c^2e^2}{2(ad^2-bde+e^2c)^2\,a}+3\frac{bcd^2}{(ad^2-bde+e^2c)^2\sqrt{4ac-b^2}}\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)-4\frac{c^2de}{(ad^2-bde+e^2c)^2\sqrt{4ac-b^2}}\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)-\frac{b^3d^2}{(ad^2-bde+e^2c)^2\,a}\arctan\left((2ax+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}+2\frac{b^2dec}{(ad^2-bde+e^2c)^2\sqrt{4ac-b^2}a}\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)-\frac{bc^2e^2}{(ad^2-bde+e^2c)^2\,a}\arctan\left((2ax+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a+c/x^2+b/x)/(e*x+d)^2,x)
```

```
[Out] d^4/(a*d^2-b*d*e+c*e^2)^2/e^2*ln(e*x+d)*a-2*d^3/(a*d^2-b*d*e+c*e^
2)^2/e*ln(e*x+d)*b+3*d^2/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*c+d^3/e^
2/(a*d^2-b*d*e+c*e^2)/(e*x+d)-1/2/(a*d^2-b*d*e+c*e^2)^2*ln(a*x^2+
b*x+c)*c*d^2+1/2/(a*d^2-b*d*e+c*e^2)^2/a*ln(a*x^2+b*x+c)*b^2*d^2-
1/(a*d^2-b*d*e+c*e^2)^2/a*ln(a*x^2+b*x+c)*b*c*d*e+1/2/(a*d^2-b*d*
e+c*e^2)^2/a*ln(a*x^2+b*x+c)*c^2*e^2+3/(a*d^2-b*d*e+c*e^2)^2/(4*a
*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c*d^2-4/(a*d^
2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(
1/2))*c^2*d*e-1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2
*a*x+b)/(4*a*c-b^2)^(1/2))*b^3/a*d^2+2/(a*d^2-b*d*e+c*e^2)^2/(4*a
*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2/a*c*d*e-1/(
a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^
2)^(1/2))*b/a*c^2*e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((e*x + d)^2*(a + b/x + c/x^2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 17.6323, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((e*x + d)^2*(a + b/x + c/x^2)),x, algorithm="fricas")
```

```
[Out] [1/2*((b*c^2*d*e^4 + (b^3 - 3*a*b*c)*d^3*e^2 - 2*(b^2*c - 2*a*c^2
)*d^2*e^3 + (b*c^2*e^5 + (b^3 - 3*a*b*c)*d^2*e^3 - 2*(b^2*c - 2*a
*c^2)*d*e^4)*x)*log((b^3 - 4*a*b*c + 2*(a*b^2 - 4*a^2*c)*x + (2*a
^2*x^2 + 2*a*b*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(a*x^2 + b*x +
 c)) + (2*a^2*d^5 - 2*a*b*d^4*e + 2*a*c*d^3*e^2 - (2*b*c*d^2*e^3
- c^2*d*e^4 - (b^2 - a*c)*d^3*e^2 + (2*b*c*d*e^4 - c^2*e^5 - (b^2
 - a*c)*d^2*e^3)*x)*log(a*x^2 + b*x + c) + 2*(a^2*d^5 - 2*a*b*d^4
*e + 3*a*c*d^3*e^2 + (a^2*d^4*e - 2*a*b*d^3*e^2 + 3*a*c*d^2*e^3)*
x)*log(e*x + d))*sqrt(b^2 - 4*a*c))/((a^3*d^5*e^2 - 2*a^2*b*d^4*e
^3 - 2*a*b*c*d^2*e^5 + a*c^2*d*e^6 + (a*b^2 + 2*a^2*c)*d^3*e^4 +
(a^3*d^4*e^3 - 2*a^2*b*d^3*e^4 - 2*a*b*c*d*e^6 + a*c^2*e^7 + (a*b
^2 + 2*a^2*c)*d^2*e^5)*x)*sqrt(b^2 - 4*a*c)), -1/2*(2*(b*c^2*d*e^
4 + (b^3 - 3*a*b*c)*d^3*e^2 - 2*(b^2*c - 2*a*c^2)*d^2*e^3 + (b*c^
2*e^5 + (b^3 - 3*a*b*c)*d^2*e^3 - 2*(b^2*c - 2*a*c^2)*d*e^4)*x)*a
rctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) - (2*a^2*d^5
 - 2*a*b*d^4*e + 2*a*c*d^3*e^2 - (2*b*c*d^2*e^3 - c^2*d*e^4 - (b^
2 - a*c)*d^3*e^2 + (2*b*c*d*e^4 - c^2*e^5 - (b^2 - a*c)*d^2*e^3)*
x)*log(a*x^2 + b*x + c) + 2*(a^2*d^5 - 2*a*b*d^4*e + 3*a*c*d^3*e^
2 + (a^2*d^4*e - 2*a*b*d^3*e^2 + 3*a*c*d^2*e^3)*x)*log(e*x + d))*
sqrt(-b^2 + 4*a*c))/((a^3*d^5*e^2 - 2*a^2*b*d^4*e^3 - 2*a*b*c*d^2
*e^5 + a*c^2*d*e^6 + (a*b^2 + 2*a^2*c)*d^3*e^4 + (a^3*d^4*e^3 - 2
*a^2*b*d^3*e^4 - 2*a*b*c*d*e^6 + a*c^2*e^7 + (a*b^2 + 2*a^2*c)*d^
2*e^5)*x)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.28371, size = 556, normalized size = 2.26

$$\frac{1}{2}\left(\frac{2\,d^3e^2}{(ad^2e^3 - bde^4 + ce^5)(xe+d)} + \frac{2\left(b^3d^2e^3 - 3\,abcd^2e^3 - 2\,b^2cde^4 + 4\,ac^2de^4 + bc^2e^5\right)\arctan\left(-\frac{\left(2\,ad - \frac{2\,ad^2}{xe+d} - be + \frac{2\,bde}{xe+d} - \frac{2\,ce^2}{xe+d}\right)}{\sqrt{-b^2+4\,ac}}\right.}{(a^3d^4 - 2\,a^2bd^3e + ab^2d^2e^2 + 2\,a^2cd^2e^2 - 2\,abcde^3 + ac^2e^4)\sqrt{-b^2+4\,ac}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((e*x + d)^2*(a + b/x + c/x^2)),x, algorithm="giac")
```

```
[Out] 1/2*(2*d^3*e^2/((a*d^2*e^3 - b*d*e^4 + c*e^5)*(x*e + d)) + 2*(b^3
*d^2*e^3 - 3*a*b*c*d^2*e^3 - 2*b^2*c*d*e^4 + 4*a*c^2*d*e^4 + b*c^
2*e^5)*arctan(-(2*a*d - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e +
d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((a^3*d
^4 - 2*a^2*b*d^3*e + a*b^2*d^2*e^2 + 2*a^2*c*d^2*e^2 - 2*a*b*c*d*
e^3 + a*c^2*e^4)*sqrt(-b^2 + 4*a*c)) + (b^2*d^2*e - a*c*d^2*e - 2
*b*c*d*e^2 + c^2*e^3)*ln(-a + 2*a*d/(x*e + d) - a*d^2/(x*e + d)^2
 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a^3*d^
4 - 2*a^2*b*d^3*e + a*b^2*d^2*e^2 + 2*a^2*c*d^2*e^2 - 2*a*b*c*d*e
^3 + a*c^2*e^4) - 2*e^(-1)*ln(abs(x*e + d)*e^(-1)/(x*e + d)^2)/a)
*e^(-1)
```

3.73 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

Optimal. Leaf size=194

$$-\frac{\left(-2c\left(ad^2-ce^2\right)+b^2d^2-2bcde\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2}-\frac{d^2}{e(d+ex)\,(ad^2-bde+ce^2)}-\frac{d(bd-2ce)\log\left(ax^2+bx+c\right)}{2\,(ad^2-e(bd-ce))^2}+\frac{d(bd-2ce)\log(d+ex)}{(ad^2-e(bd-ce))^2}$$

```
[Out] -(d^2/(e*(a*d^2 - b*d*e + c*e^2)*(d + e*x))) - ((b^2*d^2 - 2*b*c*
d*e - 2*c*(a*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]]
)/(Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + (d*(b*d - 2*c*e
)*Log[d + e*x])/(a*d^2 - e*(b*d - c*e))^2 - (d*(b*d - 2*c*e)*Log[
c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e))^2)
```

Rubi [A] time = 0.558972, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-\frac{\left(-2c\left(ad^2-ce^2\right)+b^2d^2-2bcde\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2}-\frac{d^2}{e(d+ex)\,(ad^2-bde+ce^2)}-\frac{d(bd-2ce)\log\left(ax^2+bx+c\right)}{2\,(ad^2-e(bd-ce))^2}+\frac{d(bd-2ce)\log(d+ex)}{(ad^2-e(bd-ce))^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] -(d^2/(e*(a*d^2 - b*d*e + c*e^2)*(d + e*x))) - ((b^2*d^2 - 2*b*c*
d*e - 2*c*(a*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]]
)/(Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + (d*(b*d - 2*c*e
)*Log[d + e*x])/(a*d^2 - e*(b*d - c*e))^2 - (d*(b*d - 2*c*e)*Log[
c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e))^2)
```

Rubi in Sympy [A] time = 146.522, size = 177, normalized size = 0.91

$$\frac{d(bd-2ce)\log\left(\frac{d}{x}+e\right)}{(ad^2-bde+ce^2)^2}-\frac{d(bd-2ce)\log\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}{2(ad^2-bde+ce^2)^2}+\frac{d}{\left(\frac{d}{x}+e\right)(ad^2-bde+ce^2)}+\frac{(-2acd^2+b^2d^2-2bcde+2c^2e^2)\operatorname{atanh}\left(\frac{b+\frac{2c}{x}}{\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}(ad^2-bde+ce^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] d*(b*d - 2*c*e)*log(d/x + e)/(a*d**2 - b*d*e + c*e**2)**2 - d*(b*
d - 2*c*e)*log(a + b/x + c/x**2)/(2*(a*d**2 - b*d*e + c*e**2)**2)
 + d/((d/x + e)*(a*d**2 - b*d*e + c*e**2)) + (-2*a*c*d**2 + b**2*
d**2 - 2*b*c*d*e + 2*c**2*e**2)*atanh((b + 2*c/x)/sqrt(-4*a*c + b
**2))/(sqrt(-4*a*c + b**2)*(a*d**2 - b*d*e + c*e**2)**2)
```

Mathematica [A] time = 0.406467, size = 159, normalized size = 0.82

$$\frac{\frac{2(2c(ce^2-ad^2)+b^2d^2-2bcde)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{2d^2(ad^2+e(ce-bd))}{e(d+ex)}-d(bd-2ce)\log(x(ax+b)+c)+2d(bd-2ce)\log(d+ex)}{2(ad^2+e(ce-bd))^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] ((-2*d^2*(a*d^2 + e*(-(b*d) + c*e)))/(e*(d + e*x)) + (2*(b^2*d^2
- 2*b*c*d*e + 2*c*(-(a*d^2) + c*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^
2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*d*(b*d - 2*c*e)*Log[d + e*x]
- d*(b*d - 2*c*e)*Log[c + x*(b + a*x)])/(2*(a*d^2 + e*(-(b*d) + c
*e))^2)
```

Maple [B] time = 0.013, size = 389, normalized size = 2.

$$-\frac{d^{2}}{(ad^{2}-bde+e^{2}c)\,e\,(ex+d)}+\frac{d^{2}\ln(ex+d)\,b}{(ad^{2}-bde+e^{2}c)^{2}}-2\,\frac{d\ln(ex+d)\,ce}{(ad^{2}-bde+e^{2}c)^{2}}-\frac{\ln\left(ax^{2}+bx+c\right)bd^{2}}{2\,(ad^{2}-bde+e^{2}c)^{2}}$$
$$+\frac{\ln\left(ax^{2}+bx+c\right)cde}{(ad^{2}-bde+e^{2}c)^{2}}-2\,\frac{acd^{2}}{(ad^{2}-bde+e^{2}c)^{2}\sqrt{4\,ac-b^{2}}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^{2}}}\right)$$
$$+\frac{b^{2}d^{2}}{(ad^{2}-bde+e^{2}c)^{2}}\arctan\left((2\,ax+b)\frac{1}{\sqrt{4\,ac-b^{2}}}\right)\frac{1}{\sqrt{4\,ac-b^{2}}}$$
$$-2\,\frac{bcde}{(ad^{2}-bde+e^{2}c)^{2}\sqrt{4\,ac-b^{2}}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^{2}}}\right)$$
$$+2\,\frac{c^{2}e^{2}}{(ad^{2}-bde+e^{2}c)^{2}\sqrt{4\,ac-b^{2}}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+c/x^2+b/x)/(e*x+d)^2,x)
```

```
[Out] -d^2/e/(a*d^2-b*d*e+c*e^2)/(e*x+d)+d^2/(a*d^2-b*d*e+c*e^2)^2*ln(e
*x+d)*b-2*d/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*c*e-1/2/(a*d^2-b*d*e+
c*e^2)^2*ln(a*x^2+b*x+c)*b*d^2+1/(a*d^2-b*d*e+c*e^2)^2*ln(a*x^2+b
*x+c)*c*d*e-2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a
*x+b)/(4*a*c-b^2)^(1/2))*a*c*d^2+1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b
^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*d^2-2/(a*d^2-b*
d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)
)*b*c*d*e+2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x
+b)/(4*a*c-b^2)^(1/2))*c^2*e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)^2*(a + b/x + c/x^2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 6.04516, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)^2*(a + b/x + c/x^2)),x, algorithm="fricas")
```

```
[Out] [-1/2*((2*b*c*d^2*e^2 - 2*c^2*d*e^3 - (b^2 - 2*a*c)*d^3*e + (2*b*
c*d*e^3 - 2*c^2*e^4 - (b^2 - 2*a*c)*d^2*e^2)*x)*log(-(b^3 - 4*a*b
*c + 2*(a*b^2 - 4*a^2*c)*x - (2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c)*
sqrt(b^2 - 4*a*c))/(a*x^2 + b*x + c)) + (2*a*d^4 - 2*b*d^3*e + 2*
c*d^2*e^2 + (b*d^3*e - 2*c*d^2*e^2 + (b*d^2*e^2 - 2*c*d*e^3)*x)*l
og(a*x^2 + b*x + c) - 2*(b*d^3*e - 2*c*d^2*e^2 + (b*d^2*e^2 - 2*c
*d*e^3)*x)*log(e*x + d))*sqrt(b^2 - 4*a*c))/((a^2*d^5*e - 2*a*b*d
^4*e^2 - 2*b*c*d^2*e^4 + c^2*d*e^5 + (b^2 + 2*a*c)*d^3*e^3 + (a^2
*d^4*e^2 - 2*a*b*d^3*e^3 - 2*b*c*d*e^5 + c^2*e^6 + (b^2 + 2*a*c)*
d^2*e^4)*x)*sqrt(b^2 - 4*a*c)), -1/2*(2*(2*b*c*d^2*e^2 - 2*c^2*d*
e^3 - (b^2 - 2*a*c)*d^3*e + (2*b*c*d*e^3 - 2*c^2*e^4 - (b^2 - 2*a
*c)*d^2*e^2)*x)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a
*c)) + (2*a*d^4 - 2*b*d^3*e + 2*c*d^2*e^2 + (b*d^3*e - 2*c*d^2*e^
2 + (b*d^2*e^2 - 2*c*d*e^3)*x)*log(a*x^2 + b*x + c) - 2*(b*d^3*e
- 2*c*d^2*e^2 + (b*d^2*e^2 - 2*c*d*e^3)*x)*log(e*x + d))*sqrt(-b^
2 + 4*a*c))/((a^2*d^5*e - 2*a*b*d^4*e^2 - 2*b*c*d^2*e^4 + c^2*d*e
^5 + (b^2 + 2*a*c)*d^3*e^3 + (a^2*d^4*e^2 - 2*a*b*d^3*e^3 - 2*b*c
*d*e^5 + c^2*e^6 + (b^2 + 2*a*c)*d^2*e^4)*x)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.273423, size = 455, normalized size = 2.35

$$-\frac{\left(b^{2} d^{2} e^{2} - 2\, a c d^{2} e^{2} - 2\, b c d e^{3} + 2\, c^{2} e^{4}\right) \arctan\left(-\frac{\left(2\, a d - \frac{2\, a d^{2}}{x e + d} - b e + \frac{2\, b d e}{x e + d} - \frac{2\, c e^{2}}{x e + d}\right) e^{(-1)}}{\sqrt{-b^{2} + 4\, a c}}\right) e^{(-2)}}{\left(a^{2} d^{4} - 2\, a b d^{3} e + b^{2} d^{2} e^{2} + 2\, a c d^{2} e^{2} - 2\, b c d e^{3} + c^{2} e^{4}\right) \sqrt{-b^{2} + 4\, a c}} - \frac{d^{2} e}{\left(a d^{2} e^{2} - b d e^{3} + c e^{4}\right) \left(x e + d\right)} - \frac{\left(b d^{2} - 2\, c d e\right) \ln\left(-a + \frac{2\, a d}{x e + d} - \frac{a d^{2}}{(x e + d)^{2}} - \frac{b e}{x e + d} + \frac{b d e}{(x e + d)^{2}} - \frac{c e^{2}}{(x e + d)^{2}}\right)}{2\, \left(a^{2} d^{4} - 2\, a b d^{3} e + b^{2} d^{2} e^{2} + 2\, a c d^{2} e^{2} - 2\, b c d e^{3} + c^{2} e^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)^2*(a + b/x + c/x^2)),x, algorithm="giac")
```

```
[Out] -(b^2*d^2*e^2 - 2*a*c*d^2*e^2 - 2*b*c*d*e^3 + 2*c^2*e^4)*arctan(-
(2*a*d - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x
*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((a^2*d^4 - 2*a*b*d^3*
e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*b*c*d*e^3 + c^2*e^4)*sqrt(-b^
2 + 4*a*c)) - d^2*e/((a*d^2*e^2 - b*d*e^3 + c*e^4)*(x*e + d)) - 1
/2*(b*d^2 - 2*c*d*e)*ln(-a + 2*a*d/(x*e + d) - a*d^2/(x*e + d)^2
- b*e/(x*e + d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a^2*d^4
 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*b*c*d*e^3 + c^2*
e^4)
```

3.74 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx$

Optimal. Leaf size=183

$$\frac{\left(ad(bd-4ce)+bce^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2}+\frac{\left(ad^2-ce^2\right)\log\left(ax^2+bx+c\right)}{2\,(ad^2-e(bd-ce))^2}+\frac{d}{(d+ex)\,(ad^2-bde+ce^2)}-\frac{\left(ad^2-ce^2\right)\log(d+ex)}{(ad^2-e(bd-ce))^2}$$

```
[Out] d/((a*d^2 - b*d*e + c*e^2)*(d + e*x)) + ((b*c*e^2 + a*d*(b*d - 4*
c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*
(a*d^2 - e*(b*d - c*e))^2) - ((a*d^2 - c*e^2)*Log[d + e*x])/(a*d^
2 - e*(b*d - c*e))^2 + ((a*d^2 - c*e^2)*Log[c + b*x + a*x^2])/(2*
(a*d^2 - e*(b*d - c*e))^2)
```

Rubi [A] time = 0.507696, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{\left(ad(bd-4ce)+bce^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2}+\frac{\left(ad^2-ce^2\right)\log\left(ax^2+bx+c\right)}{2\,(ad^2-e(bd-ce))^2}+\frac{d}{(d+ex)\,(ad^2-bde+ce^2)}-\frac{\left(ad^2-ce^2\right)\log(d+ex)}{(ad^2-e(bd-ce))^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + c/x^2 + b/x)*x*(d + e*x)^2),x]
```

```
[Out] d/((a*d^2 - b*d*e + c*e^2)*(d + e*x)) + ((b*c*e^2 + a*d*(b*d - 4*
c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*
(a*d^2 - e*(b*d - c*e))^2) - ((a*d^2 - c*e^2)*Log[d + e*x])/(a*d^
2 - e*(b*d - c*e))^2 + ((a*d^2 - c*e^2)*Log[c + b*x + a*x^2])/(2*
(a*d^2 - e*(b*d - c*e))^2)
```

Rubi in Sympy [A] time = 103.401, size = 167, normalized size = 0.91

$$\frac{d}{(d+ex)\,(ad^2-bde+ce^2)}-\frac{\left(ad^2-ce^2\right)\log\left(d+ex\right)}{\left(ad^2-bde+ce^2\right)^2}+\frac{\left(ad^2-ce^2\right)\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)^2}+\frac{\left(abd^2-4acde+bce^2\right)\operatorname{atanh}\left(\frac{2ax+b}{\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}\left(ad^2-bde+ce^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+c/x**2+b/x)/x/(e*x+d)**2,x)
```

```
[Out] d/((d + e*x)*(a*d**2 - b*d*e + c*e**2)) - (a*d**2 - c*e**2)*log(d
 + e*x)/(a*d**2 - b*d*e + c*e**2)**2 + (a*d**2 - c*e**2)*log(a*x*
*2 + b*x + c)/(2*(a*d**2 - b*d*e + c*e**2)**2) + (a*b*d**2 - 4*a*
c*d*e + b*c*e**2)*atanh((2*a*x + b)/sqrt(-4*a*c + b**2))/(sqrt(-4
*a*c + b**2)*(a*d**2 - b*d*e + c*e**2)**2)
```

Mathematica [A] time = 0.440807, size = 148, normalized size = 0.81

$$\frac{-\frac{2\left(ad(bd-4ce)+bce^2\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\left(ad^2-ce^2\right)\log(x(ax+b)+c)+\frac{2d\left(ad^2+e(ce-bd)\right)}{d+ex}+\left(2ce^2-2ad^2\right)\log(d+ex)}{2\left(ad^2+e(ce-bd)\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + c/x^2 + b/x)*x*(d + e*x)^2),x]
```

```
[Out] ((2*d*(a*d^2 + e*(-(b*d) + c*e)))/(d + e*x) - (2*(b*c*e^2 + a*d*(
b*d - 4*c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 +
 4*a*c] + (-2*a*d^2 + 2*c*e^2)*Log[d + e*x] + (a*d^2 - c*e^2)*Log
[c + x*(b + a*x)])/(2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

Maple [A] time = 0.012, size = 328, normalized size = 1.8

$$\frac{d}{\left(ad^2-bde+e^2c\right)(ex+d)}-\frac{\ln(ex+d)\,ad^2}{\left(ad^2-bde+e^2c\right)^2}+\frac{\ln(ex+d)\,e^2c}{\left(ad^2-bde+e^2c\right)^2}+\frac{a\ln\left(ax^2+bx+c\right)d^2}{2\left(ad^2-bde+e^2c\right)^2}-\frac{\ln\left(ax^2+bx+c\right)ce^2}{2\left(ad^2-bde+e^2c\right)^2}-\frac{abd^2}{\left(ad^2-bde+e^2c\right)^2}\arctan\left((2ax+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}+4\frac{acde}{\left(ad^2-bde+e^2c\right)^2\sqrt{4ac-b^2}}\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)-\frac{bce^2}{\left(ad^2-bde+e^2c\right)^2}\arctan\left((2ax+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x)
```

```
[Out] d/(a*d^2-b*d*e+c*e^2)/(e*x+d)-1/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*a
*d^2+1/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*e^2*c+1/2/(a*d^2-b*d*e+c*e
^2)^2*a*ln(a*x^2+b*x+c)*d^2-1/2/(a*d^2-b*d*e+c*e^2)^2*ln(a*x^2+b*
x+c)*c*e^2-1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*
x+b)/(4*a*c-b^2)^(1/2))*a*b*d^2+4/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^
2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*c*d*e-1/(a*d^2-b*d
*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))
*b*c*e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((e*x + d)^2*(a + b/x + c/x^2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.22996, size = 1, normalized size = 0.01

$$\left[\frac{\left(abd^3 - 4\,acd^2e + bcde^2 + \left(abd^2e - 4\,acde^2 + bce^3\right)x\right)\log\left(\frac{b^3-4\,abc+2\,(ab^2-4\,a^2c)\,x+(2\,a^2x^2+2\,abx+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{ax^2+bx+c}\right) + \left(2\,ad^3 - \cdots\right.}{2\,(a^2d^5 - 2\,abd^4e - 2\,bcd^2e^3 + c^2de^4 + (b^2 + 2\,ac)d^3e^2 + (a^2 \cdots}\right.$$

$$-\frac{2\left(abd^3 - 4\,acd^2e + bcde^2 + \left(abd^2e - 4\,acde^2 + bce^3\right)x\right)\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,ax+b)}{b^2-4\,ac}\right) - \left(2\,ad^3 - 2\,bd^2e + 2\,cde^2 + \left(ad^3 - c \cdots\right.\right.}{2\,(a^2d^5 - 2\,abd^4e - 2\,bcd^2e^3 + c^2de^4 + (b^2 + 2\,ac)d^3e^2 + (a^2d^4e - 2\,abd^3e \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((e*x + d)^2*(a + b/x + c/x^2)*x),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b*d^3 - 4*a*c*d^2*e + b*c*d*e^2 + (a*b*d^2*e - 4*a*c*d*e
^2 + b*c*e^3)*x)*log((b^3 - 4*a*b*c + 2*(a*b^2 - 4*a^2*c)*x + (2*
a^2*x^2 + 2*a*b*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(a*x^2 + b*x
+ c)) + (2*a*d^3 - 2*b*d^2*e + 2*c*d*e^2 + (a*d^3 - c*d*e^2 + (a*
d^2*e - c*e^3)*x)*log(a*x^2 + b*x + c) - 2*(a*d^3 - c*d*e^2 + (a*
d^2*e - c*e^3)*x)*log(e*x + d))*sqrt(b^2 - 4*a*c))/((a^2*d^5 - 2*
a*b*d^4*e - 2*b*c*d^2*e^3 + c^2*d*e^4 + (b^2 + 2*a*c)*d^3*e^2 + (
a^2*d^4*e - 2*a*b*d^3*e^2 - 2*b*c*d*e^4 + c^2*e^5 + (b^2 + 2*a*c)
*d^2*e^3)*x)*sqrt(b^2 - 4*a*c)), -1/2*(2*(a*b*d^3 - 4*a*c*d^2*e +
 b*c*d*e^2 + (a*b*d^2*e - 4*a*c*d*e^2 + b*c*e^3)*x)*arctan(-sqrt(
```

```
-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) - (2*a*d^3 - 2*b*d^2*e +
 2*c*d*e^2 + (a*d^3 - c*d*e^2 + (a*d^2*e - c*e^3)*x)*log(a*x^2 +
b*x + c) - 2*(a*d^3 - c*d*e^2 + (a*d^2*e - c*e^3)*x)*log(e*x + d)
)*sqrt(-b^2 + 4*a*c))/((a^2*d^5 - 2*a*b*d^4*e - 2*b*c*d^2*e^3 + c
^2*d*e^4 + (b^2 + 2*a*c)*d^3*e^2 + (a^2*d^4*e - 2*a*b*d^3*e^2 - 2
*b*c*d*e^4 + c^2*e^5 + (b^2 + 2*a*c)*d^2*e^3)*x)*sqrt(-b^2 + 4*a*
c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+c/x**2+b/x)/x/(e*x+d)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.276847, size = 441, normalized size = 2.41

$$\frac{1}{2}\left(\frac{2\left(abd^2e-4\,acde^2+bce^3\right)\arctan\left(-\frac{\left(2\,ad-\frac{2\,ad^2}{xe+d}-be+\frac{2\,bde}{xe+d}-\frac{2\,ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}\right)e^{(-2)}}{\left(a^2d^4-2\,abd^3e+b^2d^2e^2+2\,acd^2e^2-2\,bcde^3+c^2e^4\right)\sqrt{-b^2+4\,ac}}+\frac{\left(ad^2-ce^2\right)\ln\left(-a+\frac{2\,ad}{xe+d}-\frac{ad^2}{(xe+d)^2}-\frac{be}{xe+d}+\right)}{a^2d^4e-2\,abd^3e^2+b^2d^2e^3+2\,acd^2e^3-}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)^2*(a + b/x + c/x^2)*x),x, algorithm="giac")
```

```
[Out] 1/2*(2*(a*b*d^2*e - 4*a*c*d*e^2 + b*c*e^3)*arctan(-(2*a*d - 2*a*d
^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1
)/sqrt(-b^2 + 4*a*c))*e^(-2)/((a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^
2 + 2*a*c*d^2*e^2 - 2*b*c*d*e^3 + c^2*e^4)*sqrt(-b^2 + 4*a*c)) +
(a*d^2 - c*e^2)*ln(-a + 2*a*d/(x*e + d) - a*d^2/(x*e + d)^2 - b*e
/(x*e + d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a^2*d^4*e -
2*a*b*d^3*e^2 + b^2*d^2*e^3 + 2*a*c*d^2*e^3 - 2*b*c*d*e^4 + c^2*e
^5) + 2*d*e/((a*d^2*e^2 - b*d*e^3 + c*e^4)*(x*e + d)))*e
```

3.75 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx$

Optimal. Leaf size=189

$$-\frac{\left(2a^2d^2-2ae(bd+ce)+b^2e^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2}-\frac{e}{(d+ex)\,(ad^2-bde+ce^2)}-\frac{e(2ad-be)\log\left(ax^2+bx+c\right)}{2\,(ad^2-e(bd-ce))^2}+\frac{e(2ad-be)\log(d+ex)}{(ad^2-e(bd-ce))^2}$$

```
[Out] -(e/((a*d^2 - b*d*e + c*e^2)*(d + e*x))) - ((2*a^2*d^2 + b^2*e^2
- 2*a*e*(b*d + c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqr
t[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + (e*(2*a*d - b*e)*Log[
d + e*x])/(a*d^2 - e*(b*d - c*e))^2 - (e*(2*a*d - b*e)*Log[c + b*
x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e))^2)
```

Rubi [A] time = 0.647598, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28

$$-\frac{\left(2a^2d^2-2ae(bd+ce)+b^2e^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2}-\frac{e}{(d+ex)\,(ad^2-bde+ce^2)}-\frac{e(2ad-be)\log\left(ax^2+bx+c\right)}{2\,(ad^2-e(bd-ce))^2}+\frac{e(2ad-be)\log(d+ex)}{(ad^2-e(bd-ce))^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + c/x^2 + b/x)*x^2*(d + e*x)^2),x]
```

```
[Out] -(e/((a*d^2 - b*d*e + c*e^2)*(d + e*x))) - ((2*a^2*d^2 + b^2*e^2
- 2*a*e*(b*d + c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqr
t[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + (e*(2*a*d - b*e)*Log[
d + e*x])/(a*d^2 - e*(b*d - c*e))^2 - (e*(2*a*d - b*e)*Log[c + b*
x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e))^2)
```

Rubi in Sympy [A] time = 150.937, size = 177, normalized size = 0.94

$$\frac{e\,(2ad-be)\log\,(d+ex)}{(ad^2-bde+ce^2)^2}-\frac{e\,(2ad-be)\log\left(ax^2+bx+c\right)}{2\,(ad^2-bde+ce^2)^2}-\frac{e}{(d+ex)\,(ad^2-bde+ce^2)}-\frac{\left(2a^2d^2-2abde-2ace^2+b^2e^2\right)\operatorname{atanh}\left(\frac{2ax+b}{\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}\,(ad^2-bde+ce^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+c/x**2+b/x)/x**2/(e*x+d)**2,x)
```

```
[Out] e*(2*a*d - b*e)*log(d + e*x)/(a*d**2 - b*d*e + c*e**2)**2 - e*(2*
a*d - b*e)*log(a*x**2 + b*x + c)/(2*(a*d**2 - b*d*e + c*e**2)**2)
 - e/((d + e*x)*(a*d**2 - b*d*e + c*e**2)) - (2*a**2*d**2 - 2*a*b
*d*e - 2*a*c*e**2 + b**2*e**2)*atanh((2*a*x + b)/sqrt(-4*a*c + b*
*2))/(sqrt(-4*a*c + b**2)*(a*d**2 - b*d*e + c*e**2)**2)
```

Mathematica [A] time = 0.383783, size = 151, normalized size = 0.8

$$\frac{\frac{2\left(2a^2d^2-2ae(bd+ce)+b^2e^2\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{2e\left(ad^2+e(ce-bd)\right)}{d+ex}+e(be-2ad)\log(x(ax+b)+c)-2e(be-2ad)\log(d+ex)}{2\left(ad^2+e(ce-bd)\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + c/x^2 + b/x)*x^2*(d + e*x)^2),x]
```

```
[Out] ((-2*e*(a*d^2 + e*(-(b*d) + c*e)))/(d + e*x) + (2*(2*a^2*d^2 + b^
2*e^2 - 2*a*e*(b*d + c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]]
)/Sqrt[-b^2 + 4*a*c] - 2*e*(-2*a*d + b*e)*Log[d + e*x] + e*(-2*a*
d + b*e)*Log[c + x*(b + a*x)])/(2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

Maple [B] time = 0.013, size = 386, normalized size = 2.

$$-\frac{e}{(ad^2-bde+e^2c)(ex+d)}+2\,\frac{e\ln(ex+d)ad}{(ad^2-bde+e^2c)^2}-\frac{e^2\ln(ex+d)b}{(ad^2-bde+e^2c)^2}-\frac{a\ln\left(ax^2+bx+c\right)de}{(ad^2-bde+e^2c)^2}$$
$$+\frac{\ln\left(ax^2+bx+c\right)be^2}{2\,(ad^2-bde+e^2c)^2}+2\,\frac{a^2d^2}{(ad^2-bde+e^2c)^2\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right)$$
$$-2\,\frac{abde}{(ad^2-bde+e^2c)^2\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right)$$
$$-2\,\frac{ace^2}{(ad^2-bde+e^2c)^2\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right)$$
$$+\frac{b^2e^2}{(ad^2-bde+e^2c)^2}\arctan\left((2\,ax+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x)
```

```
[Out] -e/(a*d^2-b*d*e+c*e^2)/(e*x+d)+2*e/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d
)*a*d-e^2/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*b-1/(a*d^2-b*d*e+c*e^2)
^2*a*ln(a*x^2+b*x+c)*d*e+1/2/(a*d^2-b*d*e+c*e^2)^2*ln(a*x^2+b*x+c
)*b*e^2+2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b
)/(4*a*c-b^2)^(1/2))*a^2*d^2-2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^
(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b*d*e-2/(a*d^2-b*d*e+
c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*
c*e^2+1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/
(4*a*c-b^2)^(1/2))*b^2*e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((e*x + d)^2*(a + b/x + c/x^2)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.75109, size = 1, normalized size = 0.01

$$\left[-\frac{\left(2\,a^2d^3 - 2\,abd^2e + \left(b^2 - 2\,ac\right)de^2 + \left(2\,a^2d^2e - 2\,abde^2 + \left(b^2 - 2\,ac\right)e^3\right)x\right)\log\left(\frac{b^3-4\,abc+2\,\left(ab^2-4\,a^2c\right)x+\left(2\,a^2x^2+2\,abx+b^2-2\,a\right.}{ax^2+bx+c}\right.}{2\left(a^2d^5 - 2\,abd^4e - 2\,bcd^2e^3 + c^2de^4 + (b^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((e*x + d)^2*(a + b/x + c/x^2)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((2*a^2*d^3 - 2*a*b*d^2*e + (b^2 - 2*a*c)*d*e^2 + (2*a^2*d^
2*e - 2*a*b*d*e^2 + (b^2 - 2*a*c)*e^3)*x)*log((b^3 - 4*a*b*c + 2*
(a*b^2 - 4*a^2*c)*x + (2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c)*sqrt(b^
2 - 4*a*c))/(a*x^2 + b*x + c)) + (2*a*d^2*e - 2*b*d*e^2 + 2*c*e^3
 + (2*a*d^2*e - b*d*e^2 + (2*a*d*e^2 - b*e^3)*x)*log(a*x^2 + b*x
+ c) - 2*(2*a*d^2*e - b*d*e^2 + (2*a*d*e^2 - b*e^3)*x)*log(e*x +
d))*sqrt(b^2 - 4*a*c))/((a^2*d^5 - 2*a*b*d^4*e - 2*b*c*d^2*e^3 +
c^2*d*e^4 + (b^2 + 2*a*c)*d^3*e^2 + (a^2*d^4*e - 2*a*b*d^3*e^2 -
2*b*c*d*e^4 + c^2*e^5 + (b^2 + 2*a*c)*d^2*e^3)*x)*sqrt(b^2 - 4*a*
c)), 1/2*(2*(2*a^2*d^3 - 2*a*b*d^2*e + (b^2 - 2*a*c)*d*e^2 + (2*a
^2*d^2*e - 2*a*b*d*e^2 + (b^2 - 2*a*c)*e^3)*x)*arctan(-sqrt(-b^2
+ 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) - (2*a*d^2*e - 2*b*d*e^2 + 2*
c*e^3 + (2*a*d^2*e - b*d*e^2 + (2*a*d*e^2 - b*e^3)*x)*log(a*x^2 +
```

```
 b*x + c) - 2*(2*a*d^2*e - b*d*e^2 + (2*a*d*e^2 - b*e^3)*x)*log(e
*x + d))*sqrt(-b^2 + 4*a*c))/((a^2*d^5 - 2*a*b*d^4*e - 2*b*c*d^2*
e^3 + c^2*d*e^4 + (b^2 + 2*a*c)*d^3*e^2 + (a^2*d^4*e - 2*a*b*d^3*
e^2 - 2*b*c*d*e^4 + c^2*e^5 + (b^2 + 2*a*c)*d^2*e^3)*x)*sqrt(-b^2
 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+c/x**2+b/x)/x**2/(e*x+d)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.290329, size = 447, normalized size = 2.37

$$-\frac{\left(2\,a^2d^2e^2 - 2\,abde^3 + b^2e^4 - 2\,ace^4\right)\arctan\left(-\frac{\left(2\,ad - \frac{2\,ad^2}{xe+d} - be + \frac{2\,bde}{xe+d} - \frac{2\,ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}\right)e^{(-2)}}{(a^2d^4 - 2\,abd^3e + b^2d^2e^2 + 2\,acd^2e^2 - 2\,bcde^3 + c^2e^4)\sqrt{-b^2+4\,ac}} - \frac{\left(2\,ade - be^2\right)\ln\left(-a + \frac{2\,ad}{xe+d} - \frac{ad^2}{(xe+d)^2} - \frac{be}{xe+d} + \frac{bde}{(xe+d)^2} - \frac{ce^2}{(xe+d)^2}\right)}{2\,(a^2d^4 - 2\,abd^3e + b^2d^2e^2 + 2\,acd^2e^2 - 2\,bcde^3 + c^2e^4)} - \frac{e^3}{(ad^2e^2 - bde^3 + ce^4)(xe+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)^2*(a + b/x + c/x^2)*x^2),x, algorithm="giac")
```

```
[Out] -(2*a^2*d^2*e^2 - 2*a*b*d*e^3 + b^2*e^4 - 2*a*c*e^4)*arctan(-(2*a
*d - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e +
 d))*e^(-1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((a^2*d^4 - 2*a*b*d^3*e +
b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*b*c*d*e^3 + c^2*e^4)*sqrt(-b^2 +
4*a*c)) - 1/2*(2*a*d*e - b*e^2)*ln(-a + 2*a*d/(x*e + d) - a*d^2/(
x*e + d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^
2)/(a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*b*c*d
*e^3 + c^2*e^4) - e^3/((a*d^2*e^2 - b*d*e^3 + c*e^4)*(x*e + d))
```

3.76 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx$

Optimal. Leaf size=248

$$-\frac{\left(a^2d^2-ae(2bd+ce)+b^2e^2\right)\log\left(ax^2+bx+c\right)}{2c\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(a^2d(bd+4ce)-abe(2bd+3ce)+b^3e^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{e^2}{d(d+ex)\left(ad^2-bde+ce^2\right)}-\frac{e^2\log(d+ex)\left(3ad^2-e(2bd-ce)\right)}{d^2\left(ad^2-e(bd-ce)\right)^2}+\frac{\log(x)}{cd^2}$$

```
[Out] e^2/(d*(a*d^2 - b*d*e + c*e^2)*(d + e*x)) + ((b^3*e^2 - a*b*e*(2*
b*d + 3*c*e) + a^2*d*(b*d + 4*c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2
- 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + Log[
x]/(c*d^2) - (e^2*(3*a*d^2 - e*(2*b*d - c*e))*Log[d + e*x])/(d^2*
(a*d^2 - e*(b*d - c*e))^2) - ((a^2*d^2 + b^2*e^2 - a*e*(2*b*d + c
*e))*Log[c + b*x + a*x^2])/(2*c*(a*d^2 - e*(b*d - c*e))^2)
```

Rubi [A] time = 0.848912, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$-\frac{\left(a^2d^2-ae(2bd+ce)+b^2e^2\right)\log\left(ax^2+bx+c\right)}{2c\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(a^2d(bd+4ce)-abe(2bd+3ce)+b^3e^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{e^2}{d(d+ex)\left(ad^2-e(bd-ce)\right)}-\frac{e^2\log(d+ex)\left(3ad^2-e(2bd-ce)\right)}{d^2\left(ad^2-e(bd-ce)\right)^2}+\frac{\log(x)}{cd^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + c/x^2 + b/x)*x^3*(d + e*x)^2),x]
```

```
[Out] e^2/(d*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((b^3*e^2 - a*b*e*(2*
b*d + 3*c*e) + a^2*d*(b*d + 4*c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2
- 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + Log[
x]/(c*d^2) - (e^2*(3*a*d^2 - e*(2*b*d - c*e))*Log[d + e*x])/(d^2*
(a*d^2 - e*(b*d - c*e))^2) - ((a^2*d^2 + b^2*e^2 - a*e*(2*b*d + c
*e))*Log[c + b*x + a*x^2])/(2*c*(a*d^2 - e*(b*d - c*e))^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+c/x**2+b/x)/x**3/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.476868, size = 246, normalized size = 0.99

$$\frac{\left(-a^2d^2+ae(2bd+ce)-b^2e^2\right)\log(x(ax+b)+c)}{2c\left(ad^2+e(ce-bd)\right)^2} - \frac{\left(a^2d(bd+4ce)-abe(2bd+3ce)+b^3e^2\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{c\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)^2} + \frac{e^2}{d(d+ex)\left(ad^2+e(ce-bd)\right)} - \frac{e^2\log(d+ex)\left(3ad^2+e(ce-2bd)\right)}{\left(ad^3+de(ce-bd)\right)^2} + \frac{\log(x)}{cd^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + c/x^2 + b/x)*x^3*(d + e*x)^2),x]
```

```
[Out] e^2/(d*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) - ((b^3*e^2 - a*b*e*
(2*b*d + 3*c*e) + a^2*d*(b*d + 4*c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b
^2 + 4*a*c]])/(c*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))^2)
 + Log[x]/(c*d^2) - (e^2*(3*a*d^2 + e*(-2*b*d + c*e))*Log[d + e*x
])/(a*d^3 + d*e*(-(b*d) + c*e))^2 + ((-(a^2*d^2) - b^2*e^2 + a*e*
(2*b*d + c*e))*Log[c + x*(b + a*x)])/(2*c*(a*d^2 + e*(-(b*d) + c*
e))^2)
```

Maple [B] time = 0.019, size = 589, normalized size = 2.4

$$\frac{\ln(x)}{cd^2}+\frac{e^2}{d(ad^2-bde+e^2c)(ex+d)}-3\,\frac{e^2\ln(ex+d)\,a}{(ad^2-bde+e^2c)^2}$$
$$+2\,\frac{e^3\ln(ex+d)\,b}{d(ad^2-bde+e^2c)^2}-\frac{e^4\ln(ex+d)\,c}{(ad^2-bde+e^2c)^2\,d^2}-\frac{a^2\ln\left(ax^2+bx+c\right)d^2}{2\,(ad^2-bde+e^2c)^2\,c}$$
$$+\frac{a\ln\left(ax^2+bx+c\right)bde}{(ad^2-bde+e^2c)^2\,c}+\frac{a\ln\left(ax^2+bx+c\right)e^2}{2\,(ad^2-bde+e^2c)^2}-\frac{\ln\left(ax^2+bx+c\right)b^2e^2}{2\,(ad^2-bde+e^2c)^2\,c}$$
$$-\frac{a^2bd^2}{(ad^2-bde+e^2c)^2\,c}\arctan\left((2\,ax+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$
$$-4\,\frac{a^2de}{(ad^2-bde+e^2c)^2\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right)$$
$$+2\,\frac{ab^2de}{(ad^2-bde+e^2c)^2\,c\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right)$$
$$+3\,\frac{abe^2}{(ad^2-bde+e^2c)^2\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right)$$
$$-\frac{b^3e^2}{(ad^2-bde+e^2c)^2\,c}\arctan\left((2\,ax+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x)
```

```
[Out] ln(x)/c/d^2+e^2/d/(a*d^2-b*d*e+c*e^2)/(e*x+d)-3*e^2/(a*d^2-b*d*e+
c*e^2)^2*ln(e*x+d)*a+2*e^3/d/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*b-e^
4/d^2/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*c-1/2/(a*d^2-b*d*e+c*e^2)^2
/c*a^2*ln(a*x^2+b*x+c)*d^2+1/(a*d^2-b*d*e+c*e^2)^2/c*a*ln(a*x^2+b
*x+c)*b*d*e+1/2/(a*d^2-b*d*e+c*e^2)^2*a*ln(a*x^2+b*x+c)*e^2-1/2/(
a*d^2-b*d*e+c*e^2)^2/c*ln(a*x^2+b*x+c)*b^2*e^2-1/(a*d^2-b*d*e+c*e
^2)^2/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2
*b*d^2-4/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)
/(4*a*c-b^2)^(1/2))*a^2*d*e+2/(a*d^2-b*d*e+c*e^2)^2/c/(4*a*c-b^2)
^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*d*e+3/(a*d^2-b*d
*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))
*a*b*e^2-1/(a*d^2-b*d*e+c*e^2)^2/c/(4*a*c-b^2)^(1/2)*arctan((2*a*
x+b)/(4*a*c-b^2)^(1/2))*b^3*e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((e*x + d)^2*(a + b/x + c/x^2)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((e*x + d)^2*(a + b/x + c/x^2)*x^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**3/(e*x+d)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.297199, size = 528, normalized size = 2.13

$$-\frac{\left(a^2bd^2e^2 - 2\,ab^2de^3 + 4\,a^2cde^3 + b^3e^4 - 3\,abce^4\right)\arctan\left(\frac{\left(2\,ad - \frac{2\,ad^2}{xe+d} - be + \frac{2\,bde}{xe+d} - \frac{2\,ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}\right)e^{(-2)}}{\left(a^2cd^4 - 2\,abcd^3e + b^2cd^2e^2 + 2\,ac^2d^2e^2 - 2\,bc^2de^3 + c^3e^4\right)\sqrt{-b^2+4\,ac}} - \frac{\left(a^2d^2 - 2\,abde + b^2e^2 - ace^2\right)\ln\left(a - \frac{2\,ad}{xe+d} + \frac{ad^2}{(xe+d)^2} + \frac{be}{xe+d} - \frac{bde}{(xe+d)^2} + \frac{ce^2}{(xe+d)^2}\right)}{2\left(a^2cd^4 - 2\,abcd^3e + b^2cd^2e^2 + 2\,ac^2d^2e^2 - 2\,bc^2de^3 + c^3e^4\right)} + \frac{e^5}{\left(ad^3e^3 - bd^2e^4 + cde^5\right)(xe+d)} + \frac{\ln\left(\left|-\frac{d}{xe+d} + 1\right|\right)}{cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((e*x + d)^2*(a + b/x + c/x^2)*x^3),x, algorithm="giac")
```

```
[Out] -(a^2*b*d^2*e^2 - 2*a*b^2*d*e^3 + 4*a^2*c*d*e^3 + b^3*e^4 - 3*a*b
*c*e^4)*arctan((2*a*d - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e +
d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((a^2*c
*d^4 - 2*a*b*c*d^3*e + b^2*c*d^2*e^2 + 2*a*c^2*d^2*e^2 - 2*b*c^2*
d*e^3 + c^3*e^4)*sqrt(-b^2 + 4*a*c)) - 1/2*(a^2*d^2 - 2*a*b*d*e +
 b^2*e^2 - a*c*e^2)*ln(a - 2*a*d/(x*e + d) + a*d^2/(x*e + d)^2 +
b*e/(x*e + d) - b*d*e/(x*e + d)^2 + c*e^2/(x*e + d)^2)/(a^2*c*d^4
 - 2*a*b*c*d^3*e + b^2*c*d^2*e^2 + 2*a*c^2*d^2*e^2 - 2*b*c^2*d*e^
3 + c^3*e^4) + e^5/((a*d^3*e^3 - b*d^2*e^4 + c*d*e^5)*(x*e + d))
+ ln(abs(-d/(x*e + d) + 1))/(c*d^2)
```

3.77 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx$

Optimal. Leaf size=291

$$\frac{\left(2a^3cd^2-a^2\left(b^2d^2+6bcde+2c^2e^2\right)+2ab^2e(bd+2ce)+b^4\left(-e^2\right)\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2}+\frac{(ad-be)\left(abd+2ace+b^2(-e)\right)\log\left(ax^2+bx+c\right)}{2c^2\left(ad^2-e(bd-ce)\right)^2}-\frac{e^3}{d^2(d+ex)\left(ad^2-e(bd-ce)\right)}+\frac{e^3\log(d+ex)\left(4ad^2-e(3bd-2ce)\right)}{d^3\left(ad^2-e(bd-ce)\right)^2}-\frac{\log(x)(bd+2ce)}{c^2d^3}-\frac{1}{cd^2x}$$

```
[Out] -(1/(c*d^2*x)) - e^3/(d^2*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((
2*a^3*c*d^2 - b^4*e^2 + 2*a*b^2*e*(b*d + 2*c*e) - a^2*(b^2*d^2 +
6*b*c*d*e + 2*c^2*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(
c^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) - ((b*d + 2*c*e)
*Log[x])/(c^2*d^3) + (e^3*(4*a*d^2 - e*(3*b*d - 2*c*e))*Log[d + e
*x])/(d^3*(a*d^2 - e*(b*d - c*e))^2) + ((a*d - b*e)*(a*b*d - b^2*
e + 2*a*c*e)*Log[c + b*x + a*x^2])/(2*c^2*(a*d^2 - e*(b*d - c*e))
^2)
```

Rubi [A] time = 1.20222, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24

$$\frac{\left(2a^3cd^2-a^2\left(b^2d^2+6bcde+2c^2e^2\right)+2ab^2e(bd+2ce)+b^4\left(-e^2\right)\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2}+\frac{(ad-be)\left(abd+2ace+b^2(-e)\right)\log\left(ax^2+bx+c\right)}{2c^2\left(ad^2-e(bd-ce)\right)^2}-\frac{e^3}{d^2(d+ex)\left(ad^2-e(bd-ce)\right)}+\frac{e^3\log(d+ex)\left(4ad^2-e(3bd-2ce)\right)}{d^3\left(ad^2-e(bd-ce)\right)^2}-\frac{\log(x)(bd+2ce)}{c^2d^3}-\frac{1}{cd^2x}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + c/x^2 + b/x)*x^4*(d + e*x)^2),x]
```

```
[Out] -(1/(c*d^2*x)) - e^3/(d^2*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((
2*a^3*c*d^2 - b^4*e^2 + 2*a*b^2*e*(b*d + 2*c*e) - a^2*(b^2*d^2 +
6*b*c*d*e + 2*c^2*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(
c^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) - ((b*d + 2*c*e)
*Log[x])/(c^2*d^3) + (e^3*(4*a*d^2 - e*(3*b*d - 2*c*e))*Log[d + e
*x])/(d^3*(a*d^2 - e*(b*d - c*e))^2) + ((a*d - b*e)*(a*b*d - b^2*
e + 2*a*c*e)*Log[c + b*x + a*x^2])/(2*c^2*(a*d^2 - e*(b*d - c*e))
```

```
^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+c/x**2+b/x)/x**4/(e*x+d)**2,x)

[Out] Timed out
```

Mathematica [A] time = 0.658234, size = 287, normalized size = 0.99

$$\frac{\left(-2a^3cd^2+a^2\left(b^2d^2+6bcde+2c^2e^2\right)-2ab^2e(bd+2ce)+b^4e^2\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{c^2\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)^2}+\frac{(ad-be)\left(abd+2ace+b^2(-e)\right)\log(x(ax+b)+c)}{2c^2\left(ad^2+e(ce-bd)\right)^2}-\frac{e^3}{d^2(d+ex)\left(ad^2+e(ce-bd)\right)}+\frac{e^3\log(d+ex)\left(4ad^2+e(2ce-3bd)\right)}{d^3\left(ad^2+e(ce-bd)\right)^2}-\frac{\log(x)(bd+2ce)}{c^2d^3}-\frac{1}{cd^2x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + c/x^2 + b/x)*x^4*(d + e*x)^2),x]

[Out] -(1/(c*d^2*x)) - e^3/(d^2*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) +
 ((-2*a^3*c*d^2 + b^4*e^2 - 2*a*b^2*e*(b*d + 2*c*e) + a^2*(b^2*d^
2 + 6*b*c*d*e + 2*c^2*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]
])/(c^2*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))^2) - ((b*d
+ 2*c*e)*Log[x])/(c^2*d^3) + (e^3*(4*a*d^2 + e*(-3*b*d + 2*c*e))*
Log[d + e*x])/(d^3*(a*d^2 + e*(-(b*d) + c*e))^2) + ((a*d - b*e)*(
a*b*d - b^2*e + 2*a*c*e)*Log[c + x*(b + a*x)])/(2*c^2*(a*d^2 + e*
(-(b*d) + c*e))^2)
```

Maple [B] time = 0.023, size = 791, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x)
```

```
[Out] -1/c/d^2/x-1/c^2/d^2*ln(x)*b-2/c/d^3*ln(x)*e-e^3/d^2/(a*d^2-b*d*e
+c*e^2)/(e*x+d)+4*e^3/d/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*a-3*e^4/d
^2/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*b+2*e^5/d^3/(a*d^2-b*d*e+c*e^2
)^2*ln(e*x+d)*c+1/2/(a*d^2-b*d*e+c*e^2)^2/c^2*a^2*ln(a*x^2+b*x+c)
*b*d^2+1/(a*d^2-b*d*e+c*e^2)^2/c*a^2*ln(a*x^2+b*x+c)*d*e-1/(a*d^2
-b*d*e+c*e^2)^2/c^2*a*ln(a*x^2+b*x+c)*b^2*d*e-1/(a*d^2-b*d*e+c*e^
2)^2/c*a*ln(a*x^2+b*x+c)*b*e^2+1/2/(a*d^2-b*d*e+c*e^2)^2/c^2*ln(a
*x^2+b*x+c)*b^3*e^2-2/(a*d^2-b*d*e+c*e^2)^2/c/(4*a*c-b^2)^(1/2)*a
rctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^3*d^2+1/(a*d^2-b*d*e+c*e^2)^
2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*b
^2*d^2+6/(a*d^2-b*d*e+c*e^2)^2/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+
b)/(4*a*c-b^2)^(1/2))*a^2*b*d*e+2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^
2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*e^2-2/(a*d^2-b*d
*e+c*e^2)^2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1
/2))*a*b^3*d*e-4/(a*d^2-b*d*e+c*e^2)^2/c/(4*a*c-b^2)^(1/2)*arctan
((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*e^2+1/(a*d^2-b*d*e+c*e^2)^2/c
^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^4*e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((e*x + d)^2*(a + b/x + c/x^2)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((e*x + d)^2*(a + b/x + c/x^2)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+c/x**2+b/x)/x**4/(e*x+d)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.284107, size = 657, normalized size = 2.26

$$-\frac{\left(a^{2} b^{2} d^{2} e^{2}-2 a^{3} c d^{2} e^{2}-2 a b^{3} d e^{3}+6 a^{2} b c d e^{3}+b^{4} e^{4}-4 a b^{2} c e^{4}+2 a^{2} c^{2} e^{4}\right) \arctan \left(-\frac{\left(2 a d-\frac{2 a d^{2}}{x e+d}-b e+\frac{2 b d e}{x e+d}-\frac{2 c e^{2}}{x e+d}\right) e^{(-1)}}{\sqrt{-b^{2}+4 a c}}\right) e^{(-2)}}{\left(a^{2} c^{2} d^{4}-2 a b c^{2} d^{3} e+b^{2} c^{2} d^{2} e^{2}+2 a c^{3} d^{2} e^{2}-2 b c^{3} d e^{3}+c^{4} e^{4}\right) \sqrt{-b^{2}+4 a c}}+\frac{\left(a^{2} b d^{2}-2 a b^{2} d e+2 a^{2} c d e+b^{3} e^{2}-2 a b c e^{2}\right) \ln \left(-a+\frac{2 a d}{x e+d}-\frac{a d^{2}}{(x e+d)^{2}}-\frac{b e}{x e+d}+\frac{b d e}{(x e+d)^{2}}-\frac{c e^{2}}{(x e+d)^{2}}\right)}{2\left(a^{2} c^{2} d^{4}-2 a b c^{2} d^{3} e+b^{2} c^{2} d^{2} e^{2}+2 a c^{3} d^{2} e^{2}-2 b c^{3} d e^{3}+c^{4} e^{4}\right)}-\frac{e^{7}}{\left(a d^{4} e^{4}-b d^{3} e^{5}+c d^{2} e^{6}\right)(x e+d)}-\frac{\left(b d e+2 c e^{2}\right) e^{(-1)} \ln \left(\left|-\frac{d}{x e+d}+1\right|\right)}{c^{2} d^{3}}+\frac{e}{c d^{3}\left(\frac{d}{x e+d}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)^2*(a + b/x + c/x^2)*x^4),x, algorithm="giac")
```

```
[Out] -(a^2*b^2*d^2*e^2 - 2*a^3*c*d^2*e^2 - 2*a*b^3*d*e^3 + 6*a^2*b*c*d
*e^3 + b^4*e^4 - 4*a*b^2*c*e^4 + 2*a^2*c^2*e^4)*arctan(-(2*a*d -
2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*
e^(-1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((a^2*c^2*d^4 - 2*a*b*c^2*d^3*e
 + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 - 2*b*c^3*d*e^3 + c^4*e^4)*s
qrt(-b^2 + 4*a*c)) + 1/2*(a^2*b*d^2 - 2*a*b^2*d*e + 2*a^2*c*d*e +
 b^3*e^2 - 2*a*b*c*e^2)*ln(-a + 2*a*d/(x*e + d) - a*d^2/(x*e + d)
^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a^2*
c^2*d^4 - 2*a*b*c^2*d^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 - 2
*b*c^3*d*e^3 + c^4*e^4) - e^7/((a*d^4*e^4 - b*d^3*e^5 + c*d^2*e^6
)*(x*e + d)) - (b*d*e + 2*c*e^2)*e^(-1)*ln(abs(-d/(x*e + d) + 1))
/(c^2*d^3) + e/(c*d^3*(d/(x*e + d) - 1))
```

3.78 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx$

Optimal. Leaf size=372

$$\frac{\left(a^3cd^2-a^2\left(b^2d^2+4bcde+c^2e^2\right)+ab^2e(2bd+3ce)+b^4\left(-e^2\right)\right)\log\left(ax^2+bx+c\right)}{2c^3\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(-a^3cd(3bd+4ce)+a^2b\left(b^2d^2+8bcde+5c^2e^2\right)-ab^3e(2bd+5ce)+b^5e^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{\log(x)\left(-c\left(ad^2-3ce^2\right)+b^2d^2+2bcde\right)}{c^3d^4}-\frac{e^4\log(d+ex)\left(5ad^2-e(4bd-3ce)\right)}{d^4\left(ad^2-e(bd-ce)\right)^2}+\frac{e^4}{d^3(d+ex)\left(ad^2-e(bd-ce)\right)}+\frac{bd+2ce}{c^2d^3x}-\frac{1}{2cd^2x^2}$$

```
[Out] -1/(2*c*d^2*x^2) + (b*d + 2*c*e)/(c^2*d^3*x) + e^4/(d^3*(a*d^2 -
e*(b*d - c*e))*(d + e*x)) + ((b^5*e^2 - a^3*c*d*(3*b*d + 4*c*e) -
 a*b^3*e*(2*b*d + 5*c*e) + a^2*b*(b^2*d^2 + 8*b*c*d*e + 5*c^2*e^2
))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]
*(a*d^2 - e*(b*d - c*e))^2) + ((b^2*d^2 + 2*b*c*d*e - c*(a*d^2 -
3*c*e^2))*Log[x])/(c^3*d^4) - (e^4*(5*a*d^2 - e*(4*b*d - 3*c*e))*
Log[d + e*x])/(d^4*(a*d^2 - e*(b*d - c*e))^2) + ((a^3*c*d^2 - b^4
*e^2 + a*b^2*e*(2*b*d + 3*c*e) - a^2*(b^2*d^2 + 4*b*c*d*e + c^2*e
^2))*Log[c + b*x + a*x^2])/(2*c^3*(a*d^2 - e*(b*d - c*e))^2)
```

Rubi [A] time = 1.88739, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{\left(a^3cd^2-a^2\left(b^2d^2+4bcde+c^2e^2\right)+ab^2e(2bd+3ce)+b^4\left(-e^2\right)\right)\log\left(ax^2+bx+c\right)}{2c^3\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(-a^3cd(3bd+4ce)+a^2b\left(b^2d^2+8bcde+5c^2e^2\right)-ab^3e(2bd+5ce)+b^5e^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{\log(x)\left(-c\left(ad^2-3ce^2\right)+b^2d^2+2bcde\right)}{c^3d^4}-\frac{e^4\log(d+ex)\left(5ad^2-e(4bd-3ce)\right)}{d^4\left(ad^2-e(bd-ce)\right)^2}+\frac{e^4}{d^3(d+ex)\left(ad^2-e(bd-ce)\right)}+\frac{bd+2ce}{c^2d^3x}-\frac{1}{2cd^2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + c/x^2 + b/x)*x^5*(d + e*x)^2),x]
```

```
[Out] -1/(2*c*d^2*x^2) + (b*d + 2*c*e)/(c^2*d^3*x) + e^4/(d^3*(a*d^2 -
e*(b*d - c*e))*(d + e*x)) + ((b^5*e^2 - a^3*c*d*(3*b*d + 4*c*e) -
```

```
 a*b^3*e*(2*b*d + 5*c*e) + a^2*b*(b^2*d^2 + 8*b*c*d*e + 5*c^2*e^2
))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]
*(a*d^2 - e*(b*d - c*e))^2) + ((b^2*d^2 + 2*b*c*d*e - c*(a*d^2 -
3*c*e^2))*Log[x])/(c^3*d^4) - (e^4*(5*a*d^2 - e*(4*b*d - 3*c*e))*
Log[d + e*x])/(d^4*(a*d^2 - e*(b*d - c*e))^2) + ((a^3*c*d^2 - b^4
*e^2 + a*b^2*e*(2*b*d + 3*c*e) - a^2*(b^2*d^2 + 4*b*c*d*e + c^2*e
^2))*Log[c + b*x + a*x^2])/(2*c^3*(a*d^2 - e*(b*d - c*e))^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+c/x**2+b/x)/x**5/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.779386, size = 370, normalized size = 0.99

$$-\frac{\left(-a^3cd^2+a^2\left(b^2d^2+4bcde+c^2e^2\right)-ab^2e(2bd+3ce)+b^4e^2\right)\log(x(ax+b)+c)}{2c^3\left(ad^2+e(ce-bd)\right)^2}+\frac{\left(a^3cd(3bd+4ce)-a^2b\left(b^2d^2+8bcde+5c^2e^2\right)+ab^3e(2bd+5ce)+b^5\left(-e^2\right)\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{c^3\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)^2}+\frac{\log(x)\left(c\left(3ce^2-ad^2\right)+b^2d^2+2bcde\right)}{c^3d^4}-\frac{e^4\log(d+ex)\left(5ad^2+e(3ce-4bd)\right)}{d^4\left(ad^2+e(ce-bd)\right)^2}+\frac{e^4}{d^3(d+ex)\left(ad^2+e(ce-bd)\right)}+\frac{bd+2ce}{c^2d^3x}-\frac{1}{2cd^2x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + c/x^2 + b/x)*x^5*(d + e*x)^2),x]
```

```
[Out] -1/(2*c*d^2*x^2) + (b*d + 2*c*e)/(c^2*d^3*x) + e^4/(d^3*(a*d^2 +
e*(-(b*d) + c*e))*(d + e*x)) + ((-(b^5*e^2) + a^3*c*d*(3*b*d + 4*
c*e) + a*b^3*e*(2*b*d + 5*c*e) - a^2*b*(b^2*d^2 + 8*b*c*d*e + 5*c
^2*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(c^3*Sqrt[-b^2 +
 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))^2) + ((b^2*d^2 + 2*b*c*d*e + c
*(-(a*d^2) + 3*c*e^2))*Log[x])/(c^3*d^4) - (e^4*(5*a*d^2 + e*(-4*
b*d + 3*c*e))*Log[d + e*x])/(d^4*(a*d^2 + e*(-(b*d) + c*e))^2) -
((-(a^3*c*d^2) + b^4*e^2 - a*b^2*e*(2*b*d + 3*c*e) + a^2*(b^2*d^2
 + 4*b*c*d*e + c^2*e^2))*Log[c + x*(b + a*x)])/(2*c^3*(a*d^2 + e*
```

```
(-(b*d) + c*e))^2)
```

Maple [B] time = 0.026, size = 993, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x)
```

```
[Out] 2/c^3/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4
*a*c-b^2)^(1/2))*a*b^4*d*e-8/c^2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2
)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*b^2*d*e+3/c^2/(a*
d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)
^(1/2))*a^3*b*d^2+4/c/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arc
tan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^3*d*e-1/c^3/(a*d^2-b*d*e+c*e^2
)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*b^3
*d^2-5/c/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)
/(4*a*c-b^2)^(1/2))*a^2*b*e^2+5/c^2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-
b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b^3*e^2+1/x/c^2/
d^2*b+2/x/c/d^3*e-1/c^2/d^2*ln(x)*a+1/c^3/d^2*ln(x)*b^2+3/c/d^4*l
n(x)*e^2+e^4/d^3/(a*d^2-b*d*e+c*e^2)/(e*x+d)-1/2/x^2/c/d^2+2/c^2/
d^3*ln(x)*b*e-5*e^4/d^2/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*a-1/2/c^3
/(a*d^2-b*d*e+c*e^2)^2*a^2*ln(a*x^2+b*x+c)*b^2*d^2+3/2/c^2/(a*d^2
-b*d*e+c*e^2)^2*a*ln(a*x^2+b*x+c)*b^2*e^2-1/c^3/(a*d^2-b*d*e+c*e^
2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^5*e^
2-2/c^2/(a*d^2-b*d*e+c*e^2)^2*a^2*ln(a*x^2+b*x+c)*b*d*e+1/c^3/(a*
d^2-b*d*e+c*e^2)^2*a*ln(a*x^2+b*x+c)*b^3*d*e+4*e^5/d^3/(a*d^2-b*d
*e+c*e^2)^2*ln(e*x+d)*b-3*e^6/d^4/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)
*c+1/2/c^2/(a*d^2-b*d*e+c*e^2)^2*a^3*ln(a*x^2+b*x+c)*d^2-1/2/c/(a
*d^2-b*d*e+c*e^2)^2*a^2*ln(a*x^2+b*x+c)*e^2-1/2/c^3/(a*d^2-b*d*e+
c*e^2)^2*ln(a*x^2+b*x+c)*b^4*e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)^2*(a + b/x + c/x^2)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((e*x + d)^2*(a + b/x + c/x^2)*x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+c/x**2+b/x)/x**5/(e*x+d)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.289699, size = 792, normalized size = 2.13

$$\frac{\left(a^{2} b^{3} d^{2} e^{2} - 3\, a^{3} b c d^{2} e^{2} - 2\, a b^{4} d e^{3} + 8\, a^{2} b^{2} c d e^{3} - 4\, a^{3} c^{2} d e^{3} + b^{5} e^{4} - 5\, a b^{3} c e^{4} + 5\, a^{2} b c^{2} e^{4}\right) \arctan\left(-\frac{\left(2\, a d - \frac{2\, a d^{2}}{x e + d} - b e + \frac{2\, b d e}{x e + d} - \frac{2\, c}{x e}}{\sqrt{-b^{2} + 4\, a c}}}{\left(a^{2} c^{3} d^{4} - 2\, a b c^{3} d^{3} e + b^{2} c^{3} d^{2} e^{2} + 2\, a c^{4} d^{2} e^{2} - 2\, b c^{4} d e^{3} + c^{5} e^{4}\right) \sqrt{-b^{2} + 4\, a c}} - \frac{\left(a^{2} b^{2} d^{2} - a^{3} c d^{2} - 2\, a b^{3} d e + 4\, a^{2} b c d e + b^{4} e^{2} - 3\, a b^{2} c e^{2} + a^{2} c^{2} e^{2}\right) \ln\left(-a + \frac{2\, a d}{x e + d} - \frac{a d^{2}}{(x e + d)^{2}} - \frac{b e}{x e + d} + \frac{b d e}{(x e + d)^{2}} - \frac{c e^{2}}{(x e + d)^{2}}\right)}{2\, \left(a^{2} c^{3} d^{4} - 2\, a b c^{3} d^{3} e + b^{2} c^{3} d^{2} e^{2} + 2\, a c^{4} d^{2} e^{2} - 2\, b c^{4} d e^{3} + c^{5} e^{4}\right)} + \frac{e^{9}}{(a d^{5} e^{5} - b d^{4} e^{6} + c d^{3} e^{7})(x e + d)} + \frac{\left(b^{2} d^{2} e - a c d^{2} e + 2\, b c d e^{2} + 3\, c^{2} e^{3}\right) e^{(-1)} \ln\left(\left|-\frac{d}{x e + d} + 1\right|\right)}{c^{3} d^{4}} + \frac{2\, b c d e + 5\, c^{2} e^{2} - \frac{2\, \left(b c d^{2} e^{2} + 3\, c^{2} d e^{3}\right) e^{(-1)}}{x e + d}}{2\, c^{3} d^{4} \left(\frac{d}{x e + d} - 1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((e*x + d)^2*(a + b/x + c/x^2)*x^5),x, algorithm="giac")
```

```
[Out] (a^2*b^3*d^2*e^2 - 3*a^3*b*c*d^2*e^2 - 2*a*b^4*d*e^3 + 8*a^2*b^2*
c*d*e^3 - 4*a^3*c^2*d*e^3 + b^5*e^4 - 5*a*b^3*c*e^4 + 5*a^2*b*c^2
```

```
*e^4)*arctan(-(2*a*d - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d
) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((a^2*c^
3*d^4 - 2*a*b*c^3*d^3*e + b^2*c^3*d^2*e^2 + 2*a*c^4*d^2*e^2 - 2*b
*c^4*d*e^3 + c^5*e^4)*sqrt(-b^2 + 4*a*c)) - 1/2*(a^2*b^2*d^2 - a^
3*c*d^2 - 2*a*b^3*d*e + 4*a^2*b*c*d*e + b^4*e^2 - 3*a*b^2*c*e^2 +
 a^2*c^2*e^2)*ln(-a + 2*a*d/(x*e + d) - a*d^2/(x*e + d)^2 - b*e/(
x*e + d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a^2*c^3*d^4 -
2*a*b*c^3*d^3*e + b^2*c^3*d^2*e^2 + 2*a*c^4*d^2*e^2 - 2*b*c^4*d*e
^3 + c^5*e^4) + e^9/((a*d^5*e^5 - b*d^4*e^6 + c*d^3*e^7)*(x*e + d
)) + (b^2*d^2*e - a*c*d^2*e + 2*b*c*d*e^2 + 3*c^2*e^3)*e^(-1)*ln(
abs(-d/(x*e + d) + 1))/(c^3*d^4) + 1/2*(2*b*c*d*e + 5*c^2*e^2 - 2
*(b*c*d^2*e^2 + 3*c^2*d*e^3)*e^(-1)/(x*e + d))/(c^3*d^4*(d/(x*e +
 d) - 1)^2)
```

3.79 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx$

Optimal. Leaf size=981

$$\frac{2}{11}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}x^5+\frac{2(ad+be)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(d+ex)^{7/2}x}{99ae^4}$$

$$-\frac{2\left(29a^2d^2+8b^2e^2+ae(19bd-18ce)\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(d+ex)^{5/2}x}{693a^2e^4}$$

$$+\frac{2\left(233a^3d^3+4a^2e(18bd-37ce)d+48b^3e^3+abe^2(67bd-157ce)\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(d+ex)^{3/2}x}{3465a^3e^4}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}\left(128a^5d^5-4a^4e(14bd-27ce)d^3-a^3e^2\left(37b^2d^2-135bced+156c^2e^2\right)d+128b^5e^5-8ab^3e^4(7bd+87ce)-a\right.}{3465a^5e^5\sqrt{\frac{a(d+e}{2ad-\left(b+\sqrt{b}\right.}}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)\left(128a^4d^4+4a^3e(2bd+3ce)d^2-64b^4e^4-4ab^2e^3(7bd-69ce)-3a^2e^2\left(3b^2d^2-29bced\right.\right.}{3465a^5e^5\sqrt{d+ex}\left(ax^2\right.}$$

$$-\frac{2\left(187a^4d^4-4a^3e(2bd+3ce)d^2+64b^4e^4+4ab^2e^3(7bd-69ce)+3a^2e^2\left(3b^2d^2-29bced+50c^2e^2\right)\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}}{3465a^4e^4}$$

```
[Out] (-2*(187*a^4*d^4 + 64*b^4*e^4 + 4*a*b^2*e^3*(7*b*d - 69*c*e) - 4*
a^3*d^2*e*(2*b*d + 3*c*e) + 3*a^2*e^2*(3*b^2*d^2 - 29*b*c*d*e + 5
0*c^2*e^2))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x])/(3465*a^4*e^4)
 + (2*Sqrt[a + c/x^2 + b/x]*x^5*Sqrt[d + e*x])/11 + (2*(233*a^3*d
^3 + 48*b^3*e^3 + a*b*e^2*(67*b*d - 157*c*e) + 4*a^2*d*e*(18*b*d
- 37*c*e))*Sqrt[a + c/x^2 + b/x]*x*(d + e*x)^(3/2))/(3465*a^3*e^4
) - (2*(29*a^2*d^2 + 8*b^2*e^2 + a*e*(19*b*d - 18*c*e))*Sqrt[a +
c/x^2 + b/x]*x*(d + e*x)^(5/2))/(693*a^2*e^4) + (2*(a*d + b*e)*Sq
rt[a + c/x^2 + b/x]*x*(d + e*x)^(7/2))/(99*a*e^4) + (Sqrt[2]*Sqrt
[b^2 - 4*a*c]*(128*a^5*d^5 + 128*b^5*e^5 - 4*a^4*d^3*e*(14*b*d -
27*c*e) - 8*a*b^3*e^4*(7*b*d + 87*c*e) - a^2*b*e^3*(37*b^2*d^2 -
258*b*c*d*e - 771*c^2*e^2) - a^3*d*e^2*(37*b^2*d^2 - 135*b*c*d*e
+ 156*c^2*e^2))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(
c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
- 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3465*a^5*e^5*S
qrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x +
 a*x^2)) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))*(
128*a^4*d^4 - 64*b^4*e^4 - 4*a*b^2*e^3*(7*b*d - 69*c*e) + 4*a^3*d
^2*e*(2*b*d + 3*c*e) - 3*a^2*e^2*(3*b^2*d^2 - 29*b*c*d*e + 50*c^2
*e^2))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + S
qrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))
]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2
```

```
- 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b
^2 - 4*a*c])*e)])/(3465*a^5*e^5*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rubi [A] time = 10.1348, antiderivative size = 981, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$

$$\frac{2}{11}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}x^5+\frac{2(ad+be)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(d+ex)^{7/2}x}{99ae^4}$$
$$-\frac{2\left(29a^2d^2+8b^2e^2+ae(19bd-18ce)\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(d+ex)^{5/2}x}{693a^2e^4}$$
$$+\frac{2\left(233a^3d^3+4a^2e(18bd-37ce)d+48b^3e^3+abe^2(67bd-157ce)\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(d+ex)^{3/2}x}{3465a^3e^4}$$
$$+\frac{\sqrt{2}\sqrt{b^2-4ac}\left(128a^5d^5-4a^4e(14bd-27ce)d^3-a^3e^2\left(37b^2d^2-135bced+156c^2e^2\right)d+128b^5e^5-8ab^3e^4(7bd+87ce)-a\cdots\right.}{3465a^5e^5\sqrt{\frac{a(d+e\cdots}{2ad-\left(b+\sqrt{b\cdots}\right.}}}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)\left(128a^4d^4+4a^3e(2bd+3ce)d^2-64b^4e^4-4ab^2e^3(7bd-69ce)-3a^2e^2\left(3b^2d^2-29bced\cdots\right.\right.}{3465a^5e^5\sqrt{d+ex}\left(ax^2\cdots\right.}$$
$$-\frac{2\left(187a^4d^4-4a^3e(2bd+3ce)d^2+64b^4e^4+4ab^2e^3(7bd-69ce)+3a^2e^2\left(3b^2d^2-29bced+50c^2e^2\right)\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}x\cdots}{3465a^4e^4}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[Sqrt[a + c/x^2 + b/x]*x^4*Sqrt[d + e*x],x]
```

```
[Out] (-2*(187*a^4*d^4 + 64*b^4*e^4 + 4*a*b^2*e^3*(7*b*d - 69*c*e) - 4*
a^3*d^2*e*(2*b*d + 3*c*e) + 3*a^2*e^2*(3*b^2*d^2 - 29*b*c*d*e + 5
0*c^2*e^2))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x])/(3465*a^4*e^4)
 + (2*Sqrt[a + c/x^2 + b/x]*x^5*Sqrt[d + e*x])/11 + (2*(233*a^3*d
^3 + 48*b^3*e^3 + a*b*e^2*(67*b*d - 157*c*e) + 4*a^2*d*e*(18*b*d
- 37*c*e))*Sqrt[a + c/x^2 + b/x]*x*(d + e*x)^(3/2))/(3465*a^3*e^4
) - (2*(29*a^2*d^2 + 8*b^2*e^2 + a*e*(19*b*d - 18*c*e))*Sqrt[a +
c/x^2 + b/x]*x*(d + e*x)^(5/2))/(693*a^2*e^4) + (2*(a*d + b*e)*Sq
rt[a + c/x^2 + b/x]*x*(d + e*x)^(7/2))/(99*a*e^4) + (Sqrt[2]*Sqrt
[b^2 - 4*a*c]*(128*a^5*d^5 + 128*b^5*e^5 - 4*a^4*d^3*e*(14*b*d -
27*c*e) - 8*a*b^3*e^4*(7*b*d + 87*c*e) - a^2*b*e^3*(37*b^2*d^2 -
258*b*c*d*e - 771*c^2*e^2) - a^3*d*e^2*(37*b^2*d^2 - 135*b*c*d*e
+ 156*c^2*e^2))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(
c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
```

```
- 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3465*a^5*e^5*S
qrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x +
 a*x^2)) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))*(
128*a^4*d^4 - 64*b^4*e^4 - 4*a*b^2*e^3*(7*b*d - 69*c*e) + 4*a^3*d
^2*e*(2*b*d + 3*c*e) - 3*a^2*e^2*(3*b^2*d^2 - 29*b*c*d*e + 50*c^2
*e^2))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + S
qrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))
]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2
- 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b
^2 - 4*a*c])*e)])/(3465*a^5*e^5*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)

[Out] Timed out
```

Mathematica [C] time = 14.7852, size = 10904, normalized size = 11.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a + c/x^2 + b/x]*x^4*Sqrt[d + e*x],x]

[Out] Result too large to show
```

Maple [B] time = 0.169, size = 11938, normalized size = 12.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{ex+d}x^4\sqrt{\frac{ax^2+bx+c}{x^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*x^4*sqrt((a*x^2 + b*x + c)/x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.80 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^3\sqrt{d+ex}\,dx$

Optimal. Leaf size=778

$$-\frac{4x(d+ex)^{3/2}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(8a^2d^2+ae(4bd-7ce)+3b^2e^2\right)}{315a^2e^3}$$
$$+\frac{2x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(19a^3d^3-6a^2cde^2+3abe^2(bd-9ce)+8b^3e^3\right)}{315a^3e^3}$$
$$-\frac{2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\left(8a^4d^4-a^3d^2e(4bd-9ce)-3a^2e^2\left(b^2d^2-5bcde-7c^2e^2\right)-4ab^2e^3(\right.}{315a^4e^4\left(ax^2+bx+c\right)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}}$$
$$+\frac{2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\left(16a^3d^3+6a^2cde^2-3abe^2(bd-9ce)-8b^3e^3\right)\left(ad^2-e(bd-ce)\right)\sqrt{\frac{a(d+}{2ad-e\left(\sqrt{b^2\right.}}}}{315a^4e^4\sqrt{d+ex}\left(ax^2+bx+c\right)}$$
$$+\frac{2x(d+ex)^{5/2}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(ad+be)}{63ae^3}+\frac{2}{9}x^4\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}$$

```
[Out] (2*(19*a^3*d^3 - 6*a^2*c*d*e^2 + 8*b^3*e^3 + 3*a*b*e^2*(b*d - 9*c
*e))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x])/(315*a^3*e^3) + (2*Sq
rt[a + c/x^2 + b/x]*x^4*Sqrt[d + e*x])/9 - (4*(8*a^2*d^2 + 3*b^2*
e^2 + a*e*(4*b*d - 7*c*e))*Sqrt[a + c/x^2 + b/x]*x*(d + e*x)^(3/2
))/(315*a^2*e^3) + (2*(a*d + b*e)*Sqrt[a + c/x^2 + b/x]*x*(d + e*
x)^(5/2))/(63*a*e^3) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*a^4*d^4 +
8*b^4*e^4 - a^3*d^2*e*(4*b*d - 9*c*e) - 4*a*b^2*e^3*(b*d + 9*c*e)
 - 3*a^2*e^2*(b^2*d^2 - 5*b*c*d*e - 7*c^2*e^2))*Sqrt[a + c/x^2 +
b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]
*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^
2 - 4*a*c])*e)])/(315*a^4*e^4*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sq
rt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) + (2*Sqrt[2]*Sqrt[b^2 - 4
*a*c]*(16*a^3*d^3 + 6*a^2*c*d*e^2 - 8*b^3*e^3 - 3*a*b*e^2*(b*d -
9*c*e))*(a*d^2 - e*(b*d - c*e))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(
d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x
 + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 -
4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c
]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(315*a^4*e^4*Sqrt[d +
e*x]*(c + b*x + a*x^2))
```

Rubi [A] time = 4.66489, antiderivative size = 778, normalized size of antiderivative = 1., number of

steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$

$$-\frac{4x(d+ex)^{3/2}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(8a^2d^2+ae(4bd-7ce)+3b^2e^2\right)}{315a^2e^3}$$
$$+\frac{2x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(19a^3d^3-6a^2cde^2+3abe^2(bd-9ce)+8b^3e^3\right)}{315a^3e^3}$$
$$-\frac{2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\left(8a^4d^4-a^3d^2e(4bd-9ce)-3a^2e^2\left(b^2d^2-5bcde-7c^2e^2\right)-4ab^2e^3(\right.}{315a^4e^4\left(ax^2+bx+c\right)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}}$$
$$+\frac{2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\left(16a^3d^3+6a^2cde^2-3abe^2(bd-9ce)-8b^3e^3\right)\left(ad^2-e(bd-ce)\right)\sqrt{\frac{a(d+}{2ad-e\left(\sqrt{b^2\right.}}}}{315a^4e^4\sqrt{d+ex}\left(ax^2+bx+c\right)}$$
$$+\frac{2x(d+ex)^{5/2}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(ad+be)}{63ae^3}+\frac{2}{9}x^4\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + c/x^2 + b/x]*x^3*Sqrt[d + e*x],x]
```

```
[Out] (2*(19*a^3*d^3 - 6*a^2*c*d*e^2 + 8*b^3*e^3 + 3*a*b*e^2*(b*d - 9*c
*e))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x])/(315*a^3*e^3) + (2*Sq
rt[a + c/x^2 + b/x]*x^4*Sqrt[d + e*x])/9 - (4*(8*a^2*d^2 + 3*b^2*
e^2 + a*e*(4*b*d - 7*c*e))*Sqrt[a + c/x^2 + b/x]*x*(d + e*x)^(3/2
))/(315*a^2*e^3) + (2*(a*d + b*e)*Sqrt[a + c/x^2 + b/x]*x*(d + e*
x)^(5/2))/(63*a*e^3) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*a^4*d^4 +
8*b^4*e^4 - a^3*d^2*e*(4*b*d - 9*c*e) - 4*a*b^2*e^3*(b*d + 9*c*e)
 - 3*a^2*e^2*(b^2*d^2 - 5*b*c*d*e - 7*c^2*e^2))*Sqrt[a + c/x^2 +
b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]
*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^
2 - 4*a*c])*e)])/(315*a^4*e^4*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sq
rt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) + (2*Sqrt[2]*Sqrt[b^2 - 4
*a*c]*(16*a^3*d^3 + 6*a^2*c*d*e^2 - 8*b^3*e^3 - 3*a*b*e^2*(b*d -
9*c*e))*(a*d^2 - e*(b*d - c*e))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(
d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x
 + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 -
4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c
]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(315*a^4*e^4*Sqrt[d +
e*x]*(c + b*x + a*x^2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 14.2211, size = 7531, normalized size = 9.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a + c/x^2 + b/x]*x^3*Sqrt[d + e*x],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.074, size = 9182, normalized size = 11.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex + d}\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ex+d}x^3\sqrt{\frac{ax^2+bx+c}{x^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*x^3*sqrt((a*x^2 + b*x + c)/x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.81 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^2\sqrt{d+ex}\,dx$

Optimal. Leaf size=636

$$-\frac{2x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(4a^2d^2-ae(2bd-5ce)-3aex(ad-4be)+4b^2e^2\right)}{105a^2e^2}$$

$$-\frac{2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\left(8a^2d^2-ae(bd-10ce)-4b^2e^2\right)\left(ad^2-e(bd-ce)\right)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}F\Bigg(\sin}{105a^3e^3\sqrt{d+ex}\left(ax^2+bx+c\right)}$$

$$+\frac{\sqrt{2}x\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\left(8a^3d^3-a^2de(5bd-16ce)-abe^2(5bd+29ce)+8b^3e^3\right)E\Bigg(\sin^{-1}\Bigg(\frac{\sqrt{b+2a}}{}}{105a^3e^3\left(ax^2+bx+c\right)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$+\frac{2x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(ax^2+bx+c\right)}{7a}$$

```
[Out] (-2*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*(4*a^2*d^2 + 4*b^2*e^2
- a*e*(2*b*d - 5*c*e) - 3*a*e*(a*d - 4*b*e)*x))/(105*a^2*e^2) + (
2*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*(c + b*x + a*x^2))/(7*a)
+ (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*a^3*d^3 + 8*b^3*e^3 - a^2*d*e*(5*
b*d - 16*c*e) - a*b*e^2*(5*b*d + 29*c*e))*Sqrt[a + c/x^2 + b/x]*x
*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*Ellip
ticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c
]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*
a*c])*e)])/(105*a^3*e^3*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2
 - 4*a*c])*e)]*(c + b*x + a*x^2)) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(8*a^2*d^2 - 4*b^2*e^2 - a*e*(b*d - 10*c*e))*(a*d^2 - e*(b*d - c*
e))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt
[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*E
llipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4
*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2
- 4*a*c])*e)])/(105*a^3*e^3*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rubi [A] time = 2.5136, antiderivative size = 636, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$

$$-\frac{2x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(4a^2d^2-ae(2bd-5ce)-3aex(ad-4be)+4b^2e^2\right)}{105a^2e^2}$$

$$-\frac{2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\left(8a^2d^2-ae(bd-10ce)-4b^2e^2\right)\left(ad^2-e(bd-ce)\right)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}F\Big(\sin}{105a^3e^3\sqrt{d+ex}\,(ax^2+bx+c)}$$

$$+\frac{\sqrt{2}x\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\left(8a^3d^3-a^2de(5bd-16ce)-abe^2(5bd+29ce)+8b^3e^3\right)E\left(\sin^{-1}\left(\frac{\sqrt{b+2}}{}\right.\right.}{105a^3e^3\,(ax^2+bx+c)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$+\frac{2x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(ax^2+bx+c\right)}{7a}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + c/x^2 + b/x]*x^2*Sqrt[d + e*x],x]
```

```
[Out] (-2*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*(4*a^2*d^2 + 4*b^2*e^2
- a*e*(2*b*d - 5*c*e) - 3*a*e*(a*d - 4*b*e)*x))/(105*a^2*e^2) + (
2*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*(c + b*x + a*x^2))/(7*a)
+ (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*a^3*d^3 + 8*b^3*e^3 - a^2*d*e*(5*
b*d - 16*c*e) - a*b*e^2*(5*b*d + 29*c*e))*Sqrt[a + c/x^2 + b/x]*x
*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*Ellip
ticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c
]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*
a*c])*e)])/(105*a^3*e^3*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2
 - 4*a*c])*e)]*(c + b*x + a*x^2)) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(8*a^2*d^2 - 4*b^2*e^2 - a*e*(b*d - 10*c*e))*(a*d^2 - e*(b*d - c*
e))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt
[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*E
llipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4
*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2
- 4*a*c])*e)])/(105*a^3*e^3*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 13.577, size = 5350, normalized size = 8.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a + c/x^2 + b/x]*x^2*Sqrt[d + e*x],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.06, size = 6302, normalized size = 9.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ex+d}x^2\sqrt{\frac{ax^2+bx+c}{x^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*x^2*sqrt((a*x^2 + b*x + c)/x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.82 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\,dx$

Optimal. Leaf size=550

$$\frac{2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}(2ad-be)\left(ad^2-e(bd-ce)\right)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2a}{}\right)}{15a^2e^2\sqrt{d+ex}\,(ax^2+bx+c)}$$

$$-\frac{2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\left(a^2d^2-ae(bd+3ce)+b^2e^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}}{2ad-\left(b+\sqrt{b^2-}\right.}\right)}{15a^2e^2(ax^2+bx+c)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$+\frac{2x(d+ex)^{3/2}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{5e}-\frac{2x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(2ad-be)}{15ae}$$

```
[Out] (-2*(2*a*d - b*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x])/(15*a*e)
 + (2*Sqrt[a + c/x^2 + b/x]*x*(d + e*x)^(3/2))/(5*e) - (2*Sqrt[2]
*Sqrt[b^2 - 4*a*c]*(a^2*d^2 + b^2*e^2 - a*e*(b*d + 3*c*e))*Sqrt[a
 + c/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2
 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)
/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (
b + Sqrt[b^2 - 4*a*c])*e)])/(15*a^2*e^2*Sqrt[(a*(d + e*x))/(2*a*d
 - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) + (2*Sqrt[2]*Sq
rt[b^2 - 4*a*c]*(2*a*d - b*e)*(a*d^2 - e*(b*d - c*e))*Sqrt[a + c/
x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*
e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]]
, (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/
(15*a^2*e^2*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rubi [A] time = 1.73508, antiderivative size = 550, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\frac{2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}(2ad-be)\left(ad^2-e(bd-ce)\right)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{}{2a}\right.}{15a^2e^2\sqrt{d+ex}\,(ax^2+bx+c)}$$
$$-\frac{2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\left(a^2d^2-ae(bd+3ce)+b^2e^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}}{2ad-\left(b+\sqrt{b^2-\right.}}\right.}{15a^2e^2\,(ax^2+bx+c)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}}$$
$$+\frac{2x(d+ex)^{3/2}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{5e}-\frac{2x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(2ad-be)}{15ae}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x],x]
```

```
[Out] (-2*(2*a*d - b*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x])/(15*a*e)
 + (2*Sqrt[a + c/x^2 + b/x]*x*(d + e*x)^(3/2))/(5*e) - (2*Sqrt[2]
*Sqrt[b^2 - 4*a*c]*(a^2*d^2 + b^2*e^2 - a*e*(b*d + 3*c*e))*Sqrt[a
 + c/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2
 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)
/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (
b + Sqrt[b^2 - 4*a*c])*e)])/(15*a^2*e^2*Sqrt[(a*(d + e*x))/(2*a*d
 - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) + (2*Sqrt[2]*Sq
rt[b^2 - 4*a*c]*(2*a*d - b*e)*(a*d^2 - e*(b*d - c*e))*Sqrt[a + c/
x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*
e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]]
, (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/
(15*a^2*e^2*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 13.1612, size = 3390, normalized size = 6.16

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x],x]
```

```
[Out] ((2*(a*d + b*e))/(15*a*e) + (2*x)/5)*x*Sqrt[d + e*x]*Sqrt[a + (c
+ b*x)/x^2] - (2*x*Sqrt[a + (c + b*x)/x^2]*((2*(a^2*d^2 - a*b*d*e
 + b^2*e^2 - 3*a*c*e^2)*(d + e*x)^(3/2)*(a + (a*d^2)/(d + e*x)^2
- (b*d*e)/(d + e*x)^2 + (c*e^2)/(d + e*x)^2 - (2*a*d)/(d + e*x) +
 (b*e)/(d + e*x)))/(a*Sqrt[((d + e*x)^2*(a*(-1 + d/(d + e*x))^2 +
 (e*(b - (b*d)/(d + e*x) + (c*e)/(d + e*x)))/(d + e*x)))/e^2]) -
((a*d^2 - b*d*e + c*e^2)*(d + e*x)*Sqrt[a + (a*d^2)/(d + e*x)^2 -
 (b*d*e)/(d + e*x)^2 + (c*e^2)/(d + e*x)^2 - (2*a*d)/(d + e*x) +
(b*e)/(d + e*x)]*((I*a^2*d^2*(2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*
e^2])*Sqrt[1 - (2*(a*d^2 - b*d*e + c*e^2))/((2*a*d - b*e - Sqrt[b
^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt[1 - (2*(a*d^2 - b*d*e + c*e
^2))/((2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*(Elli
pticE[I*ArcSinh[(Sqrt[2]*Sqrt[-((a*d^2 - b*d*e + c*e^2)/(2*a*d -
b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*a*d - b*e
- Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*
e^2])] - EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[-((a*d^2 - b*d*e + c*e
^2)/(2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]],
(2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*a*d - b*e + Sqrt[b^2
*e^2 - 4*a*c*e^2])]))/(Sqrt[2]*(a*d^2 - b*d*e + c*e^2)*Sqrt[-((a*
d^2 - b*d*e + c*e^2)/(2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))]*
Sqrt[a + (a*d^2 - b*d*e + c*e^2)/(d + e*x)^2 + (-2*a*d + b*e)/(d
+ e*x)]) - (I*a*b*d*e*(2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*S
qrt[1 - (2*(a*d^2 - b*d*e + c*e^2))/((2*a*d - b*e - Sqrt[b^2*e^2
- 4*a*c*e^2])*(d + e*x))]*Sqrt[1 - (2*(a*d^2 - b*d*e + c*e^2))/((
2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*(EllipticE[I
*ArcSinh[(Sqrt[2]*Sqrt[-((a*d^2 - b*d*e + c*e^2)/(2*a*d - b*e - S
qrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*a*d - b*e - Sqrt[
b^2*e^2 - 4*a*c*e^2])/(2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])]
- EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[-((a*d^2 - b*d*e + c*e^2)/(2*
a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*a*d
- b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*a*d - b*e + Sqrt[b^2*e^2 -
4*a*c*e^2])]))/(Sqrt[2]*(a*d^2 - b*d*e + c*e^2)*Sqrt[-((a*d^2 - b
*d*e + c*e^2)/(2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[a
+ (a*d^2 - b*d*e + c*e^2)/(d + e*x)^2 + (-2*a*d + b*e)/(d + e*x)]
) + (I*b^2*e^2*(2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*Sqrt[1 -
 (2*(a*d^2 - b*d*e + c*e^2))/((2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c
*e^2])*(d + e*x))]*Sqrt[1 - (2*(a*d^2 - b*d*e + c*e^2))/((2*a*d -
 b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*(EllipticE[I*ArcSin
h[(Sqrt[2]*Sqrt[-((a*d^2 - b*d*e + c*e^2)/(2*a*d - b*e - Sqrt[b^2
*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*a*d - b*e - Sqrt[b^2*e^2
 - 4*a*c*e^2])/(2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])] - Ellip
ticF[I*ArcSinh[(Sqrt[2]*Sqrt[-((a*d^2 - b*d*e + c*e^2)/(2*a*d - b
*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*a*d - b*e -
 Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e
```

```
^2])]))/(Sqrt[2]*(a*d^2 - b*d*e + c*e^2)*Sqrt[-((a*d^2 - b*d*e +
c*e^2)/(2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[a + (a*d^
2 - b*d*e + c*e^2)/(d + e*x)^2 + (-2*a*d + b*e)/(d + e*x)]) - ((3
*I)*a*c*e^2*(2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*Sqrt[1 - (2
*(a*d^2 - b*d*e + c*e^2))/((2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^
2])*(d + e*x))]*Sqrt[1 - (2*(a*d^2 - b*d*e + c*e^2))/((2*a*d - b*
e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*(EllipticE[I*ArcSinh[(
Sqrt[2]*Sqrt[-((a*d^2 - b*d*e + c*e^2)/(2*a*d - b*e - Sqrt[b^2*e^
2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*a*d - b*e - Sqrt[b^2*e^2 -
4*a*c*e^2])/(2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])] - Elliptic
F[I*ArcSinh[(Sqrt[2]*Sqrt[-((a*d^2 - b*d*e + c*e^2)/(2*a*d - b*e
- Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*a*d - b*e - Sq
rt[b^2*e^2 - 4*a*c*e^2])/(2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2]
)]))/(Sqrt[2]*(a*d^2 - b*d*e + c*e^2)*Sqrt[-((a*d^2 - b*d*e + c*e
^2)/(2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[a + (a*d^2 -
 b*d*e + c*e^2)/(d + e*x)^2 + (-2*a*d + b*e)/(d + e*x)]) + (I*Sqr
t[2]*a^2*d*Sqrt[1 - (2*(a*d^2 - b*d*e + c*e^2))/((2*a*d - b*e - S
qrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt[1 - (2*(a*d^2 - b*d*e
+ c*e^2))/((2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*
EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[-((a*d^2 - b*d*e + c*e^2)/(2*a*
d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*a*d -
b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*a*d - b*e + Sqrt[b^2*e^2 - 4*
a*c*e^2])])/(Sqrt[-((a*d^2 - b*d*e + c*e^2)/(2*a*d - b*e - Sqrt[b
^2*e^2 - 4*a*c*e^2]))]*Sqrt[a + (a*d^2 - b*d*e + c*e^2)/(d + e*x)
^2 + (-2*a*d + b*e)/(d + e*x)]) - (I*a*b*e*Sqrt[1 - (2*(a*d^2 - b
*d*e + c*e^2))/((2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*
x))]*Sqrt[1 - (2*(a*d^2 - b*d*e + c*e^2))/((2*a*d - b*e + Sqrt[b^
2*e^2 - 4*a*c*e^2])*(d + e*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt
[-((a*d^2 - b*d*e + c*e^2)/(2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^
2]))])/Sqrt[d + e*x]], (2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/
(2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])])/(Sqrt[2]*Sqrt[-((a*d^
2 - b*d*e + c*e^2)/(2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))]*Sq
rt[a + (a*d^2 - b*d*e + c*e^2)/(d + e*x)^2 + (-2*a*d + b*e)/(d +
e*x)])))/(a*Sqrt[((d + e*x)^2*(a*(-1 + d/(d + e*x))^2 + (e*(b - (
b*d)/(d + e*x) + (c*e)/(d + e*x)))/(d + e*x)))/e^2])))/(15*a*e^3*
Sqrt[c + b*x + a*x^2])
```

Maple [B] time = 0.059, size = 4361, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x)
```

```
[Out] -1/15*((a*x^2+b*x+c)/x^2)^(1/2)*x*(e*x+d)^(1/2)*(-6*x^4*a^3*e^4-2
*a^2*c*d^2*e^2+2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+
b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2
)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^
```

```
2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a
*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e
)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*a^2
*d^3*e+2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2
)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*
e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2
*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1
/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a
*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d*e^3-2^(
1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a
*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*
(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))
^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+
b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1
/2)+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*b*c*e^4-12*2^(1/2)*(-a*
(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*
c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a
*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*El
lipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/
2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d
-b*e))^(1/2))*a^2*c*d^2*e^2+3*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)
^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*
a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/
(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*
x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1
/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*a*b^2*d^2
*e^2+8*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2
)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*
e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2
*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1
/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a
*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*a^2*b*d^3*e+8*2^(1/2)*(-a*(e*x+d
)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)
^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4
*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*Elliptic
E(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(
e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))
^(1/2))*a^2*c*d^2*e^2-8*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)
-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^
2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4
*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/(
e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*
a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*a*b^2*d^2*e^2-4
*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(
-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1
/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b
*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*
a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2
)^(1/2)+2*a*d-b*e))^(1/2))*a^3*d^4+12*2^(1/2)*(-a*(e*x+d)/(e*(-4*
a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)
/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)
^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)
*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c
+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*a
```

*c^2*e^4-2*x^2*a*b^2*e^4-8*x^3*a^3*d*e^3-8*x^3*a^2*b*e^4-2*x^2*a^
3*d^2*e^2-6*x^2*a^2*c*e^4-2*a*b*c*d*e^3+2*2^(1/2)*(-a*(e*x+d)/(e*
(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2
)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+
b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(
1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4
*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2
))*(-4*a*c+b^2)^(1/2)*a*c*d*e^3+12*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c
+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e
*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1
/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-
a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^
2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*a*b*
c*d*e^3-8*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(
1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d
-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2
)-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)
^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-
4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*a*b*c*d*e^3-3*2^(1/2)*(-a*(e*
x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b
^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+
(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*Ellip
ticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),
(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*
e))^(1/2))*(-4*a*c+b^2)^(1/2)*a*b*d^2*e^2-10*x^2*a^2*b*d*e^3-2*x*
a^2*b*d^2*e^2-8*x*a^2*c*d*e^3-2*x*a*b^2*d*e^3-2*x*a*b*c*e^4+4*2^(
1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a
*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*
(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))
^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+
b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1
/2)+2*a*d-b*e))^(1/2))*b^3*d*e^3-4*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c
+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e
*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1
/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-
a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^
2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*b^2*
c*e^4-12*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1
/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-
b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)
-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^
(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4
*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*a*c^2*e^4-3*2^(1/2)*(-a*(e*x+d
)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)
^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4
*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*Elliptic
F(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(
e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))
^(1/2))*b^3*d*e^3+3*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a
*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(
1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c
+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-
4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+
b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*b^2*c*e^4)/a^2/(a*e

```
*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)/e^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ex+d}x\sqrt{\frac{ax^2+bx+c}{x^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*x*sqrt((a*x^2 + b*x + c)/x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x,x, algorithm="giac")
```

```
[Out] Timed out
```

3.83 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx$

Optimal. Leaf size=955

$$\frac{\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)x}{3ae\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(ax^2+bx+c)}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}d(ad+be)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)x}{3ae\sqrt{d+ex}(ax^2+bx+c)}$$

$$+\frac{4\sqrt{2}\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)x}{3a\sqrt{d+ex}(ax^2+bx+c)}$$

$$-\frac{\sqrt{2}c\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\left(\frac{2ad-be+\sqrt{b^2-4ac}e}{2ad};\sin^{-1}\left(\frac{}{\sqrt{2a}}\right.\right.}{\sqrt{a}(ax^2+bx+c)}$$

$$+\frac{2}{3}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}x$$

```
[Out] (2*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x])/3 + (Sqrt[2]*Sqrt[b^2 -
 4*a*c]*(a*d + b*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-(
(a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[
b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*a*e*Sqrt
[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*
x^2)) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*(a*d + b*e)*Sqrt[a + c/x^2
 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]
*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (
-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*
a*e*Sqrt[d + e*x]*(c + b*x + a*x^2)) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*
c]*(b*d + c*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d
- (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 -
 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/S
qrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b
+ Sqrt[b^2 - 4*a*c])*e)])/(3*a*Sqrt[d + e*x]*(c + b*x + a*x^2)) -
 (Sqrt[2]*c*Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[a + c/x^
2 + b/x]*x*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b - Sqrt[b^2 - 4*a*
c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])
*e)]*EllipticPi[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), ArcS
in[(Sqrt[2]*Sqrt[a]*Sqrt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4
*a*c])*e]], (b - Sqrt[b^2 - 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4
```

```
*a*c] - (2*a*d)/e)])/(Sqrt[a]*(c + b*x + a*x^2))
```

Rubi [A] time = 7.74253, antiderivative size = 955, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$

$$\frac{\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)x}{3ae\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(ax^2+bx+c)}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}d(ad+be)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)x}{3ae\sqrt{d+ex}(ax^2+bx+c)}$$
$$+\frac{4\sqrt{2}\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)x}{3a\sqrt{d+ex}(ax^2+bx+c)}$$
$$-\frac{\sqrt{2}c\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\left(\frac{2ad-be+\sqrt{b^2-4ac}e}{2ad};\sin^{-1}\left(\frac{}{\sqrt{2a}}\right.\right.}{\sqrt{a}(ax^2+bx+c)}$$
$$+\frac{2}{3}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}x$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x])/3 + (Sqrt[2]*Sqrt[b^2 -
 4*a*c]*(a*d + b*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-(
(a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[
b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*a*e*Sqrt
[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*
x^2)) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*(a*d + b*e)*Sqrt[a + c/x^2
 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]
*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (
-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*
a*e*Sqrt[d + e*x]*(c + b*x + a*x^2)) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*
c]*(b*d + c*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d
- (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 -
 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/S
qrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b
```

```
+ Sqrt[b^2 - 4*a*c])*e)])/(3*a*Sqrt[d + e*x]*(c + b*x + a*x^2)) -
 (Sqrt[2]*c*Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[a + c/x^
2 + b/x]*x*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b - Sqrt[b^2 - 4*a*
c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])
*e)]*EllipticPi[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), ArcS
in[(Sqrt[2]*Sqrt[a]*Sqrt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4
*a*c])*e]], (b - Sqrt[b^2 - 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4
*a*c] - (2*a*d)/e)])/(Sqrt[a]*(c + b*x + a*x^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\int\frac{\sqrt{d+ex}\sqrt{ax^2+bx+c}}{x}\,dx}{\sqrt{ax^2+bx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

```
[Out] x*sqrt(a + b/x + c/x**2)*Integral(sqrt(d + e*x)*sqrt(a*x**2 + b*x
 + c)/x, x)/sqrt(a*x**2 + b*x + c)
```

Mathematica [C] time = 13.1125, size = 758, normalized size = 0.79

$$\frac{2}{3}x\sqrt{a+\frac{bx+c}{x^2}}\left(-\frac{i(d+ex)\sqrt{1-\frac{2(ad^2+e(ce-bd))}{(d+ex)\left(\sqrt{e^2(b^2-4ac)}+2ad-be\right)}}\sqrt{\frac{2(ad^2+e(ce-bd))}{(d+ex)\left(\sqrt{e^2(b^2-4ac)}-2ad+be\right)}+1}\left(-\left(a\left(d\sqrt{e^2\left(b^2-4ac\right)}+3bde\right.\right.\right.}{}\right.$$

$$\left.+\frac{ad+be}{a\sqrt{d+ex}}+\sqrt{d+ex}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x],x]
```

```
[Out] (2*x*Sqrt[a + (c + b*x)/x^2]*((a*d + b*e)/(a*Sqrt[d + e*x]) + Sqr
t[d + e*x] - ((I/2)*(d + e*x)*Sqrt[1 - (2*(a*d^2 + e*(-(b*d) + c*
e)))/((2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1
+ (2*(a*d^2 + e*(-(b*d) + c*e)))/((-2*a*d + b*e + Sqrt[(b^2 - 4*a
*c)*e^2])*(d + e*x))]*((a*d + b*e)*(2*a*d - b*e + Sqrt[(b^2 - 4*a
*c)*e^2])*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^
2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((
-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2
- 4*a*c)*e^2]))] - (b*e*(-(b*e) + Sqrt[(b^2 - 4*a*c)*e^2]) + a*(3
*b*d*e - 2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2]))*EllipticF[I*ArcSin
h[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2
 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4
*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] - 6*a*c*e^2
*EllipticPi[(d*(2*a*d - b*e - Sqrt[(b^2 - 4*a*c)*e^2]))/(2*(a*d^2
 + e*(-(b*d) + c*e))), I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c
*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]],
-((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b
^2 - 4*a*c)*e^2]))]))/(Sqrt[2]*a*e^2*Sqrt[(a*d^2 + e*(-(b*d) + c*
e))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c + x*(b + a*x))))
)/3
```

Maple [B] time = 0.06, size = 3023, normalized size = 3.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x)
```

```
[Out] 1/3*((a*x^2+b*x+c)/x^2)^(1/2)*x*(e*x+d)^(1/2)*(2^(1/2)*(-a*(e*x+d
)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)
^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4
*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*Elliptic
F(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(
e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))
^(1/2))*(-4*a*c+b^2)^(1/2)*a*d^2*e-2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c
+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e
*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1
/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-
a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^
2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*(-4*
a*c+b^2)^(1/2)*b*d*e^2-2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2
)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b
^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-
4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/
(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2
*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(
1/2)*c*e^3+3*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)
)^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*
```

a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(
1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b
^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e
*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*a*b*d^2*e+6*2^(1/2)*(-a*(e
x+d)/(e(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+
b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x
+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*Elli
pticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)
,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b
*e))^(1/2))*a*c*d*e^2-3*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)
-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^
2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4
*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(
e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*
a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*b^2*d*e^2-2*2^(
1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a
*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*
(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))
^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+
b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1
/2)+2*a*d-b*e))^(1/2))*a^2*d^3-2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b
^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(
-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2
))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*
(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)
^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*a*c*d*
e^2+2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)
(e(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e
))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*
a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/
2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*
c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*b^2*d*e^2-2*2^(1/2)*(-a*(e*x+d)/(
e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1
/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*
c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticE(2
^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(
-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1
/2))*b*c*e^3+3*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*
e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+
2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)
^(1/2)-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*
c+b^2)^(1/2)-2*a*d+b*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b
e)/a/d,(-(e(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+
2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*c*e^3-6*2^(1/2)*(-a*(e*x+d)
/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^
(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*
a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticP
i(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),-1/
2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*
a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*a*c*d*e^2+3*2^(
1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a
*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*
(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))
^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d

```
+b*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a
*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))
*b*c*e^3+2*x^3*a^2*e^3+2*x^2*a^2*d*e^2+2*x^2*a*b*e^3+2*x*a*b*d*e^
2+2*x*a*c*e^3+2*a*c*d*e^2)/a/e^2/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c
*e*x+c*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2),x, algorithm="maxima")

[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)

[Out] Timed out
```

<u>GIAC/XCAS</u> [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2),x, algorithm="giac")

[Out] Timed out
```

3.84 $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx$

Optimal. Leaf size=929

$$\frac{3\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\sqrt{d+ex}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{2}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(ax^2+bx+c)}$$

$$-\frac{3\sqrt{2}\sqrt{b^2-4ac}d\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}(ax^2+bx+c)}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{a\sqrt{d+ex}(ax^2+bx+c)}$$

$$-\frac{(bd+ce)\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\left(\frac{2ad-be+\sqrt{b^2-4ac}e}{2ad};\sin^{-1}\right.}{\sqrt{2}\sqrt{a}d(ax^2+bx+c)}$$

$$-\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}$$

```
[Out] -(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x]) + (3*Sqrt[b^2 - 4*a*c]*Sqr
t[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(
b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a
*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d
- (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[2]*Sqrt[(a*(d + e*x))/(2*a*d
 - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) - (3*Sqrt[2]*Sq
rt[b^2 - 4*a*c]*d*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a
*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^
2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x
)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d -
(b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[d + e*x]*(c + b*x + a*x^2)) +
(2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(a*d + b*e)*Sqrt[a + c/x^2 + b/x]*x*
Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a
*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sq
rt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^
2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(a*Sqrt[d + e
*x]*(c + b*x + a*x^2)) - ((b*d + c*e)*Sqrt[2*a*d - (b - Sqrt[b^2
- 4*a*c])*e]*Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d + e*x))/(2*
a*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d
 - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[(2*a*d - b*e + Sqrt[b^2
 - 4*a*c]*e)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[a]*Sqrt[d + e*x])/Sqrt
[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt[b^2 - 4*a*c] - (2
*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)])/(Sqrt[2]*Sqrt[a]*d
```

```
*(c + b*x + a*x^2))
```

Rubi [A] time = 7.19342, antiderivative size = 929, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$

$$\frac{3\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\sqrt{d+ex}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{2}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}(ax^2+bx+c)}$$
$$-\frac{3\sqrt{2}\sqrt{b^2-4ac}d\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}(ax^2+bx+c)}$$
$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{a\sqrt{d+ex}(ax^2+bx+c)}$$
$$-\frac{(bd+ce)\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\left(\frac{2ad-be+\sqrt{b^2-4ace}}{2ad};\sin^{-1}\right.}{\sqrt{2}\sqrt{a}d(ax^2+bx+c)}$$
$$-\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x,x]
```

```
[Out] -(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x]) + (3*Sqrt[b^2 - 4*a*c]*Sqr
t[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(
b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a
*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d
- (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[2]*Sqrt[(a*(d + e*x))/(2*a*d
 - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) - (3*Sqrt[2]*Sq
rt[b^2 - 4*a*c]*d*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a
*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^
2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x
)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d -
(b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[d + e*x]*(c + b*x + a*x^2)) +
(2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(a*d + b*e)*Sqrt[a + c/x^2 + b/x]*x*
Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a
*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sq
rt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^
2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(a*Sqrt[d + e
```

```
*x]*(c + b*x + a*x^2)) - ((b*d + c*e)*Sqrt[2*a*d - (b - Sqrt[b^2
- 4*a*c])*e]*Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d + e*x))/(2*
a*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d
 - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[(2*a*d - b*e + Sqrt[b^2
 - 4*a*c]*e)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[a]*Sqrt[d + e*x])/Sqrt
[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt[b^2 - 4*a*c] - (2
*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)])/(Sqrt[2]*Sqrt[a]*d
*(c + b*x + a*x^2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2)/x,x)

[Out] Timed out
```

Mathematica [C] time = 13.1193, size = 4893, normalized size = 5.27

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x,x]

[Out] -(Sqrt[d + e*x]*Sqrt[a + (c + b*x)/x^2]) + (x*Sqrt[a + (c + b*x)/
x^2]*((3*(d + e*x)^(3/2)*(a + (a*d^2)/(d + e*x)^2 - (b*d*e)/(d +
e*x)^2 + (c*e^2)/(d + e*x)^2 - (2*a*d)/(d + e*x) + (b*e)/(d + e*x
)))/Sqrt[((d + e*x)^2*(a*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d
+ e*x) + (c*e)/(d + e*x)))/(d + e*x)))/e^2] - (((3*I)/2)*a*d^2*(2
*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x)*Sqrt[a + (a*d^2
)/(d + e*x)^2 - (b*d*e)/(d + e*x)^2 + (c*e^2)/(d + e*x)^2 - (2*a*
d)/(d + e*x) + (b*e)/(d + e*x)]*Sqrt[1 - (2*(a*d^2 - b*d*e + c*e^
2))/((2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt[1
 - (2*(a*d^2 - b*d*e + c*e^2))/((2*a*d - b*e + Sqrt[b^2*e^2 - 4*a
*c*e^2])*(d + e*x))]*(EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[-((a*d^2
- b*d*e + c*e^2)/(2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqr
t[d + e*x]], (2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*a*d - b
*e + Sqrt[b^2*e^2 - 4*a*c*e^2])] - EllipticF[I*ArcSinh[(Sqrt[2]*S
qrt[-((a*d^2 - b*d*e + c*e^2)/(2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c
*e^2]))])/Sqrt[d + e*x]], (2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2
])/(2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])]))/(Sqrt[2]*(a*d^2 -
 b*d*e + c*e^2)*Sqrt[-((a*d^2 - b*d*e + c*e^2)/(2*a*d - b*e - Sqr
```

```
t[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[a + (a*d^2 - b*d*e + c*e^2)/(d + e
*x)^2 + (-2*a*d + b*e)/(d + e*x)]*Sqrt[((d + e*x)^2*(a*(-1 + d/(d
 + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (c*e)/(d + e*x)))/(d + e*x
)))/e^2]) + (((3*I)/2)*b*d*e*(2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*
e^2])*(d + e*x)*Sqrt[a + (a*d^2)/(d + e*x)^2 - (b*d*e)/(d + e*x)^
2 + (c*e^2)/(d + e*x)^2 - (2*a*d)/(d + e*x) + (b*e)/(d + e*x)]*Sq
rt[1 - (2*(a*d^2 - b*d*e + c*e^2))/((2*a*d - b*e - Sqrt[b^2*e^2 -
 4*a*c*e^2])*(d + e*x))]*Sqrt[1 - (2*(a*d^2 - b*d*e + c*e^2))/((2
*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*(EllipticE[I*
ArcSinh[(Sqrt[2]*Sqrt[-((a*d^2 - b*d*e + c*e^2)/(2*a*d - b*e - Sq
rt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*a*d - b*e - Sqrt[b
^2*e^2 - 4*a*c*e^2])/(2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])] -
 EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[-((a*d^2 - b*d*e + c*e^2)/(2*a
*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*a*d -
 b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*a*d - b*e + Sqrt[b^2*e^2 - 4
*a*c*e^2])]))/(Sqrt[2]*(a*d^2 - b*d*e + c*e^2)*Sqrt[-((a*d^2 - b*
d*e + c*e^2)/(2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[a +
 (a*d^2 - b*d*e + c*e^2)/(d + e*x)^2 + (-2*a*d + b*e)/(d + e*x)]*
Sqrt[((d + e*x)^2*(a*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*
x) + (c*e)/(d + e*x)))/(d + e*x)))/e^2]) - (((3*I)/2)*c*e^2*(2*a*
d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x)*Sqrt[a + (a*d^2)/(
d + e*x)^2 - (b*d*e)/(d + e*x)^2 + (c*e^2)/(d + e*x)^2 - (2*a*d)/
(d + e*x) + (b*e)/(d + e*x)]*Sqrt[1 - (2*(a*d^2 - b*d*e + c*e^2))
/((2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt[1 -
(2*(a*d^2 - b*d*e + c*e^2))/((2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*
e^2])*(d + e*x))]*(EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[-((a*d^2 - b
*d*e + c*e^2)/(2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d
 + e*x]], (2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*a*d - b*e
+ Sqrt[b^2*e^2 - 4*a*c*e^2])] - EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt
[-((a*d^2 - b*d*e + c*e^2)/(2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^
2]))])/Sqrt[d + e*x]], (2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/
(2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])]))/(Sqrt[2]*(a*d^2 - b*
d*e + c*e^2)*Sqrt[-((a*d^2 - b*d*e + c*e^2)/(2*a*d - b*e - Sqrt[b
^2*e^2 - 4*a*c*e^2]))]*Sqrt[a + (a*d^2 - b*d*e + c*e^2)/(d + e*x)
^2 + (-2*a*d + b*e)/(d + e*x)]*Sqrt[((d + e*x)^2*(a*(-1 + d/(d +
e*x))^2 + (e*(b - (b*d)/(d + e*x) + (c*e)/(d + e*x)))/(d + e*x)))
/e^2]) - (I*a*d*(d + e*x)*Sqrt[a + (a*d^2)/(d + e*x)^2 - (b*d*e)/
(d + e*x)^2 + (c*e^2)/(d + e*x)^2 - (2*a*d)/(d + e*x) + (b*e)/(d
+ e*x)]*Sqrt[1 - (2*(a*d^2 - b*d*e + c*e^2))/((2*a*d - b*e - Sqrt
[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt[1 - (2*(a*d^2 - b*d*e + c
*e^2))/((2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Ell
ipticF[I*ArcSinh[(Sqrt[2]*Sqrt[-((a*d^2 - b*d*e + c*e^2)/(2*a*d -
 b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*a*d - b*e
 - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c
*e^2])])/(Sqrt[2]*Sqrt[-((a*d^2 - b*d*e + c*e^2)/(2*a*d - b*e - S
qrt[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[a + (a*d^2 - b*d*e + c*e^2)/(d +
 e*x)^2 + (-2*a*d + b*e)/(d + e*x)]*Sqrt[((d + e*x)^2*(a*(-1 + d/
(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (c*e)/(d + e*x)))/(d + e
*x)))/e^2]) + (I*b*e*(d + e*x)*Sqrt[a + (a*d^2)/(d + e*x)^2 - (b*
d*e)/(d + e*x)^2 + (c*e^2)/(d + e*x)^2 - (2*a*d)/(d + e*x) + (b*e
)/(d + e*x)]*Sqrt[1 - (2*(a*d^2 - b*d*e + c*e^2))/((2*a*d - b*e -
 Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt[1 - (2*(a*d^2 - b*d*
e + c*e^2))/((2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))
]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[-((a*d^2 - b*d*e + c*e^2)/(2*
```

```
a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*a*d
- b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*a*d - b*e + Sqrt[b^2*e^2 -
4*a*c*e^2])])/(Sqrt[2]*Sqrt[-((a*d^2 - b*d*e + c*e^2)/(2*a*d - b*
e - Sqrt[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[a + (a*d^2 - b*d*e + c*e^2)
/(d + e*x)^2 + (-2*a*d + b*e)/(d + e*x)]*Sqrt[((d + e*x)^2*(a*(-1
 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (c*e)/(d + e*x)))/(
d + e*x)))/e^2]) - (I*c*e^2*(d + e*x)*Sqrt[a + (a*d^2)/(d + e*x)^
2 - (b*d*e)/(d + e*x)^2 + (c*e^2)/(d + e*x)^2 - (2*a*d)/(d + e*x)
 + (b*e)/(d + e*x)]*Sqrt[1 - (2*(a*d^2 - b*d*e + c*e^2))/((2*a*d
- b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt[1 - (2*(a*d^2
 - b*d*e + c*e^2))/((2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])*(d
+ e*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[-((a*d^2 - b*d*e + c*e
^2)/(2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]],
(2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*a*d - b*e + Sqrt[b^2
*e^2 - 4*a*c*e^2])])/(Sqrt[2]*d*Sqrt[-((a*d^2 - b*d*e + c*e^2)/(2
*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[a + (a*d^2 - b*d*e
 + c*e^2)/(d + e*x)^2 + (-2*a*d + b*e)/(d + e*x)]*Sqrt[((d + e*x)
^2*(a*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (c*e)/(d +
 e*x)))/(d + e*x)))/e^2]) + (I*b*e*(d + e*x)*Sqrt[a + (a*d^2)/(d
+ e*x)^2 - (b*d*e)/(d + e*x)^2 + (c*e^2)/(d + e*x)^2 - (2*a*d)/(d
 + e*x) + (b*e)/(d + e*x)]*Sqrt[1 - (2*(a*d^2 - b*d*e + c*e^2))/(
(2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*Sqrt[1 - (2
*(a*d^2 - b*d*e + c*e^2))/((2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^
2])*(d + e*x))]*EllipticPi[(d*(2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c
*e^2]))/(2*(a*d^2 - b*d*e + c*e^2)), I*ArcSinh[(Sqrt[2]*Sqrt[-((a
*d^2 - b*d*e + c*e^2)/(2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))]
)/Sqrt[d + e*x]], (2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*a*
d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])])/(Sqrt[2]*Sqrt[-((a*d^2 - b
*d*e + c*e^2)/(2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2]))]*Sqrt[a
+ (a*d^2 - b*d*e + c*e^2)/(d + e*x)^2 + (-2*a*d + b*e)/(d + e*x)]
*Sqrt[((d + e*x)^2*(a*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e
*x) + (c*e)/(d + e*x)))/(d + e*x)))/e^2]) + (I*c*e^2*(d + e*x)*Sq
rt[a + (a*d^2)/(d + e*x)^2 - (b*d*e)/(d + e*x)^2 + (c*e^2)/(d + e
*x)^2 - (2*a*d)/(d + e*x) + (b*e)/(d + e*x)]*Sqrt[1 - (2*(a*d^2 -
 b*d*e + c*e^2))/((2*a*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])*(d +
e*x))]*Sqrt[1 - (2*(a*d^2 - b*d*e + c*e^2))/((2*a*d - b*e + Sqrt[
b^2*e^2 - 4*a*c*e^2])*(d + e*x))]*EllipticPi[(d*(2*a*d - b*e - Sq
rt[b^2*e^2 - 4*a*c*e^2]))/(2*(a*d^2 - b*d*e + c*e^2)), I*ArcSinh[
(Sqrt[2]*Sqrt[-((a*d^2 - b*d*e + c*e^2)/(2*a*d - b*e - Sqrt[b^2*e
^2 - 4*a*c*e^2]))])/Sqrt[d + e*x]], (2*a*d - b*e - Sqrt[b^2*e^2 -
 4*a*c*e^2])/(2*a*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])])/(Sqrt[2]
*d*Sqrt[-((a*d^2 - b*d*e + c*e^2)/(2*a*d - b*e - Sqrt[b^2*e^2 - 4
*a*c*e^2]))]*Sqrt[a + (a*d^2 - b*d*e + c*e^2)/(d + e*x)^2 + (-2*a
*d + b*e)/(d + e*x)]*Sqrt[((d + e*x)^2*(a*(-1 + d/(d + e*x))^2 +
(e*(b - (b*d)/(d + e*x) + (c*e)/(d + e*x)))/(d + e*x)))/e^2])))/(
e*Sqrt[c + b*x + a*x^2])
```

Maple [B] time = 0.066, size = 3553, normalized size = 3.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x)
```

```
[Out] 1/2*((a*x^2+b*x+c)/x^2)^(1/2)*(e*x+d)^(1/2)*(2^(1/2)*(-a*(e*x+d)/
(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(
1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a
*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(
2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*
(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(
1/2))*(-4*a*c+b^2)^(1/2)*x*a*d^2*e-2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a
*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/
(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^
(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*
(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+
b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*(-
4*a*c+b^2)^(1/2)*x*b*d*e^2+4*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^
(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a
*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(
e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x
+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/
2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x*a^2*d^3-
2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-
2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/
2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*
e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a
*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)
^(1/2)+2*a*d-b*e))^(1/2))*x*a*b*d^2*e+6*2^(1/2)*(-a*(e*x+d)/(e*(-
4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-
b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^
2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/
2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a
*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))
*x*a*c*d*e^2-2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*
e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+
2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)
^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c
+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/
(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x*b^2*d*e^2-6*2^(1/2)*(-
a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*
a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2
*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*
EllipticE(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(
1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a
*d-b*e))^(1/2))*x*a^2*d^3+6*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(
1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*
c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e
*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+
d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2
)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x*a*b*d^2*e
-6*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e
*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^
(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d
```

```
+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-
2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b
^2)^(1/2)+2*a*d-b*e))^(1/2))*x*a*c*d*e^2+2^(1/2)*(-a*(e*x+d)/(e*(
-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)
-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b
^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticPi(2^(
1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),-1/2*(e*
(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b
*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*x
*b*d*e^2+2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1
/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-
b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)
-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)
^(1/2)-2*a*d+b*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/a/
d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-
b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*x*c*e^3-2*2^(1/2)*(-a*(e*x+d)/(e*
(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2
)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+
b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticPi(2^
(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),-1/2*(e
*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+
b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x*a*b*d^2*e-2*2^(1/
2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x
+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e
*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(
1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b
*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a*c
+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x
*a*c*d*e^2+2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^
(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*
d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/
2)-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^
2)^(1/2)-2*a*d+b*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/
a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*
d-b*e))^(1/2))*x*b^2*d*e^2+2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1
/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c
+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*
(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+
d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(
1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c
+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x*b*c*e^3-2*x^3*a^2*d*e^2-2*x^2*a^
2*d^2*e-2*x^2*a*b*d*e^2-2*x*a*b*d^2*e-2*a*c*d*e^2*x-2*a*c*d^2*e)/
(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)/a/e/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d + ex}\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2)/x,x)

[Out] Integral(sqrt(d + e*x)*sqrt(a + b/x + c/x**2)/x, x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x,x, algorithm="giac")

[Out] Timed out
```

3.85 $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx$

Optimal. Leaf size=1287

result too large to display

```
[Out] -((b*d + c*e)*Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/(4*c*d) - (Sqr
t[a + c/x^2 + b/x]*Sqrt[d + e*x])/(2*x) + (Sqrt[b^2 - 4*a*c]*(b*d
 + c*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x
+ a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]
*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(4*Sqrt[2]*c*d*Sqrt[(a*
(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)
) + (3*Sqrt[b^2 - 4*a*c]*e*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e
*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*
x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/
(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[2]*Sqrt[d + e*x]*(c +
 b*x + a*x^2)) - (Sqrt[b^2 - 4*a*c]*(b*d + c*e)*Sqrt[a + c/x^2 +
b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sq
rt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*
Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*Sqr
t[2]*c*Sqrt[d + e*x]*(c + b*x + a*x^2)) - ((a*d + b*e)*Sqrt[2*a*d
 - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2
*a*(d + e*x))/(2*a*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*a*
(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[(2*a*d
 - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[a]*Sq
rt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt[
b^2 - 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)])/(
Sqrt[2]*Sqrt[a]*d*(c + b*x + a*x^2)) + ((b*d + c*e)^2*Sqrt[2*a*d
- (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2*
a*(d + e*x))/(2*a*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*a*(
d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[(2*a*d
- b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[a]*Sqr
t[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt[b
^2 - 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)])/(4
*Sqrt[2]*Sqrt[a]*c*d^2*(c + b*x + a*x^2))
```

Rubi [A] time = 13.6361, antiderivative size = 1287, normalized size of antiderivative = 1., number

of steps used = 24, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$

$$\frac{\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\left(\frac{2ad-be+\sqrt{b^2-4ac}e}{2ad};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{a}}{\sqrt{2ad-\left(b-\right.}}\right.\right.}{4\sqrt{2}\sqrt{a}cd^2\left(ax^2+bx+c\right)}$$

$$+\frac{\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\sqrt{d+ex}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)(bd+ce)}{4\sqrt{2}cd\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\left(ax^2+bx+c\right)}$$

$$-\frac{\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)(bd+ce)}{2\sqrt{2}c\sqrt{d+ex}\left(ax^2+bx+c\right)}$$

$$-\frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}(bd+ce)}{4cd}$$

$$+\frac{3\sqrt{b^2-4ac}e\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{2}\sqrt{d+ex}\left(ax^2+bx+c\right)}$$

$$-\frac{(ad+be)\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\left(\frac{2ad-be+\sqrt{b^2-4ac}e}{2ad};\sin^{-1}\right.}{\sqrt{2}\sqrt{a}d\left(ax^2+bx+c\right)}$$

$$-\frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}}{2x}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x^2,x]
```

```
[Out] -((b*d + c*e)*Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/(4*c*d) - (Sqr
t[a + c/x^2 + b/x]*Sqrt[d + e*x])/(2*x) + (Sqrt[b^2 - 4*a*c]*(b*d
 + c*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x
+ a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]
*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(4*Sqrt[2]*c*d*Sqrt[(a*
(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)
) + (3*Sqrt[b^2 - 4*a*c]*e*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e
*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*
x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/
(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[2]*Sqrt[d + e*x]*(c +
 b*x + a*x^2)) - (Sqrt[b^2 - 4*a*c]*(b*d + c*e)*Sqrt[a + c/x^2 +
```

```
b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sq
rt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*
Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*Sqr
t[2]*c*Sqrt[d + e*x]*(c + b*x + a*x^2)) - ((a*d + b*e)*Sqrt[2*a*d
 - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2
*a*(d + e*x))/(2*a*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*a*
(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[(2*a*d
 - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[a]*Sq
rt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt[
b^2 - 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)])/(
Sqrt[2]*Sqrt[a]*d*(c + b*x + a*x^2)) + ((b*d + c*e)^2*Sqrt[2*a*d
- (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2*
a*(d + e*x))/(2*a*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*a*(
d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[(2*a*d
- b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[a]*Sqr
t[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt[b
^2 - 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)])/(4
*Sqrt[2]*Sqrt[a]*c*d^2*(c + b*x + a*x^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\sqrt{d+ex}\int^{\frac{1}{\sqrt{x}}}\sqrt{dx^2+e}\sqrt{a+bx^2+cx^4}\,dx}{\sqrt{x}\sqrt{\frac{d}{x}+e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2)/x**2,x)
```

```
[Out] -2*sqrt(d + e*x)*Integral(sqrt(d*x**2 + e)*sqrt(a + b*x**2 + c*x*
*4), (x, 1/sqrt(x)))/(sqrt(x)*sqrt(d/x + e))
```

Mathematica [C] time = 13.6071, size = 6206, normalized size = 4.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x^2,x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.071, size = 4957, normalized size = 3.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x^2,x)
```

```
[Out] 1/8*((a*x^2+b*x+c)/x^2)^(1/2)*(e*x+d)^(1/2)*(-4*x^2*a^2*c*d^3*e-6
*x^3*a^2*c*d^2*e^2-2*x^3*a*b^2*d^2*e^2-2*x^2*a*c^2*d*e^3-6*x*a*c^
2*d^2*e^2-2*x^4*a^2*b*d^2*e^2-2*x^4*a^2*c*d*e^3-2*x^3*a^2*b*d^3*e
-2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(
-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1
/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b
*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2
*a*d+b*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(
-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1
/2))*(-4*a*c+b^2)^(1/2)*x^2*c^2*e^4-2*2^(1/2)*(-a*(e*x+d)/(e*(-4*
a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)
/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)
^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)
*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c
+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x
^2*a^2*b*d^4-2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)
)^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*
a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(
1/2)-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+
b^2)^(1/2)-2*a*d+b*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e
)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*
a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*x^2*b^2*d^2*e^2+12*2^(1/2)*(-
a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*
a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2
*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*
EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(
1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a
*d-b*e))^(1/2))*x^2*a^2*c*d^3*e-2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^
2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-
4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2)
)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(
e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^
(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x^2*a*b
^2*d^3*e-2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^
(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*
d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/
2)-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2
)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(
-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x^2*a*c^2*d*e^3+2*2^(1/2)*(-
a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*
a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2
*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*
EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(
1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a
```

*d-b*e))^(1/2))*x^2*a*c^2*d*e^3-8*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+
b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*
(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/
2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-
a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),-1/2*(e*(-4*a*c
+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*
(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x^2*a^2*c*d^3*e+2*2^(1/2)*(
-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4
*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+
2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)
EllipticPi(2^(1/2)(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))
^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a*c+b^2
)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x^2*a
*b^2*d^3*e+2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)
)^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*
a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(
1/2)-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+
b^2)^(1/2)-2*a*d+b*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e
)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*
a*d-b*e))^(1/2))*x^2*a*c^2*d*e^3+2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c
+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e
*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1
/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(
-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),-1/2*(e*(-4*a*
c+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e
*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x^2*b^2*c*d*e^3-2*2^(1/2)*
(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-
4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b
+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2
)*EllipticE(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))
^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2
*a*d-b*e))^(1/2))*x^2*a^2*c*d^3*e+2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*
c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(
e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(
1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(
-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b
^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x^2
*a*b^2*d^3*e-4*a*c^2*d^3*e-2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1
/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c
+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*
(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+
d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(
1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c
+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x^2*b^3*d^2*e^2-2^(1/2)*(-a*(e*x+d
)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)
^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4
*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*Elliptic
Pi(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),-1
/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2
*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x^2*b*c^2*e^4-
8*x^2*a*b*c*d^2*e^2-6*x*a*b*c*d^3*e+4*2^(1/2)*(-a*(e*x+d)/(e*(-4*
a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)
/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)
^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2

```
)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),-1/2*(e*(-4
*a*c+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)
/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*x^2*
a*c*d^2*e^2+2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e
))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2
*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^
(1/2)-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c
+b^2)^(1/2)-2*a*d+b*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*
e)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2
*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*x^2*b*c*d*e^3+2^(1/2)*(-a*(e
*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+
b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x
+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*Elli
pticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)
,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b
*e))^(1/2))*(-4*a*c+b^2)^(1/2)*x^2*a*b*d^3*e-5*2^(1/2)*(-a*(e*x+d
)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)
^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4
*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*Elliptic
F(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(
e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))
^(1/2))*(-4*a*c+b^2)^(1/2)*x^2*a*c*d^2*e^2-5*2^(1/2)*(-a*(e*x+d)/
(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(
1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a
*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(
2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*
(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(
1/2))*x^2*a*b*c*d^2*e^2-2*x^3*a*b*c*d*e^3-2*x^2*a*b^2*d^3*e)/x/a/
e/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)/c/d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.86 $\int (fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=29

$$\text{Int}\left((fx)^m \left(a+cx^{2n}\right)^p (d+ex^n)^q, x\right)$$

```
[Out] Unintegrable[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x]
```

Rubi [A] time = 0.0529469, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left((fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p, x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p,x]
```

```
[Out] Defer[Int][(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (fx)^m \left(a+cx^{2n}\right)^p (d+ex^n)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(d+e*x**n)**q*(a+c*x**(2*n))**p,x)
```

```
[Out] Integral((f*x)**m*(a + c*x**(2*n))**p*(d + e*x**n)**q, x)
```

Mathematica [A] time = 0.244799, size = 0, normalized size = 0.

$$\int (fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p,x]
```

```
[Out] Integrate[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x]
```

Maple [A] time = 0.647, size = 0, normalized size = 0.

$$\int (fx)^{m} (d + ex^{n})^{q} \left(a + cx^{2n}\right)^{p} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n} + a\right)^{p} (ex^{n} + d)^{q} (fx)^{m} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q*(f*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q*(f*x)^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^{2n} + a\right)^{p} (ex^{n} + d)^{q} (fx)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q*(f*x)^m,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + a)^p*(e*x^n + d)^q*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(d+e*x**n)**q*(a+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n} + a\right)^{p} (ex^{n} + d)^{q} \,(fx)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q*(f*x)^m,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q*(f*x)^m, x)
```

3.87 $\int (fx)^m (d+ex^n)^3 \left(a+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=358

$$\frac{d^3(fx)^{m+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)}{f(m+1)}+\frac{3d^2ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+n+1}{2n},-p;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+n+1}+\frac{3de^2x^{2n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+2n+1}{2n},-p;\frac{m+4n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+2n+1}+\frac{e^3x^{3n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+3n+1}{2n},-p;\frac{m+5n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+3n+1}$$

```
[Out] (d^3*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2
*n), -p, 1 + (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c
*x^(2*n))/a)^p) + (3*d^2*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hy
pergeometric2F1[(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*
x^(2*n))/a)])/((1 + m + n)*(1 + (c*x^(2*n))/a)^p) + (3*d*e^2*x^(1
 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m + 2*n)
/(2*n), -p, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + 2*n
)*(1 + (c*x^(2*n))/a)^p) + (e^3*x^(1 + 3*n)*(f*x)^m*(a + c*x^(2*n
))^p*Hypergeometric2F1[(1 + m + 3*n)/(2*n), -p, (1 + m + 5*n)/(2*
n), -((c*x^(2*n))/a)])/((1 + m + 3*n)*(1 + (c*x^(2*n))/a)^p)
```

Rubi [A] time = 0.506772, antiderivative size = 358, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{d^3(fx)^{m+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)}{f(m+1)}+\frac{3d^2ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+n+1}{2n},-p;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+n+1}+\frac{3de^2x^{2n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+2n+1}{2n},-p;\frac{m+4n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+2n+1}+\frac{e^3x^{3n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+3n+1}{2n},-p;\frac{m+5n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+3n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(f*x)^m*(d + e*x^n)^3*(a + c*x^(2*n))^p,x]
```

```
[Out] (d^3*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2
*n), -p, 1 + (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c
*x^(2*n))/a)^p) + (3*d^2*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hy
pergeometric2F1[(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*
x^(2*n))/a)])/((1 + m + n)*(1 + (c*x^(2*n))/a)^p) + (3*d*e^2*x^(1
 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m + 2*n)
/(2*n), -p, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + 2*n
)*(1 + (c*x^(2*n))/a)^p) + (e^3*x^(1 + 3*n)*(f*x)^m*(a + c*x^(2*n
))^p*Hypergeometric2F1[(1 + m + 3*n)/(2*n), -p, (1 + m + 5*n)/(2*
n), -((c*x^(2*n))/a)])/((1 + m + 3*n)*(1 + (c*x^(2*n))/a)^p)
```

Rubi in Sympy [A] time = 65.938, size = 320, normalized size = 0.89

$$\frac{d^{3}\left(fx\right)^{m+1}\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^{p}{}_{2}F_{1}\left(\begin{matrix}-p,\ \frac{m+1}{2n}\\1+\frac{m+1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{f\left(m+1\right)}+\frac{3d^{2}ex^{n}\left(fx\right)^{-n}\left(fx\right)^{m+n+1}\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^{p}{}_{2}F_{1}\left(\begin{matrix}-p,\ \frac{m+n+1}{2n}\\\frac{m+3n+1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{f\left(m+n+1\right)}+\frac{3de^{2}x^{2n}\left(fx\right)^{-2n}\left(fx\right)^{m+2n+1}\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^{p}{}_{2}F_{1}\left(\begin{matrix}-p,\ \frac{m+2n+1}{2n}\\\frac{m+4n+1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{f\left(m+2n+1\right)}+\frac{e^{3}x^{3n}\left(fx\right)^{-3n}\left(fx\right)^{m+3n+1}\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^{p}{}_{2}F_{1}\left(\begin{matrix}-p,\ \frac{m+3n+1}{2n}\\\frac{m+5n+1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{f\left(m+3n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(d+e*x**n)**3*(a+c*x**(2*n))**p,x)
```

```
[Out] d**3*(f*x)**(m + 1)*(1 + c*x**(2*n)/a)**(-p)*(a + c*x**(2*n))**p*
hyper((-p, (m + 1)/(2*n)), (1 + (m + 1)/(2*n),), -c*x**(2*n)/a)/(
f*(m + 1)) + 3*d**2*e*x**n*(f*x)**(-n)*(f*x)**(m + n + 1)*(1 + c*
x**(2*n)/a)**(-p)*(a + c*x**(2*n))**p*hyper((-p, (m + n + 1)/(2*n
)), ((m + 3*n + 1)/(2*n),), -c*x**(2*n)/a)/(f*(m + n + 1)) + 3*d*
e**2*x**(2*n)*(f*x)**(-2*n)*(f*x)**(m + 2*n + 1)*(1 + c*x**(2*n)/
a)**(-p)*(a + c*x**(2*n))**p*hyper((-p, (m + 2*n + 1)/(2*n)), ((m
 + 4*n + 1)/(2*n),), -c*x**(2*n)/a)/(f*(m + 2*n + 1)) + e**3*x**(
3*n)*(f*x)**(-3*n)*(f*x)**(m + 3*n + 1)*(1 + c*x**(2*n)/a)**(-p)*
(a + c*x**(2*n))**p*hyper((-p, (m + 3*n + 1)/(2*n)), ((m + 5*n +
1)/(2*n),), -c*x**(2*n)/a)/(f*(m + 3*n + 1))
```

Mathematica [A] time = 0.685681, size = 249, normalized size = 0.7

$$x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\left(\frac{d^3\,{}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)}{m+1}+ex^n\left(\frac{3d^2\,{}_2F_1\left(\frac{m+n+1}{2n},-p;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+n+1}+ex^n\left(\frac{3d\,{}_2F_1\left(\frac{m+2n+1}{2n},-p;\frac{m+4n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+2n+1}+\frac{ex^n\,{}_2F_1\left(\frac{m+3n+1}{2n},-p;\frac{m+5n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+3n+1}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(f*x)^m*(d + e*x^n)^3*(a + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*((d^3*Hypergeometric2F1[(1 + m)/(2*n
), -p, 1 + (1 + m)/(2*n), -((c*x^(2*n))/a)])/(1 + m) + e*x^n*((3*
d^2*Hypergeometric2F1[(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n),
 -((c*x^(2*n))/a)])/(1 + m + n) + e*x^n*((3*d*Hypergeometric2F1[(
1 + m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a)])/(
1 + m + 2*n) + (e*x^n*Hypergeometric2F1[(1 + m + 3*n)/(2*n), -p,
(1 + m + 5*n)/(2*n), -((c*x^(2*n))/a)])/(1 + m + 3*n)))))/(1 + (c
*x^(2*n))/a)^p
```

Maple [F] time = 0.113, size = 0, normalized size = 0.

$$\int (fx)^m\,(d+ex^n)^3\left(a+cx^{2\,n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^n+d)^3\left(cx^{2\,n}+a\right)^p(fx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^3*(c*x^(2*n) + a)^p*(f*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^3*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^{3}x^{3\,n}+3\,de^{2}x^{2\,n}+3\,d^{2}ex^{n}+d^{3}\right)\left(cx^{2\,n}+a\right)^{p}(fx)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^3*(c*x^(2*n) + a)^p*(f*x)^m,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)*(c*x
^(2*n) + a)^p*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(d+e*x**n)**3*(a+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^3*(c*x^(2*n) + a)^p*(f*x)^m,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.88 $\int (fx)^m (d+ex^n)^2 \left(a+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=262

$$\frac{d^2(fx)^{m+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)}{f(m+1)}+\frac{2dex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+n+1}{2n},-p;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+n+1}+\frac{e^2x^{2n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+2n+1}{2n},-p;\frac{m+4n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+2n+1}$$

```
[Out] (d^2*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2
*n), -p, 1 + (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c
*x^(2*n))/a)^p) + (2*d*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hype
rgeometric2F1[(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^
(2*n))/a)])/((1 + m + n)*(1 + (c*x^(2*n))/a)^p) + (e^2*x^(1 + 2*n
)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m + 2*n)/(2*n)
, -p, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + 2*n)*(1 +
 (c*x^(2*n))/a)^p)
```

Rubi [A] time = 0.348406, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{d^2(fx)^{m+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)}{f(m+1)}+\frac{2dex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+n+1}{2n},-p;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+n+1}+\frac{e^2x^{2n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+2n+1}{2n},-p;\frac{m+4n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+2n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(f*x)^m*(d + e*x^n)^2*(a + c*x^(2*n))^p,x]
```

```
[Out] (d^2*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2
*n), -p, 1 + (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c
*x^(2*n))/a)^p) + (2*d*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hype
rgeometric2F1[(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^
(2*n))/a)])/((1 + m + n)*(1 + (c*x^(2*n))/a)^p) + (e^2*x^(1 + 2*n
)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m + 2*n)/(2*n)
, -p, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + 2*n)*(1 +
```

```
(c*x^(2*n))/a)^p)
```

Rubi in Sympy [A] time = 46.7903, size = 228, normalized size = 0.87

$$\frac{d^2 (fx)^{m+1} \left(1+\frac{cx^{2n}}{a}\right)^{-p} \left(a+cx^{2n}\right)^p {}_2F_1\left(\left.\begin{matrix} -p, \frac{m+1}{2n} \\ 1+\frac{m+1}{2n} \end{matrix}\right| -\frac{cx^{2n}}{a}\right)}{f(m+1)} + \frac{2dex^n (fx)^{-n} (fx)^{m+n+1} \left(1+\frac{cx^{2n}}{a}\right)^{-p} \left(a+cx^{2n}\right)^p {}_2F_1\left(\left.\begin{matrix} -p, \frac{m+n+1}{2n} \\ \frac{m+3n+1}{2n} \end{matrix}\right| -\frac{cx^{2n}}{a}\right)}{f(m+n+1)} + \frac{e^2x^{2n} (fx)^{-2n} (fx)^{m+2n+1} \left(1+\frac{cx^{2n}}{a}\right)^{-p} \left(a+cx^{2n}\right)^p {}_2F_1\left(\left.\begin{matrix} -p, \frac{m+2n+1}{2n} \\ \frac{m+4n+1}{2n} \end{matrix}\right| -\frac{cx^{2n}}{a}\right)}{f(m+2n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(d+e*x**n)**2*(a+c*x**(2*n))**p,x)
```

```
[Out] d**2*(f*x)**(m + 1)*(1 + c*x**(2*n)/a)**(-p)*(a + c*x**(2*n))**p*
hyper((-p, (m + 1)/(2*n)), (1 + (m + 1)/(2*n),), -c*x**(2*n)/a)/(
f*(m + 1)) + 2*d*e*x**n*(f*x)**(-n)*(f*x)**(m + n + 1)*(1 + c*x**
(2*n)/a)**(-p)*(a + c*x**(2*n))**p*hyper((-p, (m + n + 1)/(2*n)),
 ((m + 3*n + 1)/(2*n),), -c*x**(2*n)/a)/(f*(m + n + 1)) + e**2*x*
*(2*n)*(f*x)**(-2*n)*(f*x)**(m + 2*n + 1)*(1 + c*x**(2*n)/a)**(-p
)*(a + c*x**(2*n))**p*hyper((-p, (m + 2*n + 1)/(2*n)), ((m + 4*n
+ 1)/(2*n),), -c*x**(2*n)/a)/(f*(m + 2*n + 1))
```

Mathematica [A] time = 0.335145, size = 189, normalized size = 0.72

$$x(fx)^m \left(a+cx^{2n}\right)^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} \left(\frac{d^2\, {}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)}{m+1} + ex^n \left(\frac{2d\, {}_2F_1\left(\frac{m+n+1}{2n},-p;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+n+1} + \frac{ex^n\, {}_2F_1\left(\frac{m+2n+1}{2n},-p;\frac{m+4n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+2n+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(f*x)^m*(d + e*x^n)^2*(a + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*((d^2*Hypergeometric2F1[(1 + m)/(2*n
), -p, 1 + (1 + m)/(2*n), -((c*x^(2*n))/a)])/(1 + m) + e*x^n*((2*
d*Hypergeometric2F1[(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -
((c*x^(2*n))/a)])/(1 + m + n) + (e*x^n*Hypergeometric2F1[(1 + m +
 2*n)/(2*n), -p, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a)])/(1 + m +
 2*n))))/(1 + (c*x^(2*n))/a)^p
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int (fx)^m (d+ex^n)^2 \left(a+cx^{2n}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^n + d)^2 \left(cx^{2n} + a\right)^p (fx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^2*(c*x^(2*n) + a)^p*(f*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^2*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^{2n} + 2\,dex^n + d^2\right)\left(cx^{2n} + a\right)^p (fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^2*(c*x^(2*n) + a)^p*(f*x)^m,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)*(c*x^(2*n) + a)^p*(f*x)^
m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(d+e*x**n)**2*(a+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^2*(c*x^(2*n) + a)^p*(f*x)^m,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.89 $\int (fx)^m (d + ex^n) \left(a + cx^{2n}\right)^p \, dx$

Optimal. Leaf size=166

$$\frac{d(fx)^{m+1} \left(a + cx^{2n}\right)^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{2n}, -p; \frac{m+1}{2n} + 1; -\frac{cx^{2n}}{a}\right)}{f(m+1)} + \frac{ex^{n+1}(fx)^m \left(a + cx^{2n}\right)^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+n+1}{2n}, -p; \frac{m+3n+1}{2n}; -\frac{cx^{2n}}{a}\right)}{m+n+1}$$

```
[Out] (d*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2*n
), -p, 1 + (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c*x
^(2*n))/a)^p) + (e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeome
tric2F1[(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))
/a)])/((1 + m + n)*(1 + (c*x^(2*n))/a)^p)
```

Rubi [A] time = 0.216018, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{d(fx)^{m+1} \left(a + cx^{2n}\right)^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{2n}, -p; \frac{m+1}{2n} + 1; -\frac{cx^{2n}}{a}\right)}{f(m+1)} + \frac{ex^{n+1}(fx)^m \left(a + cx^{2n}\right)^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+n+1}{2n}, -p; \frac{m+3n+1}{2n}; -\frac{cx^{2n}}{a}\right)}{m+n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(f*x)^m*(d + e*x^n)*(a + c*x^(2*n))^p,x]
```

```
[Out] (d*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2*n
), -p, 1 + (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c*x
^(2*n))/a)^p) + (e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeome
tric2F1[(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))
/a)])/((1 + m + n)*(1 + (c*x^(2*n))/a)^p)
```

Rubi in Sympy [A] time = 27.4712, size = 136, normalized size = 0.82

$$\frac{d\,(fx)^{m+1}\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^{p}{}_2F_1\left(\begin{matrix}-p,\frac{m+1}{2n}\\1+\frac{m+1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{f\,(m+1)}+\frac{ex^{n}\,(fx)^{-n}\,(fx)^{m+n+1}\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^{p}{}_2F_1\left(\begin{matrix}-p,\frac{m+n+1}{2n}\\\frac{m+3n+1}{2n}\end{matrix}\middle|-\frac{cx^{2n}}{a}\right)}{f\,(m+n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(d+e*x**n)*(a+c*x**(2*n))**p,x)
```

```
[Out] d*(f*x)**(m + 1)*(1 + c*x**(2*n)/a)**(-p)*(a + c*x**(2*n))**p*hyp
er((-p, (m + 1)/(2*n)), (1 + (m + 1)/(2*n),), -c*x**(2*n)/a)/(f*(
m + 1)) + e*x**n*(f*x)**(-n)*(f*x)**(m + n + 1)*(1 + c*x**(2*n)/a
)**(-p)*(a + c*x**(2*n))**p*hyper((-p, (m + n + 1)/(2*n)), ((m +
3*n + 1)/(2*n),), -c*x**(2*n)/a)/(f*(m + n + 1))
```

Mathematica [A] time = 0.147541, size = 136, normalized size = 0.82

$$\frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\left(d(m+n+1)\,{}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)+e(m+1)x^n\,{}_2F_1\left(\frac{m+n+1}{2n},-p;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a}\right)\right)}{(m+1)(m+n+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(f*x)^m*(d + e*x^n)*(a + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*(d*(1 + m + n)*Hypergeometric2F1[(1
+ m)/(2*n), -p, 1 + (1 + m)/(2*n), -((c*x^(2*n))/a)] + e*(1 + m)*
x^n*Hypergeometric2F1[(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n),
 -((c*x^(2*n))/a)]))/((1 + m)*(1 + m + n)*(1 + (c*x^(2*n))/a)^p)
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int (fx)^m\,(d+ex^n)\left(a+cx^{2\,n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x)

[Out] int((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^n + d)(cx^{2n} + a)^p (fx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)*(c*x^(2*n) + a)^p*(f*x)^m,x, algorithm="maxima")

[Out] integrate((e*x^n + d)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex^n + d)(cx^{2n} + a)^p (fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)*(c*x^(2*n) + a)^p*(f*x)^m,x, algorithm="fricas")

[Out] integral((e*x^n + d)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(d+e*x**n)*(a+c*x**(2*n))**p,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^n + d)\left(cx^{2n} + a\right)^p (fx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)*(c*x^(2*n) + a)^p*(f*x)^m,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

3.90 $\int \frac{(fx)^m(a+cx^{2n})^p}{d+ex^n}\,dx$

Optimal. Leaf size=194

$$\frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2n};-p,1;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d(m+1)}-\frac{ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+n+1}{2n};-p,1;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(m+n+1)}$$

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 1, 1 + (
1 + m)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d*(1 + m)*(1
 + (c*x^(2*n))/a)^p) - (e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*App
ellF1[(1 + m + n)/(2*n), -p, 1, (1 + m + 3*n)/(2*n), -((c*x^(2*n)
)/a), (e^2*x^(2*n))/d^2])/(d^2*(1 + m + n)*(1 + (c*x^(2*n))/a)^p)
```

Rubi [A] time = 0.504238, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2n};-p,1;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d(m+1)}-\frac{ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+n+1}{2n};-p,1;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(m+n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n),x]
```


```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 1, 1 + (
1 + m)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d*(1 + m)*(1
 + (c*x^(2*n))/a)^p) - (e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*App
ellF1[(1 + m + n)/(2*n), -p, 1, (1 + m + 3*n)/(2*n), -((c*x^(2*n)
)/a), (e^2*x^(2*n))/d^2])/(d^2*(1 + m + n)*(1 + (c*x^(2*n))/a)^p)
```

Rubi in Sympy [A] time = 88.1715, size = 162, normalized size = 0.84

$$\frac{x^{-m}x^{m+1}\left(fx\right)^m\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^p\operatorname{appellf_1}\left(\frac{m+1}{2n},1,-p,1+\frac{m+1}{2n},\frac{e^2x^{2n}}{d^2},-\frac{cx^{2n}}{a}\right)}{d\left(m+1\right)}-\frac{ex^{-m}x^{m+n+1}\left(fx\right)^m\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^p\operatorname{appellf_1}\left(\frac{m+n+1}{2n},1,-p,\frac{m+3n+1}{2n},\frac{e^2x^{2n}}{d^2},-\frac{cx^{2n}}{a}\right)}{d^2\left(m+n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n),x)
```

```
[Out] x**(-m)*x**(m + 1)*(f*x)**m*(1 + c*x**(2*n)/a)**(-p)*(a + c*x**(2
*n))**p*appellf1((m + 1)/(2*n), 1, -p, 1 + (m + 1)/(2*n), e**2*x*
*(2*n)/d**2, -c*x**(2*n)/a)/(d*(m + 1)) - e*x**(-m)*x**(m + n + 1
)*(f*x)**m*(1 + c*x**(2*n)/a)**(-p)*(a + c*x**(2*n))**p*appellf1(
(m + n + 1)/(2*n), 1, -p, (m + 3*n + 1)/(2*n), e**2*x**(2*n)/d**2
, -c*x**(2*n)/a)/(d**2*(m + n + 1))
```

Mathematica [A] time = 0.095658, size = 0, normalized size = 0.

$$\int \frac{(fx)^m \left(a+cx^{2n}\right)^p}{d+ex^n}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n),x]
```

```
[Out] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n), x]
```

Maple [F] time = 0.131, size = 0, normalized size = 0.

$$\int \frac{(fx)^m \left(a+cx^{2\,n}\right)^p}{d+ex^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x)
```

```
[Out] int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2\,n}+a\right)^p (fx)^m}{ex^n+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^{2n}+a\right)^{p}(fx)^{m}}{ex^{n}+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d),x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n}+a\right)^{p}(fx)^{m}}{ex^{n}+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d), x)
```

3.91 $\int \frac{(fx)^m(a+cx^{2n})^p}{(d+ex^n)^2}\,dx$

Optimal. Leaf size=302

$$\frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2n};-p,2;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(m+1)}+\frac{e^2x^{2n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+2n+1}{2n};-p,2;\frac{m+4n+1}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(m+2n+1)}-\frac{2ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+n+1}{2n};-p,2;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(m+n+1)}$$

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 2, 1 + (
1 + m)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2*(1 + m)*
(1 + (c*x^(2*n))/a)^p) - (2*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p
*AppellF1[(1 + m + n)/(2*n), -p, 2, (1 + m + 3*n)/(2*n), -((c*x^(
2*n))/a), (e^2*x^(2*n))/d^2])/(d^3*(1 + m + n)*(1 + (c*x^(2*n))/a
)^p) + (e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m
 + 2*n)/(2*n), -p, 2, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a), (e^2
*x^(2*n))/d^2])/(d^4*(1 + m + 2*n)*(1 + (c*x^(2*n))/a)^p)
```

Rubi [A] time = 0.763223, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2n};-p,2;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(m+1)}+\frac{e^2x^{2n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+2n+1}{2n};-p,2;\frac{m+4n+1}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(m+2n+1)}-\frac{2ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+n+1}{2n};-p,2;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(m+n+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^2,x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 2, 1 + (
1 + m)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2*(1 + m)*
(1 + (c*x^(2*n))/a)^p) - (2*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p
*AppellF1[(1 + m + n)/(2*n), -p, 2, (1 + m + 3*n)/(2*n), -((c*x^(
2*n))/a), (e^2*x^(2*n))/d^2])/(d^3*(1 + m + n)*(1 + (c*x^(2*n))/a
)^p) + (e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m
```

```
 + 2*n)/(2*n), -p, 2, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a), (e^2
*x^(2*n))/d^2])/(d^4*(1 + m + 2*n)*(1 + (c*x^(2*n))/a)^p)
```

Rubi in Sympy [A] time = 155.763, size = 258, normalized size = 0.85

$$\frac{x^{-m}x^{m+1}(fx)^m\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^p \operatorname{appellf}_1\left(\frac{m+1}{2n},2,-p,1+\frac{m+1}{2n},\frac{e^2x^{2n}}{d^2},-\frac{cx^{2n}}{a}\right)}{d^2(m+1)} - \frac{2ex^{-m}x^{m+n+1}(fx)^m\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^p \operatorname{appellf}_1\left(\frac{m+n+1}{2n},2,-p,\frac{m+3n+1}{2n},\frac{e^2x^{2n}}{d^2},-\frac{cx^{2n}}{a}\right)}{d^3(m+n+1)} + \frac{e^2x^{-m}x^{m+2n+1}(fx)^m\left(1+\frac{cx^{2n}}{a}\right)^{-p}\left(a+cx^{2n}\right)^p \operatorname{appellf}_1\left(\frac{m+2n+1}{2n},2,-p,\frac{m+4n+1}{2n},\frac{e^2x^{2n}}{d^2},-\frac{cx^{2n}}{a}\right)}{d^4(m+2n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n)**2,x)
```

```
[Out] x**(-m)*x**(m + 1)*(f*x)**m*(1 + c*x**(2*n)/a)**(-p)*(a + c*x**(2
*n))**p*appellf1((m + 1)/(2*n), 2, -p, 1 + (m + 1)/(2*n), e**2*x*
*(2*n)/d**2, -c*x**(2*n)/a)/(d**2*(m + 1)) - 2*e*x**(-m)*x**(m +
n + 1)*(f*x)**m*(1 + c*x**(2*n)/a)**(-p)*(a + c*x**(2*n))**p*appe
llf1((m + n + 1)/(2*n), 2, -p, (m + 3*n + 1)/(2*n), e**2*x**(2*n)
/d**2, -c*x**(2*n)/a)/(d**3*(m + n + 1)) + e**2*x**(-m)*x**(m + 2
*n + 1)*(f*x)**m*(1 + c*x**(2*n)/a)**(-p)*(a + c*x**(2*n))**p*app
ellf1((m + 2*n + 1)/(2*n), 2, -p, (m + 4*n + 1)/(2*n), e**2*x**(2
*n)/d**2, -c*x**(2*n)/a)/(d**4*(m + 2*n + 1))
```

Mathematica [A] time = 0.195435, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^2,x]
```

```
[Out] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^2, x]
```

Maple [F] time = 0.131, size = 0, normalized size = 0.

$$\int \frac{(fx)^m \left(a + cx^{2n}\right)^p}{(d + ex^n)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x)

[Out] int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n} + a\right)^p (fx)^m}{(ex^n + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^2,x, algorithm="maxima")

[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^{2n} + a\right)^p (fx)^m}{e^2x^{2n} + 2\,dex^n + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^2,x, algorithm="fricas")

[Out] integral((c*x^(2*n) + a)^p*(f*x)^m/(e^2*x^(2*n) + 2*d*e*x^n + d^2
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n} + a\right)^p (fx)^m}{(ex^n + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^2, x)
```

3.92 $\int \frac{(fx)^m\left(a+cx^{2n}\right)^p}{(d+ex^n)^3}\,dx$

Optimal. Leaf size=412

$$-\frac{e^3x^{3n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+3n+1}{2n};-p,3;\frac{m+5n+1}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^6(m+3n+1)}+\frac{3e^2x^{2n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+2n+1}{2n};-p,3;\frac{m+4n+1}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^5(m+2n+1)}-\frac{3ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+n+1}{2n};-p,3;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(m+n+1)}+\frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2n};-p,3;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(m+1)}$$

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 3, 1 + (
1 + m)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^3*(1 + m)*
(1 + (c*x^(2*n))/a)^p) - (3*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p
*AppellF1[(1 + m + n)/(2*n), -p, 3, (1 + m + 3*n)/(2*n), -((c*x^(
2*n))/a), (e^2*x^(2*n))/d^2])/(d^4*(1 + m + n)*(1 + (c*x^(2*n))/a
)^p) + (3*e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 +
 m + 2*n)/(2*n), -p, 3, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a), (e
^2*x^(2*n))/d^2])/(d^5*(1 + m + 2*n)*(1 + (c*x^(2*n))/a)^p) - (e^
3*x^(1 + 3*n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + 3*n)/(2
*n), -p, 3, (1 + m + 5*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/
d^2])/(d^6*(1 + m + 3*n)*(1 + (c*x^(2*n))/a)^p)
```

Rubi [A] time = 1.05371, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$-\frac{e^3x^{3n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+3n+1}{2n};-p,3;\frac{m+5n+1}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^6(m+3n+1)}+\frac{3e^2x^{2n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+2n+1}{2n};-p,3;\frac{m+4n+1}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^5(m+2n+1)}-\frac{3ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+n+1}{2n};-p,3;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(m+n+1)}+\frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2n};-p,3;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^3,x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 3, 1 + (
1 + m)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^3*(1 + m)*
(1 + (c*x^(2*n))/a)^p) - (3*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p
*AppellF1[(1 + m + n)/(2*n), -p, 3, (1 + m + 3*n)/(2*n), -((c*x^(
2*n))/a), (e^2*x^(2*n))/d^2])/(d^4*(1 + m + n)*(1 + (c*x^(2*n))/a
)^p) + (3*e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 +
 m + 2*n)/(2*n), -p, 3, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a), (e
^2*x^(2*n))/d^2])/(d^5*(1 + m + 2*n)*(1 + (c*x^(2*n))/a)^p) - (e^
3*x^(1 + 3*n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + 3*n)/(2
*n), -p, 3, (1 + m + 5*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/
d^2])/(d^6*(1 + m + 3*n)*(1 + (c*x^(2*n))/a)^p)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.878973, size = 0, normalized size = 0.

$$\int \frac{(fx)^m \left(a + cx^{2n}\right)^p}{(d + ex^n)^3} \, dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^3,x]
```

```
[Out] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^3, x]
```

Maple [F] time = 0.131, size = 0, normalized size = 0.

$$\int \frac{(fx)^m \left(a + cx^{2\,n}\right)^p}{(d + ex^n)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

```
[Out] int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2\,n} + a\right)^{p} \left(fx\right)^{m}}{\left(ex^{n} + d\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^{2\,n} + a\right)^{p} \left(fx\right)^{m}}{e^{3}x^{3\,n} + 3\,de^{2}x^{2\,n} + 3\,d^{2}ex^{n} + d^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + a)^p*(f*x)^m/(e^3*x^(3*n) + 3*d*e^2*x^(2*n)
 + 3*d^2*e*x^n + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n} + a\right)^{p} (fx)^{m}}{(ex^{n} + d)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^3, x)
```

3.93 $\int (b+2cx)\left(a+bx+cx^2\right)^{13} dx$

Optimal. Leaf size=16

$$\frac{1}{14}\left(a+bx+cx^2\right)^{14}$$

```
[Out] (a + b*x + c*x^2)^14/14
```

Rubi [A] time = 0.0345534, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{1}{14}\left(a+bx+cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(a + b*x + c*x^2)^13,x]
```

```
[Out] (a + b*x + c*x^2)^14/14
```

Rubi in Sympy [A] time = 3.67173, size = 12, normalized size = 0.75

$$\frac{\left(a+bx+cx^2\right)^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2*c*x+b)*(c*x**2+b*x+a)**13,x)
```

```
[Out] (a + b*x + c*x**2)**14/14
```

Mathematica [B] time = 0.273274, size = 201, normalized size = 12.56

$$\frac{1}{14}x(b+cx)\left(14a^{13}+91a^{12}x(b+cx)+364a^{11}x^2(b+cx)^2+1001a^{10}x^3(b+cx)^3+2002a^9x^4(b+cx)^4+3003a^8x^5(b+cx)^5+3432a^7x^6(b+cx)^6+3003a^6x^7(b+cx)^7+2002a^5x^8(b+cx)^8+1001a^4x^9(b+cx)^9+364a^3x^{10}(b+cx)^{10}+91a^2x^{11}(b+cx)^{11}+14ax^{12}(b+cx)^{12}+x^{13}(b+cx)^{13}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*c*x)*(a + b*x + c*x^2)^13,x]
```

```
[Out] (x*(b + c*x)*(14*a^13 + 91*a^12*x*(b + c*x) + 364*a^11*x^2*(b + c
*x)^2 + 1001*a^10*x^3*(b + c*x)^3 + 2002*a^9*x^4*(b + c*x)^4 + 30
03*a^8*x^5*(b + c*x)^5 + 3432*a^7*x^6*(b + c*x)^6 + 3003*a^6*x^7*
(b + c*x)^7 + 2002*a^5*x^8*(b + c*x)^8 + 1001*a^4*x^9*(b + c*x)^9
 + 364*a^3*x^10*(b + c*x)^10 + 91*a^2*x^11*(b + c*x)^11 + 14*a*x^
12*(b + c*x)^12 + x^13*(b + c*x)^13))/14
```

Maple [B] time = 0.007, size = 46548, normalized size = 2909.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*c*x+b)*(c*x^2+b*x+a)^13,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 0.767148, size = 19, normalized size = 1.19

$$\frac{1}{14}\left(cx^2 + bx + a\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)^13*(2*c*x + b),x, algorithm="maxima")
```

```
[Out] 1/14*(c*x^2 + b*x + a)^14
```

Fricas [A] time = 0.264036, size = 1, normalized size = 0.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)^13*(2*c*x + b),x, algorithm="fricas")
```

```
[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 + x^26*c^13*a +
 26*x^25*c^11*b^3 + 13*x^25*c^12*b*a + 143/2*x^24*c^10*b^4 + 78*x
```

```
^24*c^11*b^2*a + 13/2*x^24*c^12*a^2 + 143*x^23*c^9*b^5 + 286*x^23
*c^10*b^3*a + 78*x^23*c^11*b*a^2 + 429/2*x^22*c^8*b^6 + 715*x^22*
c^9*b^4*a + 429*x^22*c^10*b^2*a^2 + 26*x^22*c^11*a^3 + 1716/7*x^2
1*c^7*b^7 + 1287*x^21*c^8*b^5*a + 1430*x^21*c^9*b^3*a^2 + 286*x^2
1*c^10*b*a^3 + 429/2*x^20*c^6*b^8 + 1716*x^20*c^7*b^6*a + 6435/2*
x^20*c^8*b^4*a^2 + 1430*x^20*c^9*b^2*a^3 + 143/2*x^20*c^10*a^4 +
143*x^19*c^5*b^9 + 1716*x^19*c^6*b^7*a + 5148*x^19*c^7*b^5*a^2 +
4290*x^19*c^8*b^3*a^3 + 715*x^19*c^9*b*a^4 + 143/2*x^18*c^4*b^10
+ 1287*x^18*c^5*b^8*a + 6006*x^18*c^6*b^6*a^2 + 8580*x^18*c^7*b^4
*a^3 + 6435/2*x^18*c^8*b^2*a^4 + 143*x^18*c^9*a^5 + 26*x^17*c^3*b
^11 + 715*x^17*c^4*b^9*a + 5148*x^17*c^5*b^7*a^2 + 12012*x^17*c^6
*b^5*a^3 + 8580*x^17*c^7*b^3*a^4 + 1287*x^17*c^8*b*a^5 + 13/2*x^1
6*c^2*b^12 + 286*x^16*c^3*b^10*a + 6435/2*x^16*c^4*b^8*a^2 + 1201
2*x^16*c^5*b^6*a^3 + 15015*x^16*c^6*b^4*a^4 + 5148*x^16*c^7*b^2*a
^5 + 429/2*x^16*c^8*a^6 + x^15*c*b^13 + 78*x^15*c^2*b^11*a + 1430
*x^15*c^3*b^9*a^2 + 8580*x^15*c^4*b^7*a^3 + 18018*x^15*c^5*b^5*a^
4 + 12012*x^15*c^6*b^3*a^5 + 1716*x^15*c^7*b*a^6 + 1/14*x^14*b^14
 + 13*x^14*c*b^12*a + 429*x^14*c^2*b^10*a^2 + 4290*x^14*c^3*b^8*a
^3 + 15015*x^14*c^4*b^6*a^4 + 18018*x^14*c^5*b^4*a^5 + 6006*x^14*
c^6*b^2*a^6 + 1716/7*x^14*c^7*a^7 + x^13*b^13*a + 78*x^13*c*b^11*
a^2 + 1430*x^13*c^2*b^9*a^3 + 8580*x^13*c^3*b^7*a^4 + 18018*x^13*
c^4*b^5*a^5 + 12012*x^13*c^5*b^3*a^6 + 1716*x^13*c^6*b*a^7 + 13/2
*x^12*b^12*a^2 + 286*x^12*c*b^10*a^3 + 6435/2*x^12*c^2*b^8*a^4 +
12012*x^12*c^3*b^6*a^5 + 15015*x^12*c^4*b^4*a^6 + 5148*x^12*c^5*b
^2*a^7 + 429/2*x^12*c^6*a^8 + 26*x^11*b^11*a^3 + 715*x^11*c*b^9*a
^4 + 5148*x^11*c^2*b^7*a^5 + 12012*x^11*c^3*b^5*a^6 + 8580*x^11*c
^4*b^3*a^7 + 1287*x^11*c^5*b*a^8 + 143/2*x^10*b^10*a^4 + 1287*x^1
0*c*b^8*a^5 + 6006*x^10*c^2*b^6*a^6 + 8580*x^10*c^3*b^4*a^7 + 643
5/2*x^10*c^4*b^2*a^8 + 143*x^10*c^5*a^9 + 143*x^9*b^9*a^5 + 1716*
x^9*c*b^7*a^6 + 5148*x^9*c^2*b^5*a^7 + 4290*x^9*c^3*b^3*a^8 + 715
*x^9*c^4*b*a^9 + 429/2*x^8*b^8*a^6 + 1716*x^8*c*b^6*a^7 + 6435/2*
x^8*c^2*b^4*a^8 + 1430*x^8*c^3*b^2*a^9 + 143/2*x^8*c^4*a^10 + 171
6/7*x^7*b^7*a^7 + 1287*x^7*c*b^5*a^8 + 1430*x^7*c^2*b^3*a^9 + 286
*x^7*c^3*b*a^10 + 429/2*x^6*b^6*a^8 + 715*x^6*c*b^4*a^9 + 429*x^6
*c^2*b^2*a^10 + 26*x^6*c^3*a^11 + 143*x^5*b^5*a^9 + 286*x^5*c*b^3
*a^10 + 78*x^5*c^2*b*a^11 + 143/2*x^4*b^4*a^10 + 78*x^4*c*b^2*a^1
1 + 13/2*x^4*c^2*a^12 + 26*x^3*b^3*a^11 + 13*x^3*c*b*a^12 + 13/2*
x^2*b^2*a^12 + x^2*c*a^13 + x*b*a^13
```

Sympy [A] time = 0.966792, size = 1326, normalized size = 82.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x+b)*(c*x**2+b*x+a)**13,x)
```

```
[Out] a**13*b*x + b*c**13*x**27 + c**14*x**28/14 + x**26*(a*c**13 + 13*
b**2*c**12/2) + x**25*(13*a*b*c**12 + 26*b**3*c**11) + x**24*(13*
```

```
a**2*c**12/2 + 78*a*b**2*c**11 + 143*b**4*c**10/2) + x**23*(78*a*
*2*b*c**11 + 286*a*b**3*c**10 + 143*b**5*c**9) + x**22*(26*a**3*c
**11 + 429*a**2*b**2*c**10 + 715*a*b**4*c**9 + 429*b**6*c**8/2) +
 x**21*(286*a**3*b*c**10 + 1430*a**2*b**3*c**9 + 1287*a*b**5*c**8
 + 1716*b**7*c**7/7) + x**20*(143*a**4*c**10/2 + 1430*a**3*b**2*c
**9 + 6435*a**2*b**4*c**8/2 + 1716*a*b**6*c**7 + 429*b**8*c**6/2)
 + x**19*(715*a**4*b*c**9 + 4290*a**3*b**3*c**8 + 5148*a**2*b**5*
c**7 + 1716*a*b**7*c**6 + 143*b**9*c**5) + x**18*(143*a**5*c**9 +
 6435*a**4*b**2*c**8/2 + 8580*a**3*b**4*c**7 + 6006*a**2*b**6*c**
6 + 1287*a*b**8*c**5 + 143*b**10*c**4/2) + x**17*(1287*a**5*b*c**
8 + 8580*a**4*b**3*c**7 + 12012*a**3*b**5*c**6 + 5148*a**2*b**7*c
**5 + 715*a*b**9*c**4 + 26*b**11*c**3) + x**16*(429*a**6*c**8/2 +
 5148*a**5*b**2*c**7 + 15015*a**4*b**4*c**6 + 12012*a**3*b**6*c**
5 + 6435*a**2*b**8*c**4/2 + 286*a*b**10*c**3 + 13*b**12*c**2/2) +
 x**15*(1716*a**6*b*c**7 + 12012*a**5*b**3*c**6 + 18018*a**4*b**5
*c**5 + 8580*a**3*b**7*c**4 + 1430*a**2*b**9*c**3 + 78*a*b**11*c*
*2 + b**13*c) + x**14*(1716*a**7*c**7/7 + 6006*a**6*b**2*c**6 + 1
8018*a**5*b**4*c**5 + 15015*a**4*b**6*c**4 + 4290*a**3*b**8*c**3
+ 429*a**2*b**10*c**2 + 13*a*b**12*c + b**14/14) + x**13*(1716*a*
*7*b*c**6 + 12012*a**6*b**3*c**5 + 18018*a**5*b**5*c**4 + 8580*a*
*4*b**7*c**3 + 1430*a**3*b**9*c**2 + 78*a**2*b**11*c + a*b**13) +
 x**12*(429*a**8*c**6/2 + 5148*a**7*b**2*c**5 + 15015*a**6*b**4*c
**4 + 12012*a**5*b**6*c**3 + 6435*a**4*b**8*c**2/2 + 286*a**3*b**
10*c + 13*a**2*b**12/2) + x**11*(1287*a**8*b*c**5 + 8580*a**7*b**
3*c**4 + 12012*a**6*b**5*c**3 + 5148*a**5*b**7*c**2 + 715*a**4*b*
*9*c + 26*a**3*b**11) + x**10*(143*a**9*c**5 + 6435*a**8*b**2*c**
4/2 + 8580*a**7*b**4*c**3 + 6006*a**6*b**6*c**2 + 1287*a**5*b**8*
c + 143*a**4*b**10/2) + x**9*(715*a**9*b*c**4 + 4290*a**8*b**3*c*
*3 + 5148*a**7*b**5*c**2 + 1716*a**6*b**7*c + 143*a**5*b**9) + x*
*8*(143*a**10*c**4/2 + 1430*a**9*b**2*c**3 + 6435*a**8*b**4*c**2/
2 + 1716*a**7*b**6*c + 429*a**6*b**8/2) + x**7*(286*a**10*b*c**3
+ 1430*a**9*b**3*c**2 + 1287*a**8*b**5*c + 1716*a**7*b**7/7) + x*
*6*(26*a**11*c**3 + 429*a**10*b**2*c**2 + 715*a**9*b**4*c + 429*a
**8*b**6/2) + x**5*(78*a**11*b*c**2 + 286*a**10*b**3*c + 143*a**9
*b**5) + x**4*(13*a**12*c**2/2 + 78*a**11*b**2*c + 143*a**10*b**4
/2) + x**3*(13*a**12*b*c + 26*a**11*b**3) + x**2*(a**13*c + 13*a*
*12*b**2/2)
```

GIAC/XCAS [A] time = 0.266729, size = 1, normalized size = 0.06

$$\textit{Done}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)^13*(2*c*x + b),x, algorithm="giac")
```

```
[Out] Done
```

3.94 $\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{13}\,dx$

Optimal. Leaf size=18

$$\frac{1}{28}\left(a+bx^2+cx^4\right)^{14}$$

```
[Out] (a + b*x^2 + c*x^4)^14/28
```

Rubi [A] time = 0.0659763, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{1}{28}\left(a+bx^2+cx^4\right)^{14}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^13,x]
```

```
[Out] (a + b*x^2 + c*x^4)^14/28
```

Rubi in Sympy [A] time = 4.94578, size = 14, normalized size = 0.78

$$\frac{\left(a+bx^2+cx^4\right)^{14}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2+a)**13,x)
```

```
[Out] (a + b*x**2 + c*x**4)**14/28
```

Mathematica [B] time = 0.286706, size = 233, normalized size = 12.94

$$\frac{1}{28}x^2\left(b+cx^2\right)\left(14a^{13}+91a^{12}x^2\left(b+cx^2\right)+364a^{11}x^4\left(b+cx^2\right)^2+1001a^{10}x^6\left(b+cx^2\right)^3\right.$$
$$+2002a^9x^8\left(b+cx^2\right)^4+3003a^8x^{10}\left(b+cx^2\right)^5+3432a^7x^{12}\left(b+cx^2\right)^6$$
$$+3003a^6x^{14}\left(b+cx^2\right)^7+2002a^5x^{16}\left(b+cx^2\right)^8+1001a^4x^{18}\left(b+cx^2\right)^9$$
$$\left.+364a^3x^{20}\left(b+cx^2\right)^{10}+91a^2x^{22}\left(b+cx^2\right)^{11}+14ax^{24}\left(b+cx^2\right)^{12}+x^{26}\left(b+cx^2\right)^{13}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^13,x]
```

```
[Out] (x^2*(b + c*x^2)*(14*a^13 + 91*a^12*x^2*(b + c*x^2) + 364*a^11*x^
4*(b + c*x^2)^2 + 1001*a^10*x^6*(b + c*x^2)^3 + 2002*a^9*x^8*(b +
 c*x^2)^4 + 3003*a^8*x^10*(b + c*x^2)^5 + 3432*a^7*x^12*(b + c*x^
2)^6 + 3003*a^6*x^14*(b + c*x^2)^7 + 2002*a^5*x^16*(b + c*x^2)^8
+ 1001*a^4*x^18*(b + c*x^2)^9 + 364*a^3*x^20*(b + c*x^2)^10 + 91*
a^2*x^22*(b + c*x^2)^11 + 14*a*x^24*(b + c*x^2)^12 + x^26*(b + c*
x^2)^13))/28
```

Maple [B]   time = 0.004, size = 46552, normalized size = 2586.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x)
```

```
[Out] result too large to display
```

Maxima [A]   time = 0.793231, size = 1674, normalized size = 93.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^13*(2*c*x^2 + b)*x,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 1/4*(13*b^2*c^12 + 2*a*c^13)*x
^52 + 13/2*(2*b^3*c^11 + a*b*c^12)*x^50 + 13/4*(11*b^4*c^10 + 12*
a*b^2*c^11 + a^2*c^12)*x^48 + 13/2*(11*b^5*c^9 + 22*a*b^3*c^10 +
6*a^2*b*c^11)*x^46 + 13/4*(33*b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^
2*c^10 + 4*a^3*c^11)*x^44 + 143/14*(12*b^7*c^7 + 63*a*b^5*c^8 + 7
0*a^2*b^3*c^9 + 14*a^3*b*c^10)*x^42 + 143/4*(3*b^8*c^6 + 24*a*b^6
*c^7 + 45*a^2*b^4*c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^40 + 143/2*(
b^9*c^5 + 12*a*b^7*c^6 + 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*
b*c^9)*x^38 + 143/4*(b^10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 1
20*a^3*b^4*c^7 + 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^36 + 13/2*(2*b^11*
c^3 + 55*a*b^9*c^4 + 396*a^2*b^7*c^5 + 924*a^3*b^5*c^6 + 660*a^4*
b^3*c^7 + 99*a^5*b*c^8)*x^34 + 13/4*(b^12*c^2 + 44*a*b^10*c^3 + 4
95*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*b^4*c^6 + 792*a^5*b^
```

```
2*c^7 + 33*a^6*c^8)*x^32 + 1/2*(b^13*c + 78*a*b^11*c^2 + 1430*a^2
*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^30 + 1/28*(b^14 + 182*a*b^12*c + 6006*a^2*
b^10*c^2 + 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^
4*c^5 + 84084*a^6*b^2*c^6 + 3432*a^7*c^7)*x^28 + 1/2*(a*b^13 + 78
*a^2*b^11*c + 1430*a^3*b^9*c^2 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5
*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^6)*x^26 + 13/4*(a^2*b^12
+ 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c^3 + 2310*a^6*b
^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^24 + 13/2*(2*a^3*b^11 +
55*a^4*b^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^
4 + 99*a^8*b*c^5)*x^22 + 143/4*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*
b^6*c^2 + 120*a^7*b^4*c^3 + 45*a^8*b^2*c^4 + 2*a^9*c^5)*x^20 + 14
3/2*(a^5*b^9 + 12*a^6*b^7*c + 36*a^7*b^5*c^2 + 30*a^8*b^3*c^3 + 5
*a^9*b*c^4)*x^18 + 143/4*(3*a^6*b^8 + 24*a^7*b^6*c + 45*a^8*b^4*c
^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^16 + 1/2*a^13*b*x^2 + 143/14*(1
2*a^7*b^7 + 63*a^8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c^3)*x^14 +
 13/4*(33*a^8*b^6 + 110*a^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)
*x^12 + 13/2*(11*a^9*b^5 + 22*a^10*b^3*c + 6*a^11*b*c^2)*x^10 + 1
3/4*(11*a^10*b^4 + 12*a^11*b^2*c + a^12*c^2)*x^8 + 13/2*(2*a^11*b
^3 + a^12*b*c)*x^6 + 1/4*(13*a^12*b^2 + 2*a^13*c)*x^4
```

Fricas [A] time = 0.277022, size = 1, normalized size = 0.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^13*(2*c*x^2 + b)*x,x, algorithm="fricas")
```

```
[Out] 1/28*x^56*c^14 + 1/2*x^54*c^13*b + 13/4*x^52*c^12*b^2 + 1/2*x^52*
c^13*a + 13*x^50*c^11*b^3 + 13/2*x^50*c^12*b*a + 143/4*x^48*c^10*
b^4 + 39*x^48*c^11*b^2*a + 13/4*x^48*c^12*a^2 + 143/2*x^46*c^9*b^
5 + 143*x^46*c^10*b^3*a + 39*x^46*c^11*b*a^2 + 429/4*x^44*c^8*b^6
 + 715/2*x^44*c^9*b^4*a + 429/2*x^44*c^10*b^2*a^2 + 13*x^44*c^11*
a^3 + 858/7*x^42*c^7*b^7 + 1287/2*x^42*c^8*b^5*a + 715*x^42*c^9*b
^3*a^2 + 143*x^42*c^10*b*a^3 + 429/4*x^40*c^6*b^8 + 858*x^40*c^7*
b^6*a + 6435/4*x^40*c^8*b^4*a^2 + 715*x^40*c^9*b^2*a^3 + 143/4*x^
40*c^10*a^4 + 143/2*x^38*c^5*b^9 + 858*x^38*c^6*b^7*a + 2574*x^38
*c^7*b^5*a^2 + 2145*x^38*c^8*b^3*a^3 + 715/2*x^38*c^9*b*a^4 + 143
/4*x^36*c^4*b^10 + 1287/2*x^36*c^5*b^8*a + 3003*x^36*c^6*b^6*a^2
+ 4290*x^36*c^7*b^4*a^3 + 6435/4*x^36*c^8*b^2*a^4 + 143/2*x^36*c^
9*a^5 + 13*x^34*c^3*b^11 + 715/2*x^34*c^4*b^9*a + 2574*x^34*c^5*b
^7*a^2 + 6006*x^34*c^6*b^5*a^3 + 4290*x^34*c^7*b^3*a^4 + 1287/2*x
^34*c^8*b*a^5 + 13/4*x^32*c^2*b^12 + 143*x^32*c^3*b^10*a + 6435/4
*x^32*c^4*b^8*a^2 + 6006*x^32*c^5*b^6*a^3 + 15015/2*x^32*c^6*b^4*
a^4 + 2574*x^32*c^7*b^2*a^5 + 429/4*x^32*c^8*a^6 + 1/2*x^30*c*b^1
3 + 39*x^30*c^2*b^11*a + 715*x^30*c^3*b^9*a^2 + 4290*x^30*c^4*b^7
*a^3 + 9009*x^30*c^5*b^5*a^4 + 6006*x^30*c^6*b^3*a^5 + 858*x^30*c
^7*b*a^6 + 1/28*x^28*b^14 + 13/2*x^28*c*b^12*a + 429/2*x^28*c^2*b
^10*a^2 + 2145*x^28*c^3*b^8*a^3 + 15015/2*x^28*c^4*b^6*a^4 + 9009
```

```
*x^28*c^5*b^4*a^5 + 3003*x^28*c^6*b^2*a^6 + 858/7*x^28*c^7*a^7 +
1/2*x^26*b^13*a + 39*x^26*c*b^11*a^2 + 715*x^26*c^2*b^9*a^3 + 429
0*x^26*c^3*b^7*a^4 + 9009*x^26*c^4*b^5*a^5 + 6006*x^26*c^5*b^3*a^
6 + 858*x^26*c^6*b*a^7 + 13/4*x^24*b^12*a^2 + 143*x^24*c*b^10*a^3
 + 6435/4*x^24*c^2*b^8*a^4 + 6006*x^24*c^3*b^6*a^5 + 15015/2*x^24
*c^4*b^4*a^6 + 2574*x^24*c^5*b^2*a^7 + 429/4*x^24*c^6*a^8 + 13*x^
22*b^11*a^3 + 715/2*x^22*c*b^9*a^4 + 2574*x^22*c^2*b^7*a^5 + 6006
*x^22*c^3*b^5*a^6 + 4290*x^22*c^4*b^3*a^7 + 1287/2*x^22*c^5*b*a^8
 + 143/4*x^20*b^10*a^4 + 1287/2*x^20*c*b^8*a^5 + 3003*x^20*c^2*b^
6*a^6 + 4290*x^20*c^3*b^4*a^7 + 6435/4*x^20*c^4*b^2*a^8 + 143/2*x
^20*c^5*a^9 + 143/2*x^18*b^9*a^5 + 858*x^18*c*b^7*a^6 + 2574*x^18
*c^2*b^5*a^7 + 2145*x^18*c^3*b^3*a^8 + 715/2*x^18*c^4*b*a^9 + 429
/4*x^16*b^8*a^6 + 858*x^16*c*b^6*a^7 + 6435/4*x^16*c^2*b^4*a^8 +
715*x^16*c^3*b^2*a^9 + 143/4*x^16*c^4*a^10 + 858/7*x^14*b^7*a^7 +
 1287/2*x^14*c*b^5*a^8 + 715*x^14*c^2*b^3*a^9 + 143*x^14*c^3*b*a^
10 + 429/4*x^12*b^6*a^8 + 715/2*x^12*c*b^4*a^9 + 429/2*x^12*c^2*b
^2*a^10 + 13*x^12*c^3*a^11 + 143/2*x^10*b^5*a^9 + 143*x^10*c*b^3*
a^10 + 39*x^10*c^2*b*a^11 + 143/4*x^8*b^4*a^10 + 39*x^8*c*b^2*a^1
1 + 13/4*x^8*c^2*a^12 + 13*x^6*b^3*a^11 + 13/2*x^6*c*b*a^12 + 13/
4*x^4*b^2*a^12 + 1/2*x^4*c*a^13 + 1/2*x^2*b*a^13
```

Sympy [A] time = 0.916498, size = 1384, normalized size = 76.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2+a)**13,x)
```

```
[Out] a**13*b*x**2/2 + b*c**13*x**54/2 + c**14*x**56/28 + x**52*(a*c**1
3/2 + 13*b**2*c**12/4) + x**50*(13*a*b*c**12/2 + 13*b**3*c**11) +
 x**48*(13*a**2*c**12/4 + 39*a*b**2*c**11 + 143*b**4*c**10/4) + x
**46*(39*a**2*b*c**11 + 143*a*b**3*c**10 + 143*b**5*c**9/2) + x**
44*(13*a**3*c**11 + 429*a**2*b**2*c**10/2 + 715*a*b**4*c**9/2 + 4
29*b**6*c**8/4) + x**42*(143*a**3*b*c**10 + 715*a**2*b**3*c**9 +
1287*a*b**5*c**8/2 + 858*b**7*c**7/7) + x**40*(143*a**4*c**10/4 +
 715*a**3*b**2*c**9 + 6435*a**2*b**4*c**8/4 + 858*a*b**6*c**7 + 4
29*b**8*c**6/4) + x**38*(715*a**4*b*c**9/2 + 2145*a**3*b**3*c**8
+ 2574*a**2*b**5*c**7 + 858*a*b**7*c**6 + 143*b**9*c**5/2) + x**3
6*(143*a**5*c**9/2 + 6435*a**4*b**2*c**8/4 + 4290*a**3*b**4*c**7
+ 3003*a**2*b**6*c**6 + 1287*a*b**8*c**5/2 + 143*b**10*c**4/4) +
x**34*(1287*a**5*b*c**8/2 + 4290*a**4*b**3*c**7 + 6006*a**3*b**5*
c**6 + 2574*a**2*b**7*c**5 + 715*a*b**9*c**4/2 + 13*b**11*c**3) +
 x**32*(429*a**6*c**8/4 + 2574*a**5*b**2*c**7 + 15015*a**4*b**4*c
**6/2 + 6006*a**3*b**6*c**5 + 6435*a**2*b**8*c**4/4 + 143*a*b**10
*c**3 + 13*b**12*c**2/4) + x**30*(858*a**6*b*c**7 + 6006*a**5*b**
3*c**6 + 9009*a**4*b**5*c**5 + 4290*a**3*b**7*c**4 + 715*a**2*b**
9*c**3 + 39*a*b**11*c**2 + b**13*c/2) + x**28*(858*a**7*c**7/7 +
3003*a**6*b**2*c**6 + 9009*a**5*b**4*c**5 + 15015*a**4*b**6*c**4/
```

```
2 + 2145*a**3*b**8*c**3 + 429*a**2*b**10*c**2/2 + 13*a*b**12*c/2
+ b**14/28) + x**26*(858*a**7*b*c**6 + 6006*a**6*b**3*c**5 + 9009
*a**5*b**5*c**4 + 4290*a**4*b**7*c**3 + 715*a**3*b**9*c**2 + 39*a
**2*b**11*c + a*b**13/2) + x**24*(429*a**8*c**6/4 + 2574*a**7*b**
2*c**5 + 15015*a**6*b**4*c**4/2 + 6006*a**5*b**6*c**3 + 6435*a**4
*b**8*c**2/4 + 143*a**3*b**10*c + 13*a**2*b**12/4) + x**22*(1287*
a**8*b*c**5/2 + 4290*a**7*b**3*c**4 + 6006*a**6*b**5*c**3 + 2574*
a**5*b**7*c**2 + 715*a**4*b**9*c/2 + 13*a**3*b**11) + x**20*(143*
a**9*c**5/2 + 6435*a**8*b**2*c**4/4 + 4290*a**7*b**4*c**3 + 3003*
a**6*b**6*c**2 + 1287*a**5*b**8*c/2 + 143*a**4*b**10/4) + x**18*(
715*a**9*b*c**4/2 + 2145*a**8*b**3*c**3 + 2574*a**7*b**5*c**2 + 8
58*a**6*b**7*c + 143*a**5*b**9/2) + x**16*(143*a**10*c**4/4 + 715
*a**9*b**2*c**3 + 6435*a**8*b**4*c**2/4 + 858*a**7*b**6*c + 429*a
**6*b**8/4) + x**14*(143*a**10*b*c**3 + 715*a**9*b**3*c**2 + 1287
*a**8*b**5*c/2 + 858*a**7*b**7/7) + x**12*(13*a**11*c**3 + 429*a*
*10*b**2*c**2/2 + 715*a**9*b**4*c/2 + 429*a**8*b**6/4) + x**10*(3
9*a**11*b*c**2 + 143*a**10*b**3*c + 143*a**9*b**5/2) + x**8*(13*a
**12*c**2/4 + 39*a**11*b**2*c + 143*a**10*b**4/4) + x**6*(13*a**1
2*b*c/2 + 13*a**11*b**3) + x**4*(a**13*c/2 + 13*a**12*b**2/4)
```

GIAC/XCAS [A] time = 0.275046, size = 1, normalized size = 0.06

$$Done$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^13*(2*c*x^2 + b)*x,x, algorithm="giac")
```

```
[Out] Done
```

3.95 $\int x^2 \left(b+2cx^3\right) \left(a+bx^3+cx^6\right)^{13} \, dx$

Optimal. Leaf size=18

$$\frac{1}{42} \left(a+bx^3+cx^6\right)^{14}$$

```
[Out] (a + b*x^3 + c*x^6)^14/42
```

Rubi [A] time = 0.0701099, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$

$$\frac{1}{42} \left(a+bx^3+cx^6\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^13,x]
```

```
[Out] (a + b*x^3 + c*x^6)^14/42
```

Rubi in Sympy [A] time = 4.9908, size = 14, normalized size = 0.78

$$\frac{\left(a+bx^3+cx^6\right)^{14}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3+a)**13,x)
```

```
[Out] (a + b*x**3 + c*x**6)**14/42
```

Mathematica [B] time = 0.287124, size = 233, normalized size = 12.94

$$\frac{1}{42}x^3 \left(b+cx^3\right) \left(14a^{13}+91a^{12}x^3 \left(b+cx^3\right)+364a^{11}x^6 \left(b+cx^3\right)^2+1001a^{10}x^9 \left(b+cx^3\right)^3 \right.$$
$$+2002a^9x^{12} \left(b+cx^3\right)^4+3003a^8x^{15} \left(b+cx^3\right)^5+3432a^7x^{18} \left(b+cx^3\right)^6$$
$$+3003a^6x^{21} \left(b+cx^3\right)^7+2002a^5x^{24} \left(b+cx^3\right)^8+1001a^4x^{27} \left(b+cx^3\right)^9$$
$$\left.+364a^3x^{30} \left(b+cx^3\right)^{10}+91a^2x^{33} \left(b+cx^3\right)^{11}+14ax^{36} \left(b+cx^3\right)^{12}+x^{39} \left(b+cx^3\right)^{13}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^13,x]
```

```
[Out] (x^3*(b + c*x^3)*(14*a^13 + 91*a^12*x^3*(b + c*x^3) + 364*a^11*x^
6*(b + c*x^3)^2 + 1001*a^10*x^9*(b + c*x^3)^3 + 2002*a^9*x^12*(b
+ c*x^3)^4 + 3003*a^8*x^15*(b + c*x^3)^5 + 3432*a^7*x^18*(b + c*x
^3)^6 + 3003*a^6*x^21*(b + c*x^3)^7 + 2002*a^5*x^24*(b + c*x^3)^8
 + 1001*a^4*x^27*(b + c*x^3)^9 + 364*a^3*x^30*(b + c*x^3)^10 + 91
*a^2*x^33*(b + c*x^3)^11 + 14*a*x^36*(b + c*x^3)^12 + x^39*(b + c
*x^3)^13))/42
```

Maple [B] time = 0.004, size = 46552, normalized size = 2586.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 0.779615, size = 1674, normalized size = 93.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^13*(2*c*x^3 + b)*x^2,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 1/6*(13*b^2*c^12 + 2*a*c^13)*x
^78 + 13/3*(2*b^3*c^11 + a*b*c^12)*x^75 + 13/6*(11*b^4*c^10 + 12*
a*b^2*c^11 + a^2*c^12)*x^72 + 13/3*(11*b^5*c^9 + 22*a*b^3*c^10 +
6*a^2*b*c^11)*x^69 + 13/6*(33*b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^
2*c^10 + 4*a^3*c^11)*x^66 + 143/21*(12*b^7*c^7 + 63*a*b^5*c^8 + 7
0*a^2*b^3*c^9 + 14*a^3*b*c^10)*x^63 + 143/6*(3*b^8*c^6 + 24*a*b^6
*c^7 + 45*a^2*b^4*c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^60 + 143/3*(
b^9*c^5 + 12*a*b^7*c^6 + 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*
b*c^9)*x^57 + 143/6*(b^10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 1
20*a^3*b^4*c^7 + 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^54 + 13/3*(2*b^11*
c^3 + 55*a*b^9*c^4 + 396*a^2*b^7*c^5 + 924*a^3*b^5*c^6 + 660*a^4*
b^3*c^7 + 99*a^5*b*c^8)*x^51 + 13/6*(b^12*c^2 + 44*a*b^10*c^3 + 4
95*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*b^4*c^6 + 792*a^5*b^
```

```
2*c^7 + 33*a^6*c^8)*x^48 + 1/3*(b^13*c + 78*a*b^11*c^2 + 1430*a^2
*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^45 + 1/42*(b^14 + 182*a*b^12*c + 6006*a^2*
b^10*c^2 + 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^
4*c^5 + 84084*a^6*b^2*c^6 + 3432*a^7*c^7)*x^42 + 1/3*(a*b^13 + 78
*a^2*b^11*c + 1430*a^3*b^9*c^2 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5
*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^6)*x^39 + 13/6*(a^2*b^12
+ 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c^3 + 2310*a^6*b
^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^36 + 13/3*(2*a^3*b^11 +
55*a^4*b^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^
4 + 99*a^8*b*c^5)*x^33 + 143/6*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*
b^6*c^2 + 120*a^7*b^4*c^3 + 45*a^8*b^2*c^4 + 2*a^9*c^5)*x^30 + 14
3/3*(a^5*b^9 + 12*a^6*b^7*c + 36*a^7*b^5*c^2 + 30*a^8*b^3*c^3 + 5
*a^9*b*c^4)*x^27 + 143/6*(3*a^6*b^8 + 24*a^7*b^6*c + 45*a^8*b^4*c
^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^24 + 143/21*(12*a^7*b^7 + 63*a^
8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c^3)*x^21 + 13/6*(33*a^8*b^6
 + 110*a^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)*x^18 + 1/3*a^13*
b*x^3 + 13/3*(11*a^9*b^5 + 22*a^10*b^3*c + 6*a^11*b*c^2)*x^15 + 1
3/6*(11*a^10*b^4 + 12*a^11*b^2*c + a^12*c^2)*x^12 + 13/3*(2*a^11*
b^3 + a^12*b*c)*x^9 + 1/6*(13*a^12*b^2 + 2*a^13*c)*x^6
```

Fricas [A] time = 0.265948, size = 1, normalized size = 0.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6 + b*x^3 + a)^13*(2*c*x^3 + b)*x^2,x, algorithm="fricas")
```

```
[Out] 1/42*x^84*c^14 + 1/3*x^81*c^13*b + 13/6*x^78*c^12*b^2 + 1/3*x^78*
c^13*a + 26/3*x^75*c^11*b^3 + 13/3*x^75*c^12*b*a + 143/6*x^72*c^1
0*b^4 + 26*x^72*c^11*b^2*a + 13/6*x^72*c^12*a^2 + 143/3*x^69*c^9*
b^5 + 286/3*x^69*c^10*b^3*a + 26*x^69*c^11*b*a^2 + 143/2*x^66*c^8
*b^6 + 715/3*x^66*c^9*b^4*a + 143*x^66*c^10*b^2*a^2 + 26/3*x^66*c
^11*a^3 + 572/7*x^63*c^7*b^7 + 429*x^63*c^8*b^5*a + 1430/3*x^63*c
^9*b^3*a^2 + 286/3*x^63*c^10*b*a^3 + 143/2*x^60*c^6*b^8 + 572*x^6
0*c^7*b^6*a + 2145/2*x^60*c^8*b^4*a^2 + 1430/3*x^60*c^9*b^2*a^3 +
 143/6*x^60*c^10*a^4 + 143/3*x^57*c^5*b^9 + 572*x^57*c^6*b^7*a +
1716*x^57*c^7*b^5*a^2 + 1430*x^57*c^8*b^3*a^3 + 715/3*x^57*c^9*b*
a^4 + 143/6*x^54*c^4*b^10 + 429*x^54*c^5*b^8*a + 2002*x^54*c^6*b^
6*a^2 + 2860*x^54*c^7*b^4*a^3 + 2145/2*x^54*c^8*b^2*a^4 + 143/3*x
^54*c^9*a^5 + 26/3*x^51*c^3*b^11 + 715/3*x^51*c^4*b^9*a + 1716*x^
51*c^5*b^7*a^2 + 4004*x^51*c^6*b^5*a^3 + 2860*x^51*c^7*b^3*a^4 +
429*x^51*c^8*b*a^5 + 13/6*x^48*c^2*b^12 + 286/3*x^48*c^3*b^10*a +
 2145/2*x^48*c^4*b^8*a^2 + 4004*x^48*c^5*b^6*a^3 + 5005*x^48*c^6*
b^4*a^4 + 1716*x^48*c^7*b^2*a^5 + 143/2*x^48*c^8*a^6 + 1/3*x^45*c
*b^13 + 26*x^45*c^2*b^11*a + 1430/3*x^45*c^3*b^9*a^2 + 2860*x^45*
c^4*b^7*a^3 + 6006*x^45*c^5*b^5*a^4 + 4004*x^45*c^6*b^3*a^5 + 572
*x^45*c^7*b*a^6 + 1/42*x^42*b^14 + 13/3*x^42*c*b^12*a + 143*x^42*
c^2*b^10*a^2 + 1430*x^42*c^3*b^8*a^3 + 5005*x^42*c^4*b^6*a^4 + 60
```

```
06*x^42*c^5*b^4*a^5 + 2002*x^42*c^6*b^2*a^6 + 572/7*x^42*c^7*a^7
+ 1/3*x^39*b^13*a + 26*x^39*c*b^11*a^2 + 1430/3*x^39*c^2*b^9*a^3
+ 2860*x^39*c^3*b^7*a^4 + 6006*x^39*c^4*b^5*a^5 + 4004*x^39*c^5*b
^3*a^6 + 572*x^39*c^6*b*a^7 + 13/6*x^36*b^12*a^2 + 286/3*x^36*c*b
^10*a^3 + 2145/2*x^36*c^2*b^8*a^4 + 4004*x^36*c^3*b^6*a^5 + 5005*
x^36*c^4*b^4*a^6 + 1716*x^36*c^5*b^2*a^7 + 143/2*x^36*c^6*a^8 + 2
6/3*x^33*b^11*a^3 + 715/3*x^33*c*b^9*a^4 + 1716*x^33*c^2*b^7*a^5
+ 4004*x^33*c^3*b^5*a^6 + 2860*x^33*c^4*b^3*a^7 + 429*x^33*c^5*b*
a^8 + 143/6*x^30*b^10*a^4 + 429*x^30*c*b^8*a^5 + 2002*x^30*c^2*b^
6*a^6 + 2860*x^30*c^3*b^4*a^7 + 2145/2*x^30*c^4*b^2*a^8 + 143/3*x
^30*c^5*a^9 + 143/3*x^27*b^9*a^5 + 572*x^27*c*b^7*a^6 + 1716*x^27
*c^2*b^5*a^7 + 1430*x^27*c^3*b^3*a^8 + 715/3*x^27*c^4*b*a^9 + 143
/2*x^24*b^8*a^6 + 572*x^24*c*b^6*a^7 + 2145/2*x^24*c^2*b^4*a^8 +
1430/3*x^24*c^3*b^2*a^9 + 143/6*x^24*c^4*a^10 + 572/7*x^21*b^7*a^
7 + 429*x^21*c*b^5*a^8 + 1430/3*x^21*c^2*b^3*a^9 + 286/3*x^21*c^3
*b*a^10 + 143/2*x^18*b^6*a^8 + 715/3*x^18*c*b^4*a^9 + 143*x^18*c^
2*b^2*a^10 + 26/3*x^18*c^3*a^11 + 143/3*x^15*b^5*a^9 + 286/3*x^15
*c*b^3*a^10 + 26*x^15*c^2*b*a^11 + 143/6*x^12*b^4*a^10 + 26*x^12*
c*b^2*a^11 + 13/6*x^12*c^2*a^12 + 26/3*x^9*b^3*a^11 + 13/3*x^9*c*
b*a^12 + 13/6*x^6*b^2*a^12 + 1/3*x^6*c*a^13 + 1/3*x^3*b*a^13
```

Sympy [A] time = 0.923851, size = 1394, normalized size = 77.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3+a)**13,x)
```

```
[Out] a**13*b*x**3/3 + b*c**13*x**81/3 + c**14*x**84/42 + x**78*(a*c**1
3/3 + 13*b**2*c**12/6) + x**75*(13*a*b*c**12/3 + 26*b**3*c**11/3)
 + x**72*(13*a**2*c**12/6 + 26*a*b**2*c**11 + 143*b**4*c**10/6) +
 x**69*(26*a**2*b*c**11 + 286*a*b**3*c**10/3 + 143*b**5*c**9/3) +
 x**66*(26*a**3*c**11/3 + 143*a**2*b**2*c**10 + 715*a*b**4*c**9/3
 + 143*b**6*c**8/2) + x**63*(286*a**3*b*c**10/3 + 1430*a**2*b**3*
c**9/3 + 429*a*b**5*c**8 + 572*b**7*c**7/7) + x**60*(143*a**4*c**
10/6 + 1430*a**3*b**2*c**9/3 + 2145*a**2*b**4*c**8/2 + 572*a*b**6
*c**7 + 143*b**8*c**6/2) + x**57*(715*a**4*b*c**9/3 + 1430*a**3*b
**3*c**8 + 1716*a**2*b**5*c**7 + 572*a*b**7*c**6 + 143*b**9*c**5/
3) + x**54*(143*a**5*c**9/3 + 2145*a**4*b**2*c**8/2 + 2860*a**3*b
**4*c**7 + 2002*a**2*b**6*c**6 + 429*a*b**8*c**5 + 143*b**10*c**4
/6) + x**51*(429*a**5*b*c**8 + 2860*a**4*b**3*c**7 + 4004*a**3*b*
*5*c**6 + 1716*a**2*b**7*c**5 + 715*a*b**9*c**4/3 + 26*b**11*c**3
/3) + x**48*(143*a**6*c**8/2 + 1716*a**5*b**2*c**7 + 5005*a**4*b*
*4*c**6 + 4004*a**3*b**6*c**5 + 2145*a**2*b**8*c**4/2 + 286*a*b**
10*c**3/3 + 13*b**12*c**2/6) + x**45*(572*a**6*b*c**7 + 4004*a**5
*b**3*c**6 + 6006*a**4*b**5*c**5 + 2860*a**3*b**7*c**4 + 1430*a**
2*b**9*c**3/3 + 26*a*b**11*c**2 + b**13*c/3) + x**42*(572*a**7*c*
*7/7 + 2002*a**6*b**2*c**6 + 6006*a**5*b**4*c**5 + 5005*a**4*b**6
```

```
*c**4 + 1430*a**3*b**8*c**3 + 143*a**2*b**10*c**2 + 13*a*b**12*c/
3 + b**14/42) + x**39*(572*a**7*b*c**6 + 4004*a**6*b**3*c**5 + 60
06*a**5*b**5*c**4 + 2860*a**4*b**7*c**3 + 1430*a**3*b**9*c**2/3 +
 26*a**2*b**11*c + a*b**13/3) + x**36*(143*a**8*c**6/2 + 1716*a**
7*b**2*c**5 + 5005*a**6*b**4*c**4 + 4004*a**5*b**6*c**3 + 2145*a*
*4*b**8*c**2/2 + 286*a**3*b**10*c/3 + 13*a**2*b**12/6) + x**33*(4
29*a**8*b*c**5 + 2860*a**7*b**3*c**4 + 4004*a**6*b**5*c**3 + 1716
*a**5*b**7*c**2 + 715*a**4*b**9*c/3 + 26*a**3*b**11/3) + x**30*(1
43*a**9*c**5/3 + 2145*a**8*b**2*c**4/2 + 2860*a**7*b**4*c**3 + 20
02*a**6*b**6*c**2 + 429*a**5*b**8*c + 143*a**4*b**10/6) + x**27*(
715*a**9*b*c**4/3 + 1430*a**8*b**3*c**3 + 1716*a**7*b**5*c**2 + 5
72*a**6*b**7*c + 143*a**5*b**9/3) + x**24*(143*a**10*c**4/6 + 143
0*a**9*b**2*c**3/3 + 2145*a**8*b**4*c**2/2 + 572*a**7*b**6*c + 14
3*a**6*b**8/2) + x**21*(286*a**10*b*c**3/3 + 1430*a**9*b**3*c**2/
3 + 429*a**8*b**5*c + 572*a**7*b**7/7) + x**18*(26*a**11*c**3/3 +
 143*a**10*b**2*c**2 + 715*a**9*b**4*c/3 + 143*a**8*b**6/2) + x**
15*(26*a**11*b*c**2 + 286*a**10*b**3*c/3 + 143*a**9*b**5/3) + x**
12*(13*a**12*c**2/6 + 26*a**11*b**2*c + 143*a**10*b**4/6) + x**9*
(13*a**12*b*c/3 + 26*a**11*b**3/3) + x**6*(a**13*c/3 + 13*a**12*b
**2/6)
```

GIAC/XCAS [A] time = 0.277393, size = 1, normalized size = 0.06

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 + a)^13*(2*c*x^3 + b)*x^2,x, algorithm="giac")
```

```
[Out] Done
```

3.96 $\int x^{-1+n}(b+2cx^n)(a+bx^n+cx^{2n})^{13}\,dx$

Optimal. Leaf size=23

$$\frac{(a+bx^n+cx^{2n})^{14}}{14n}$$

```
[Out] (a + b*x^n + c*x^(2*n))^14/(14*n)
```

Rubi [A] time = 0.0690415, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{(a+bx^n+cx^{2n})^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^13,x]

[Out] (a + b*x^n + c*x^(2*n))^14/(14*n)
```

Rubi in Sympy [A] time = 14.2302, size = 17, normalized size = 0.74

$$\frac{(a+bx^n+cx^{2n})^{14}}{14n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**(-1+n)*(b+2*c*x**n)*(a+b*x**n+c*x**(2*n))**13,x)

[Out] (a + b*x**n + c*x**(2*n))**14/(14*n)
```

Mathematica [B] time = 0.726732, size = 260, normalized size = 11.3

$$x^n(b+cx^n)\left(14a^{13}+91a^{12}x^n(b+cx^n)+364a^{11}x^{2n}(b+cx^n)^2+1001a^{10}x^{3n}(b+cx^n)^3+2002a^9x^{4n}(b+cx^n)^4+3003a^8x^{5n}\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (x^n*(b + c*x^n)*(14*a^13 + 91*a^12*x^n*(b + c*x^n) + 364*a^11*x^
(2*n)*(b + c*x^n)^2 + 1001*a^10*x^(3*n)*(b + c*x^n)^3 + 2002*a^9*
x^(4*n)*(b + c*x^n)^4 + 3003*a^8*x^(5*n)*(b + c*x^n)^5 + 3432*a^7
*x^(6*n)*(b + c*x^n)^6 + 3003*a^6*x^(7*n)*(b + c*x^n)^7 + 2002*a^
5*x^(8*n)*(b + c*x^n)^8 + 1001*a^4*x^(9*n)*(b + c*x^n)^9 + 364*a^
3*x^(10*n)*(b + c*x^n)^10 + 91*a^2*x^(11*n)*(b + c*x^n)^11 + 14*a
*x^(12*n)*(b + c*x^n)^12 + x^(13*n)*(b + c*x^n)^13))/(14*n)
```

Maple [B] time = 0.099, size = 2042, normalized size = 88.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x)
```

```
[Out] 143*b^5*a^9/n*(x^n)^5+b^13*c/n*(x^n)^15+26*b^11*a^3/n*(x^n)^11+26
*a^11*b^3/n*(x^n)^3+143*a^5*b^9/n*(x^n)^9+143*b^5*c^9/n*(x^n)^23+
26*c^11*b^3/n*(x^n)^25+a*b^13/n*(x^n)^13+b*a^13/n*x^n+b*c^13/n*(x
^n)^27+143*c^9/n*(x^n)^18*a^5+143/2*c^4/n*(x^n)^18*b^10+26*c^11/n
*(x^n)^22*a^3+429/2*c^8/n*(x^n)^22*b^6+26*b^11*c^3/n*(x^n)^17+c^1
3/n*(x^n)^26*a+13/2*c^12/n*(x^n)^26*b^2+1716/7*b^7*a^7/n*(x^n)^7+
143*c^5*b^9/n*(x^n)^19+1716/7*b^7*c^7/n*(x^n)^21+13/2*c^12/n*(x^n
)^24*a^2+143/2*c^10/n*(x^n)^24*b^4+143*a^9/n*(x^n)^10*c^5+143/2*a
^4/n*(x^n)^10*b^10+a^13/n*(x^n)^2*c+13/2*a^12/n*(x^n)^2*b^2+6006/
n*(x^n)^14*a^6*b^2*c^6+18018/n*(x^n)^14*a^5*b^4*c^5+15015/n*(x^n)
^14*b^6*a^4*c^4+4290/n*(x^n)^14*a^3*b^8*c^3+429/n*(x^n)^14*a^2*b^
10*c^2+13/n*(x^n)^14*a*b^12*c+1/14*c^14/n*(x^n)^28+13*a^12*b/n*(x
^n)^3*c+1287*a^5/n*(x^n)^10*b^8*c+286*b*a^10/n*(x^n)^7*c^3+1430*b
^3*a^9/n*(x^n)^7*c^2+1287*b^5*a^8/n*(x^n)^7*c+715*c^9*b/n*(x^n)^1
9*a^4+4290*c^8*b^3/n*(x^n)^19*a^3+5148*c^7*b^5/n*(x^n)^19*a^2+171
6*c^6*b^7/n*(x^n)^19*a+78*c^11/n*(x^n)^24*a*b^2+6435/2*a^8/n*(x^n
)^10*b^2*c^4+8580*a^7/n*(x^n)^10*b^4*c^3+6006*a^6/n*(x^n)^10*b^6*
c^2+143/2*a^10/n*(x^n)^8*c^4+429/2*a^6/n*(x^n)^8*b^8+26*a^11/n*(x
^n)^6*c^3+429/2*a^8/n*(x^n)^6*b^6+143/2*c^10/n*(x^n)^20*a^4+429/2
*c^6/n*(x^n)^20*b^8+429/2*c^8/n*(x^n)^16*a^6+13/2*c^2/n*(x^n)^16*
b^12+429/2*a^8/n*(x^n)^12*c^6+13/2*a^2/n*(x^n)^12*b^12+13/2*a^12/
n*(x^n)^4*c^2+143/2*a^10/n*(x^n)^4*b^4+1716/7/n*(x^n)^14*a^7*c^7+
1430*c^9/n*(x^n)^20*a^3*b^2+6435/2*c^8/n*(x^n)^20*a^2*b^4+1716*c^
7/n*(x^n)^20*a*b^6+5148*c^7/n*(x^n)^16*a^5*b^2+15015*c^6/n*(x^n)^
16*a^4*b^4+12012*c^5/n*(x^n)^16*a^3*b^6+6435/2*c^4/n*(x^n)^16*a^2
*b^8+286*c^3/n*(x^n)^16*a*b^10+6435/2*c^8/n*(x^n)^18*b^2*a^4+8580
*c^7/n*(x^n)^18*b^4*a^3+6006*c^6/n*(x^n)^18*b^6*a^2+1287*c^5/n*(x
^n)^18*a*b^8+429*c^10/n*(x^n)^22*a^2*b^2+715*c^9/n*(x^n)^22*a*b^4
+1/14/n*(x^n)^14*b^14+13*c^12*b/n*(x^n)^25*a+1716*a^7*b/n*(x^n)^1
3*c^6+12012*a^6*b^3/n*(x^n)^13*c^5+18018*a^5*b^5/n*(x^n)^13*c^4+8
580*a^4*b^7/n*(x^n)^13*c^3+1430*a^3*b^9/n*(x^n)^13*c^2+78*a^2*b^1
```

```
1/n*(x^n)^13*c+1430*a^9/n*(x^n)^8*b^2*c^3+6435/2*a^8/n*(x^n)^8*b^
4*c^2+1716*a^7/n*(x^n)^8*b^6*c+429*a^10/n*(x^n)^6*b^2*c^2+715*a^9
/n*(x^n)^6*b^4*c+12012*b^5*c^6/n*(x^n)^17*a^3+5148*b^7*c^5/n*(x^n
)^17*a^2+715*b^9*c^4/n*(x^n)^17*a+5148*a^7/n*(x^n)^12*b^2*c^5+150
15*a^6/n*(x^n)^12*b^4*c^4+12012*a^5/n*(x^n)^12*b^6*c^3+6435/2*a^4
/n*(x^n)^12*b^8*c^2+286*a^3/n*(x^n)^12*b^10*c+78*a^11/n*(x^n)^4*b
^2*c+78*b*a^11/n*(x^n)^5*c^2+286*b^3*a^10/n*(x^n)^5*c+1716*b*c^7/
n*(x^n)^15*a^6+12012*b^3*c^6/n*(x^n)^15*a^5+18018*b^5*c^5/n*(x^n)
^15*a^4+8580*b^7*c^4/n*(x^n)^15*a^3+1430*b^9*c^3/n*(x^n)^15*a^2+7
8*b^11*c^2/n*(x^n)^15*a+1287*b*a^8/n*(x^n)^11*c^5+8580*b^3*a^7/n*
(x^n)^11*c^4+12012*b^5*a^6/n*(x^n)^11*c^3+5148*b^7*a^5/n*(x^n)^11
*c^2+715*b^9*a^4/n*(x^n)^11*c+715*a^9*b/n*(x^n)^9*c^4+4290*a^8*b^
3/n*(x^n)^9*c^3+5148*a^7*b^5/n*(x^n)^9*c^2+1716*a^6*b^7/n*(x^n)^9
*c+78*b*c^11/n*(x^n)^23*a^2+286*b^3*c^10/n*(x^n)^23*a+1287*b*c^8/
n*(x^n)^17*a^5+8580*b^3*c^7/n*(x^n)^17*a^4+286*b*c^10/n*(x^n)^21*
a^3+1430*b^3*c^9/n*(x^n)^21*a^2+1287*b^5*c^8/n*(x^n)^21*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + b*x^n + a)^13*(2*c*x^n + b)*x^(n - 1),x, algorithm="maxima

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.333561, size = 1751, normalized size = 76.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + b*x^n + a)^13*(2*c*x^n + b)*x^(n - 1),x, algorithm="fricas

[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 14*a^13*b*x^n + 7*(13*
b^2*c^12 + 2*a*c^13)*x^(26*n) + 182*(2*b^3*c^11 + a*b*c^12)*x^(25
*n) + 91*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c^12)*x^(24*n) + 182*
(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^(23*n) + 91*(33*b^6
*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 + 4*a^3*c^11)*x^(22*n) + 2
86*(12*b^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 14*a^3*b*c^10)*x
^(21*n) + 1001*(3*b^8*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 + 20*a^
3*b^2*c^9 + a^4*c^10)*x^(20*n) + 2002*(b^9*c^5 + 12*a*b^7*c^6 + 3
6*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9)*x^(19*n) + 1001*(b^
10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7 + 45*a^4
```

```
*b^2*c^8 + 2*a^5*c^9)*x^(18*n) + 182*(2*b^11*c^3 + 55*a*b^9*c^4 +
 396*a^2*b^7*c^5 + 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 + 99*a^5*b*c
^8)*x^(17*n) + 91*(b^12*c^2 + 44*a*b^10*c^3 + 495*a^2*b^8*c^4 + 1
848*a^3*b^6*c^5 + 2310*a^4*b^4*c^6 + 792*a^5*b^2*c^7 + 33*a^6*c^8
)*x^(16*n) + 14*(b^13*c + 78*a*b^11*c^2 + 1430*a^2*b^9*c^3 + 8580
*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c^6 + 1716*a^6*b
*c^7)*x^(15*n) + (b^14 + 182*a*b^12*c + 6006*a^2*b^10*c^2 + 60060
*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 84084*a^
6*b^2*c^6 + 3432*a^7*c^7)*x^(14*n) + 14*(a*b^13 + 78*a^2*b^11*c +
 1430*a^3*b^9*c^2 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*
a^6*b^3*c^5 + 1716*a^7*b*c^6)*x^(13*n) + 91*(a^2*b^12 + 44*a^3*b^
10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c^3 + 2310*a^6*b^4*c^4 + 79
2*a^7*b^2*c^5 + 33*a^8*c^6)*x^(12*n) + 182*(2*a^3*b^11 + 55*a^4*b
^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 + 99*a
^8*b*c^5)*x^(11*n) + 1001*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*b^6*c
^2 + 120*a^7*b^4*c^3 + 45*a^8*b^2*c^4 + 2*a^9*c^5)*x^(10*n) + 200
2*(a^5*b^9 + 12*a^6*b^7*c + 36*a^7*b^5*c^2 + 30*a^8*b^3*c^3 + 5*a
^9*b*c^4)*x^(9*n) + 1001*(3*a^6*b^8 + 24*a^7*b^6*c + 45*a^8*b^4*c
^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^(8*n) + 286*(12*a^7*b^7 + 63*a^
8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c^3)*x^(7*n) + 91*(33*a^8*b^
6 + 110*a^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)*x^(6*n) + 182*(
11*a^9*b^5 + 22*a^10*b^3*c + 6*a^11*b*c^2)*x^(5*n) + 91*(11*a^10*
b^4 + 12*a^11*b^2*c + a^12*c^2)*x^(4*n) + 182*(2*a^11*b^3 + a^12*
b*c)*x^(3*n) + 7*(13*a^12*b^2 + 2*a^13*c)*x^(2*n))/n
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+n)*(b+2*c*x**n)*(a+b*x**n+c*x**(2*n))**13,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.325759, size = 1, normalized size = 0.04

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + b*x^n + a)^13*(2*c*x^n + b)*x^(n - 1),x, algorithm="giac")

[Out] Done
```

3.97 $\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx$

Optimal. Leaf size=18

$$\frac{1}{14}\left(a-bx-cx^2\right)^{14}$$

```
[Out] (a - b*x - c*x^2)^14/14
```

Rubi [A] time = 0.0352768, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$

$$\frac{1}{14}\left(a-bx-cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(-a + b*x + c*x^2)^13,x]
```

```
[Out] (a - b*x - c*x^2)^14/14
```

Rubi in Sympy [A] time = 4.02151, size = 12, normalized size = 0.67

$$\frac{\left(-a+bx+cx^2\right)^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2*c*x+b)*(c*x**2+b*x-a)**13,x)
```

```
[Out] (-a + b*x + c*x**2)**14/14
```

Mathematica [B] time = 0.302325, size = 201, normalized size = 11.17

$$\frac{1}{14}x(b+cx)\left(-14a^{13}+91a^{12}x(b+cx)-364a^{11}x^2(b+cx)^2+1001a^{10}x^3(b+cx)^3-2002a^9x^4(b+cx)^4+3003a^8x^5(b+cx)^5-3432a^7x^6(b+cx)^6+3003a^6x^7(b+cx)^7-2002a^5x^8(b+cx)^8+1001a^4x^9(b+cx)^9-364a^3x^{10}(b+cx)^{10}+91a^2x^{11}(b+cx)^{11}-14ax^{12}(b+cx)^{12}+x^{13}(b+cx)^{13}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*c*x)*(-a + b*x + c*x^2)^13,x]
```

```
[Out] (x*(b + c*x)*(-14*a^13 + 91*a^12*x*(b + c*x) - 364*a^11*x^2*(b +
c*x)^2 + 1001*a^10*x^3*(b + c*x)^3 - 2002*a^9*x^4*(b + c*x)^4 + 3
003*a^8*x^5*(b + c*x)^5 - 3432*a^7*x^6*(b + c*x)^6 + 3003*a^6*x^7
*(b + c*x)^7 - 2002*a^5*x^8*(b + c*x)^8 + 1001*a^4*x^9*(b + c*x)^
9 - 364*a^3*x^10*(b + c*x)^10 + 91*a^2*x^11*(b + c*x)^11 - 14*a*x
^12*(b + c*x)^12 + x^13*(b + c*x)^13))/14
```

Maple [B] time = 0.007, size = 47685, normalized size = 2649.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*c*x+b)*(c*x^2+b*x-a)^13,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 0.770251, size = 22, normalized size = 1.22

$$\frac{1}{14}\left(cx^2 + bx - a\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x - a)^13*(2*c*x + b),x, algorithm="maxima")
```

```
[Out] 1/14*(c*x^2 + b*x - a)^14
```

Fricas [A] time = 0.258895, size = 1, normalized size = 0.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x - a)^13*(2*c*x + b),x, algorithm="fricas")
```

```
[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 - x^26*c^13*a +
 26*x^25*c^11*b^3 - 13*x^25*c^12*b*a + 143/2*x^24*c^10*b^4 - 78*x
```

```
^24*c^11*b^2*a + 13/2*x^24*c^12*a^2 + 143*x^23*c^9*b^5 - 286*x^23
*c^10*b^3*a + 78*x^23*c^11*b*a^2 + 429/2*x^22*c^8*b^6 - 715*x^22*
c^9*b^4*a + 429*x^22*c^10*b^2*a^2 - 26*x^22*c^11*a^3 + 1716/7*x^2
1*c^7*b^7 - 1287*x^21*c^8*b^5*a + 1430*x^21*c^9*b^3*a^2 - 286*x^2
1*c^10*b*a^3 + 429/2*x^20*c^6*b^8 - 1716*x^20*c^7*b^6*a + 6435/2*
x^20*c^8*b^4*a^2 - 1430*x^20*c^9*b^2*a^3 + 143/2*x^20*c^10*a^4 +
143*x^19*c^5*b^9 - 1716*x^19*c^6*b^7*a + 5148*x^19*c^7*b^5*a^2 -
4290*x^19*c^8*b^3*a^3 + 715*x^19*c^9*b*a^4 + 143/2*x^18*c^4*b^10
- 1287*x^18*c^5*b^8*a + 6006*x^18*c^6*b^6*a^2 - 8580*x^18*c^7*b^4
*a^3 + 6435/2*x^18*c^8*b^2*a^4 - 143*x^18*c^9*a^5 + 26*x^17*c^3*b
^11 - 715*x^17*c^4*b^9*a + 5148*x^17*c^5*b^7*a^2 - 12012*x^17*c^6
*b^5*a^3 + 8580*x^17*c^7*b^3*a^4 - 1287*x^17*c^8*b*a^5 + 13/2*x^1
6*c^2*b^12 - 286*x^16*c^3*b^10*a + 6435/2*x^16*c^4*b^8*a^2 - 1201
2*x^16*c^5*b^6*a^3 + 15015*x^16*c^6*b^4*a^4 - 5148*x^16*c^7*b^2*a
^5 + 429/2*x^16*c^8*a^6 + x^15*c*b^13 - 78*x^15*c^2*b^11*a + 1430
*x^15*c^3*b^9*a^2 - 8580*x^15*c^4*b^7*a^3 + 18018*x^15*c^5*b^5*a^
4 - 12012*x^15*c^6*b^3*a^5 + 1716*x^15*c^7*b*a^6 + 1/14*x^14*b^14
 - 13*x^14*c*b^12*a + 429*x^14*c^2*b^10*a^2 - 4290*x^14*c^3*b^8*a
^3 + 15015*x^14*c^4*b^6*a^4 - 18018*x^14*c^5*b^4*a^5 + 6006*x^14*
c^6*b^2*a^6 - 1716/7*x^14*c^7*a^7 - x^13*b^13*a + 78*x^13*c*b^11*
a^2 - 1430*x^13*c^2*b^9*a^3 + 8580*x^13*c^3*b^7*a^4 - 18018*x^13*
c^4*b^5*a^5 + 12012*x^13*c^5*b^3*a^6 - 1716*x^13*c^6*b*a^7 + 13/2
*x^12*b^12*a^2 - 286*x^12*c*b^10*a^3 + 6435/2*x^12*c^2*b^8*a^4 -
12012*x^12*c^3*b^6*a^5 + 15015*x^12*c^4*b^4*a^6 - 5148*x^12*c^5*b
^2*a^7 + 429/2*x^12*c^6*a^8 - 26*x^11*b^11*a^3 + 715*x^11*c*b^9*a
^4 - 5148*x^11*c^2*b^7*a^5 + 12012*x^11*c^3*b^5*a^6 - 8580*x^11*c
^4*b^3*a^7 + 1287*x^11*c^5*b*a^8 + 143/2*x^10*b^10*a^4 - 1287*x^1
0*c*b^8*a^5 + 6006*x^10*c^2*b^6*a^6 - 8580*x^10*c^3*b^4*a^7 + 643
5/2*x^10*c^4*b^2*a^8 - 143*x^10*c^5*a^9 - 143*x^9*b^9*a^5 + 1716*
x^9*c*b^7*a^6 - 5148*x^9*c^2*b^5*a^7 + 4290*x^9*c^3*b^3*a^8 - 715
*x^9*c^4*b*a^9 + 429/2*x^8*b^8*a^6 - 1716*x^8*c*b^6*a^7 + 6435/2*
x^8*c^2*b^4*a^8 - 1430*x^8*c^3*b^2*a^9 + 143/2*x^8*c^4*a^10 - 171
6/7*x^7*b^7*a^7 + 1287*x^7*c*b^5*a^8 - 1430*x^7*c^2*b^3*a^9 + 286
*x^7*c^3*b*a^10 + 429/2*x^6*b^6*a^8 - 715*x^6*c*b^4*a^9 + 429*x^6
*c^2*b^2*a^10 - 26*x^6*c^3*a^11 - 143*x^5*b^5*a^9 + 286*x^5*c*b^3
*a^10 - 78*x^5*c^2*b*a^11 + 143/2*x^4*b^4*a^10 - 78*x^4*c*b^2*a^1
1 + 13/2*x^4*c^2*a^12 - 26*x^3*b^3*a^11 + 13*x^3*c*b*a^12 + 13/2*
x^2*b^2*a^12 - x^2*c*a^13 - x*b*a^13
```

Sympy [A] time = 0.956125, size = 1326, normalized size = 73.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x+b)*(c*x**2+b*x-a)**13,x)
```

```
[Out] -a**13*b*x + b*c**13*x**27 + c**14*x**28/14 + x**26*(-a*c**13 + 1
3*b**2*c**12/2) + x**25*(-13*a*b*c**12 + 26*b**3*c**11) + x**24*(
```

```
13*a**2*c**12/2 - 78*a*b**2*c**11 + 143*b**4*c**10/2) + x**23*(78
*a**2*b*c**11 - 286*a*b**3*c**10 + 143*b**5*c**9) + x**22*(-26*a*
*3*c**11 + 429*a**2*b**2*c**10 - 715*a*b**4*c**9 + 429*b**6*c**8/
2) + x**21*(-286*a**3*b*c**10 + 1430*a**2*b**3*c**9 - 1287*a*b**5
*c**8 + 1716*b**7*c**7/7) + x**20*(143*a**4*c**10/2 - 1430*a**3*b
**2*c**9 + 6435*a**2*b**4*c**8/2 - 1716*a*b**6*c**7 + 429*b**8*c*
*6/2) + x**19*(715*a**4*b*c**9 - 4290*a**3*b**3*c**8 + 5148*a**2*
b**5*c**7 - 1716*a*b**7*c**6 + 143*b**9*c**5) + x**18*(-143*a**5*
c**9 + 6435*a**4*b**2*c**8/2 - 8580*a**3*b**4*c**7 + 6006*a**2*b*
*6*c**6 - 1287*a*b**8*c**5 + 143*b**10*c**4/2) + x**17*(-1287*a**
5*b*c**8 + 8580*a**4*b**3*c**7 - 12012*a**3*b**5*c**6 + 5148*a**2
*b**7*c**5 - 715*a*b**9*c**4 + 26*b**11*c**3) + x**16*(429*a**6*c
**8/2 - 5148*a**5*b**2*c**7 + 15015*a**4*b**4*c**6 - 12012*a**3*b
**6*c**5 + 6435*a**2*b**8*c**4/2 - 286*a*b**10*c**3 + 13*b**12*c*
*2/2) + x**15*(1716*a**6*b*c**7 - 12012*a**5*b**3*c**6 + 18018*a*
*4*b**5*c**5 - 8580*a**3*b**7*c**4 + 1430*a**2*b**9*c**3 - 78*a*b
**11*c**2 + b**13*c) + x**14*(-1716*a**7*c**7/7 + 6006*a**6*b**2*
c**6 - 18018*a**5*b**4*c**5 + 15015*a**4*b**6*c**4 - 4290*a**3*b*
*8*c**3 + 429*a**2*b**10*c**2 - 13*a*b**12*c + b**14/14) + x**13*
(-1716*a**7*b*c**6 + 12012*a**6*b**3*c**5 - 18018*a**5*b**5*c**4
+ 8580*a**4*b**7*c**3 - 1430*a**3*b**9*c**2 + 78*a**2*b**11*c - a
*b**13) + x**12*(429*a**8*c**6/2 - 5148*a**7*b**2*c**5 + 15015*a*
*6*b**4*c**4 - 12012*a**5*b**6*c**3 + 6435*a**4*b**8*c**2/2 - 286
*a**3*b**10*c + 13*a**2*b**12/2) + x**11*(1287*a**8*b*c**5 - 8580
*a**7*b**3*c**4 + 12012*a**6*b**5*c**3 - 5148*a**5*b**7*c**2 + 71
5*a**4*b**9*c - 26*a**3*b**11) + x**10*(-143*a**9*c**5 + 6435*a**
8*b**2*c**4/2 - 8580*a**7*b**4*c**3 + 6006*a**6*b**6*c**2 - 1287*
a**5*b**8*c + 143*a**4*b**10/2) + x**9*(-715*a**9*b*c**4 + 4290*a
**8*b**3*c**3 - 5148*a**7*b**5*c**2 + 1716*a**6*b**7*c - 143*a**5
*b**9) + x**8*(143*a**10*c**4/2 - 1430*a**9*b**2*c**3 + 6435*a**8
*b**4*c**2/2 - 1716*a**7*b**6*c + 429*a**6*b**8/2) + x**7*(286*a*
*10*b*c**3 - 1430*a**9*b**3*c**2 + 1287*a**8*b**5*c - 1716*a**7*b
**7/7) + x**6*(-26*a**11*c**3 + 429*a**10*b**2*c**2 - 715*a**9*b*
*4*c + 429*a**8*b**6/2) + x**5*(-78*a**11*b*c**2 + 286*a**10*b**3
*c - 143*a**9*b**5) + x**4*(13*a**12*c**2/2 - 78*a**11*b**2*c + 1
43*a**10*b**4/2) + x**3*(13*a**12*b*c - 26*a**11*b**3) + x**2*(-a
**13*c + 13*a**12*b**2/2)
```

GIAC/XCAS [A] time = 0.268694, size = 1, normalized size = 0.06

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x - a)^13*(2*c*x + b),x, algorithm="giac")

[Out] Done
```

3.98 $\int x \left(b+2 c x^2\right) \left(-a+b x^2+c x^4\right)^{13} \, dx$

Optimal. Leaf size=20

$$\frac{1}{28} \left(a-b x^2-c x^4\right)^{14}$$

```
[Out] (a - b*x^2 - c*x^4)^14/28
```

Rubi [A] time = 0.0664352, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{1}{28} \left(a-b x^2-c x^4\right)^{14}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^13,x]

[Out] (a - b*x^2 - c*x^4)^14/28
```

Rubi in Sympy [A] time = 5.44535, size = 14, normalized size = 0.7

$$\frac{\left(a-b x^2-c x^4\right)^{14}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2-a)**13,x)

[Out] (a - b*x**2 - c*x**4)**14/28
```

Mathematica [B] time = 0.304435, size = 233, normalized size = 11.65

$$\frac{1}{28} x^2 \left(b+c x^2\right) \left(-14 a^{13}+91 a^{12} x^2 \left(b+c x^2\right)-364 a^{11} x^4 \left(b+c x^2\right)^2+1001 a^{10} x^6 \left(b+c x^2\right)^3 -2002 a^9 x^8 \left(b+c x^2\right)^4+3003 a^8 x^{10} \left(b+c x^2\right)^5-3432 a^7 x^{12} \left(b+c x^2\right)^6 +3003 a^6 x^{14} \left(b+c x^2\right)^7-2002 a^5 x^{16} \left(b+c x^2\right)^8+1001 a^4 x^{18} \left(b+c x^2\right)^9 -364 a^3 x^{20} \left(b+c x^2\right)^{10}+91 a^2 x^{22} \left(b+c x^2\right)^{11}-14 a x^{24} \left(b+c x^2\right)^{12}+x^{26} \left(b+c x^2\right)^{13}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^13,x]
```

```
[Out] (x^2*(b + c*x^2)*(-14*a^13 + 91*a^12*x^2*(b + c*x^2) - 364*a^11*x
^4*(b + c*x^2)^2 + 1001*a^10*x^6*(b + c*x^2)^3 - 2002*a^9*x^8*(b
+ c*x^2)^4 + 3003*a^8*x^10*(b + c*x^2)^5 - 3432*a^7*x^12*(b + c*x
^2)^6 + 3003*a^6*x^14*(b + c*x^2)^7 - 2002*a^5*x^16*(b + c*x^2)^8
 + 1001*a^4*x^18*(b + c*x^2)^9 - 364*a^3*x^20*(b + c*x^2)^10 + 91
*a^2*x^22*(b + c*x^2)^11 - 14*a*x^24*(b + c*x^2)^12 + x^26*(b + c
*x^2)^13))/28
```

Maple [B] time = 0.004, size = 47688, normalized size = 2384.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 0.787075, size = 1677, normalized size = 83.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 - a)^13*(2*c*x^2 + b)*x,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 1/4*(13*b^2*c^12 - 2*a*c^13)*x
^52 + 13/2*(2*b^3*c^11 - a*b*c^12)*x^50 + 13/4*(11*b^4*c^10 - 12*
a*b^2*c^11 + a^2*c^12)*x^48 + 13/2*(11*b^5*c^9 - 22*a*b^3*c^10 +
6*a^2*b*c^11)*x^46 + 13/4*(33*b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^
2*c^10 - 4*a^3*c^11)*x^44 + 143/14*(12*b^7*c^7 - 63*a*b^5*c^8 + 7
0*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^42 + 143/4*(3*b^8*c^6 - 24*a*b^6
*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^40 + 143/2*(
b^9*c^5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*
b*c^9)*x^38 + 143/4*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 1
20*a^3*b^4*c^7 + 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^36 + 13/2*(2*b^11*
c^3 - 55*a*b^9*c^4 + 396*a^2*b^7*c^5 - 924*a^3*b^5*c^6 + 660*a^4*
b^3*c^7 - 99*a^5*b*c^8)*x^34 + 13/4*(b^12*c^2 - 44*a*b^10*c^3 + 4
95*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*b^4*c^6 - 792*a^5*b^
```

```
2*c^7 + 33*a^6*c^8)*x^32 + 1/2*(b^13*c - 78*a*b^11*c^2 + 1430*a^2
*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^30 + 1/28*(b^14 - 182*a*b^12*c + 6006*a^2*
b^10*c^2 - 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^
4*c^5 + 84084*a^6*b^2*c^6 - 3432*a^7*c^7)*x^28 - 1/2*(a*b^13 - 78
*a^2*b^11*c + 1430*a^3*b^9*c^2 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5
*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^6)*x^26 + 13/4*(a^2*b^12
- 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c^3 + 2310*a^6*b
^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^24 - 13/2*(2*a^3*b^11 -
55*a^4*b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^
4 - 99*a^8*b*c^5)*x^22 + 143/4*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*
b^6*c^2 - 120*a^7*b^4*c^3 + 45*a^8*b^2*c^4 - 2*a^9*c^5)*x^20 - 14
3/2*(a^5*b^9 - 12*a^6*b^7*c + 36*a^7*b^5*c^2 - 30*a^8*b^3*c^3 + 5
*a^9*b*c^4)*x^18 + 143/4*(3*a^6*b^8 - 24*a^7*b^6*c + 45*a^8*b^4*c
^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^16 - 1/2*a^13*b*x^2 - 143/14*(1
2*a^7*b^7 - 63*a^8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c^3)*x^14 +
 13/4*(33*a^8*b^6 - 110*a^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)
*x^12 - 13/2*(11*a^9*b^5 - 22*a^10*b^3*c + 6*a^11*b*c^2)*x^10 + 1
3/4*(11*a^10*b^4 - 12*a^11*b^2*c + a^12*c^2)*x^8 - 13/2*(2*a^11*b
^3 - a^12*b*c)*x^6 + 1/4*(13*a^12*b^2 - 2*a^13*c)*x^4
```

Fricas [A] time = 0.259067, size = 1, normalized size = 0.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 - a)^13*(2*c*x^2 + b)*x,x, algorithm="fricas")
```

```
[Out] 1/28*x^56*c^14 + 1/2*x^54*c^13*b + 13/4*x^52*c^12*b^2 - 1/2*x^52*
c^13*a + 13*x^50*c^11*b^3 - 13/2*x^50*c^12*b*a + 143/4*x^48*c^10*
b^4 - 39*x^48*c^11*b^2*a + 13/4*x^48*c^12*a^2 + 143/2*x^46*c^9*b^
5 - 143*x^46*c^10*b^3*a + 39*x^46*c^11*b*a^2 + 429/4*x^44*c^8*b^6
 - 715/2*x^44*c^9*b^4*a + 429/2*x^44*c^10*b^2*a^2 - 13*x^44*c^11*
a^3 + 858/7*x^42*c^7*b^7 - 1287/2*x^42*c^8*b^5*a + 715*x^42*c^9*b
^3*a^2 - 143*x^42*c^10*b*a^3 + 429/4*x^40*c^6*b^8 - 858*x^40*c^7*
b^6*a + 6435/4*x^40*c^8*b^4*a^2 - 715*x^40*c^9*b^2*a^3 + 143/4*x^
40*c^10*a^4 + 143/2*x^38*c^5*b^9 - 858*x^38*c^6*b^7*a + 2574*x^38
*c^7*b^5*a^2 - 2145*x^38*c^8*b^3*a^3 + 715/2*x^38*c^9*b*a^4 + 143
/4*x^36*c^4*b^10 - 1287/2*x^36*c^5*b^8*a + 3003*x^36*c^6*b^6*a^2
- 4290*x^36*c^7*b^4*a^3 + 6435/4*x^36*c^8*b^2*a^4 - 143/2*x^36*c^
9*a^5 + 13*x^34*c^3*b^11 - 715/2*x^34*c^4*b^9*a + 2574*x^34*c^5*b
^7*a^2 - 6006*x^34*c^6*b^5*a^3 + 4290*x^34*c^7*b^3*a^4 - 1287/2*x
^34*c^8*b*a^5 + 13/4*x^32*c^2*b^12 - 143*x^32*c^3*b^10*a + 6435/4
*x^32*c^4*b^8*a^2 - 6006*x^32*c^5*b^6*a^3 + 15015/2*x^32*c^6*b^4*
a^4 - 2574*x^32*c^7*b^2*a^5 + 429/4*x^32*c^8*a^6 + 1/2*x^30*c*b^1
3 - 39*x^30*c^2*b^11*a + 715*x^30*c^3*b^9*a^2 - 4290*x^30*c^4*b^7
*a^3 + 9009*x^30*c^5*b^5*a^4 - 6006*x^30*c^6*b^3*a^5 + 858*x^30*c
^7*b*a^6 + 1/28*x^28*b^14 - 13/2*x^28*c*b^12*a + 429/2*x^28*c^2*b
^10*a^2 - 2145*x^28*c^3*b^8*a^3 + 15015/2*x^28*c^4*b^6*a^4 - 9009
```

```
*x^28*c^5*b^4*a^5 + 3003*x^28*c^6*b^2*a^6 - 858/7*x^28*c^7*a^7 -
1/2*x^26*b^13*a + 39*x^26*c*b^11*a^2 - 715*x^26*c^2*b^9*a^3 + 429
0*x^26*c^3*b^7*a^4 - 9009*x^26*c^4*b^5*a^5 + 6006*x^26*c^5*b^3*a^
6 - 858*x^26*c^6*b*a^7 + 13/4*x^24*b^12*a^2 - 143*x^24*c*b^10*a^3
 + 6435/4*x^24*c^2*b^8*a^4 - 6006*x^24*c^3*b^6*a^5 + 15015/2*x^24
*c^4*b^4*a^6 - 2574*x^24*c^5*b^2*a^7 + 429/4*x^24*c^6*a^8 - 13*x^
22*b^11*a^3 + 715/2*x^22*c*b^9*a^4 - 2574*x^22*c^2*b^7*a^5 + 6006
*x^22*c^3*b^5*a^6 - 4290*x^22*c^4*b^3*a^7 + 1287/2*x^22*c^5*b*a^8
 + 143/4*x^20*b^10*a^4 - 1287/2*x^20*c*b^8*a^5 + 3003*x^20*c^2*b^
6*a^6 - 4290*x^20*c^3*b^4*a^7 + 6435/4*x^20*c^4*b^2*a^8 - 143/2*x
^20*c^5*a^9 - 143/2*x^18*b^9*a^5 + 858*x^18*c*b^7*a^6 - 2574*x^18
*c^2*b^5*a^7 + 2145*x^18*c^3*b^3*a^8 - 715/2*x^18*c^4*b*a^9 + 429
/4*x^16*b^8*a^6 - 858*x^16*c*b^6*a^7 + 6435/4*x^16*c^2*b^4*a^8 -
715*x^16*c^3*b^2*a^9 + 143/4*x^16*c^4*a^10 - 858/7*x^14*b^7*a^7 +
 1287/2*x^14*c*b^5*a^8 - 715*x^14*c^2*b^3*a^9 + 143*x^14*c^3*b*a^
10 + 429/4*x^12*b^6*a^8 - 715/2*x^12*c*b^4*a^9 + 429/2*x^12*c^2*b
^2*a^10 - 13*x^12*c^3*a^11 - 143/2*x^10*b^5*a^9 + 143*x^10*c*b^3*
a^10 - 39*x^10*c^2*b*a^11 + 143/4*x^8*b^4*a^10 - 39*x^8*c*b^2*a^1
1 + 13/4*x^8*c^2*a^12 - 13*x^6*b^3*a^11 + 13/2*x^6*c*b*a^12 + 13/
4*x^4*b^2*a^12 - 1/2*x^4*c*a^13 - 1/2*x^2*b*a^13
```

Sympy [A] time = 0.947443, size = 1384, normalized size = 69.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2-a)**13,x)
```

```
[Out] -a**13*b*x**2/2 + b*c**13*x**54/2 + c**14*x**56/28 + x**52*(-a*c*
*13/2 + 13*b**2*c**12/4) + x**50*(-13*a*b*c**12/2 + 13*b**3*c**11
) + x**48*(13*a**2*c**12/4 - 39*a*b**2*c**11 + 143*b**4*c**10/4)
+ x**46*(39*a**2*b*c**11 - 143*a*b**3*c**10 + 143*b**5*c**9/2) +
x**44*(-13*a**3*c**11 + 429*a**2*b**2*c**10/2 - 715*a*b**4*c**9/2
 + 429*b**6*c**8/4) + x**42*(-143*a**3*b*c**10 + 715*a**2*b**3*c*
*9 - 1287*a*b**5*c**8/2 + 858*b**7*c**7/7) + x**40*(143*a**4*c**1
0/4 - 715*a**3*b**2*c**9 + 6435*a**2*b**4*c**8/4 - 858*a*b**6*c**
7 + 429*b**8*c**6/4) + x**38*(715*a**4*b*c**9/2 - 2145*a**3*b**3*
c**8 + 2574*a**2*b**5*c**7 - 858*a*b**7*c**6 + 143*b**9*c**5/2) +
 x**36*(-143*a**5*c**9/2 + 6435*a**4*b**2*c**8/4 - 4290*a**3*b**4
*c**7 + 3003*a**2*b**6*c**6 - 1287*a*b**8*c**5/2 + 143*b**10*c**4
/4) + x**34*(-1287*a**5*b*c**8/2 + 4290*a**4*b**3*c**7 - 6006*a**
3*b**5*c**6 + 2574*a**2*b**7*c**5 - 715*a*b**9*c**4/2 + 13*b**11*
c**3) + x**32*(429*a**6*c**8/4 - 2574*a**5*b**2*c**7 + 15015*a**4
*b**4*c**6/2 - 6006*a**3*b**6*c**5 + 6435*a**2*b**8*c**4/4 - 143*
a*b**10*c**3 + 13*b**12*c**2/4) + x**30*(858*a**6*b*c**7 - 6006*a
**5*b**3*c**6 + 9009*a**4*b**5*c**5 - 4290*a**3*b**7*c**4 + 715*a
**2*b**9*c**3 - 39*a*b**11*c**2 + b**13*c/2) + x**28*(-858*a**7*c
**7/7 + 3003*a**6*b**2*c**6 - 9009*a**5*b**4*c**5 + 15015*a**4*b*
```

```
*6*c**4/2 - 2145*a**3*b**8*c**3 + 429*a**2*b**10*c**2/2 - 13*a*b*
*12*c/2 + b**14/28) + x**26*(-858*a**7*b*c**6 + 6006*a**6*b**3*c*
*5 - 9009*a**5*b**5*c**4 + 4290*a**4*b**7*c**3 - 715*a**3*b**9*c*
*2 + 39*a**2*b**11*c - a*b**13/2) + x**24*(429*a**8*c**6/4 - 2574
*a**7*b**2*c**5 + 15015*a**6*b**4*c**4/2 - 6006*a**5*b**6*c**3 +
6435*a**4*b**8*c**2/4 - 143*a**3*b**10*c + 13*a**2*b**12/4) + x**
22*(1287*a**8*b*c**5/2 - 4290*a**7*b**3*c**4 + 6006*a**6*b**5*c**
3 - 2574*a**5*b**7*c**2 + 715*a**4*b**9*c/2 - 13*a**3*b**11) + x*
*20*(-143*a**9*c**5/2 + 6435*a**8*b**2*c**4/4 - 4290*a**7*b**4*c*
*3 + 3003*a**6*b**6*c**2 - 1287*a**5*b**8*c/2 + 143*a**4*b**10/4)
 + x**18*(-715*a**9*b*c**4/2 + 2145*a**8*b**3*c**3 - 2574*a**7*b*
*5*c**2 + 858*a**6*b**7*c - 143*a**5*b**9/2) + x**16*(143*a**10*c
**4/4 - 715*a**9*b**2*c**3 + 6435*a**8*b**4*c**2/4 - 858*a**7*b**
6*c + 429*a**6*b**8/4) + x**14*(143*a**10*b*c**3 - 715*a**9*b**3*
c**2 + 1287*a**8*b**5*c/2 - 858*a**7*b**7/7) + x**12*(-13*a**11*c
**3 + 429*a**10*b**2*c**2/2 - 715*a**9*b**4*c/2 + 429*a**8*b**6/4
) + x**10*(-39*a**11*b*c**2 + 143*a**10*b**3*c - 143*a**9*b**5/2)
 + x**8*(13*a**12*c**2/4 - 39*a**11*b**2*c + 143*a**10*b**4/4) +
x**6*(13*a**12*b*c/2 - 13*a**11*b**3) + x**4*(-a**13*c/2 + 13*a**
12*b**2/4)
```

GIAC/XCAS [A] time = 0.275438, size = 1, normalized size = 0.05

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 - a)^13*(2*c*x^2 + b)*x,x, algorithm="giac")
```

```
[Out] Done
```

3.99 $\int x^2 \left(b+2cx^3\right) \left(-a+bx^3+cx^6\right)^{13} \, dx$

Optimal. Leaf size=20

$$\frac{1}{42} \left(a-bx^3-cx^6\right)^{14}$$

```
[Out] (a - b*x^3 - c*x^6)^14/42
```

Rubi [A] time = 0.0697057, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$

$$\frac{1}{42} \left(a-bx^3-cx^6\right)^{14}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^13,x]

[Out] (a - b*x^3 - c*x^6)^14/42
```

Rubi in Sympy [A] time = 5.48683, size = 14, normalized size = 0.7

$$\frac{\left(a-bx^3-cx^6\right)^{14}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3-a)**13,x)

[Out] (a - b*x**3 - c*x**6)**14/42
```

Mathematica [B] time = 0.314556, size = 233, normalized size = 11.65

$$\frac{1}{42}x^3 \left(b+cx^3\right) \left(-14a^{13}+91a^{12}x^3 \left(b+cx^3\right)-364a^{11}x^6 \left(b+cx^3\right)^2+1001a^{10}x^9 \left(b+cx^3\right)^3 -2002a^9x^{12} \left(b+cx^3\right)^4+3003a^8x^{15} \left(b+cx^3\right)^5-3432a^7x^{18} \left(b+cx^3\right)^6 +3003a^6x^{21} \left(b+cx^3\right)^7-2002a^5x^{24} \left(b+cx^3\right)^8+1001a^4x^{27} \left(b+cx^3\right)^9 -364a^3x^{30} \left(b+cx^3\right)^{10}+91a^2x^{33} \left(b+cx^3\right)^{11}-14ax^{36} \left(b+cx^3\right)^{12}+x^{39} \left(b+cx^3\right)^{13}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^13,x]
```

```
[Out] (x^3*(b + c*x^3)*(-14*a^13 + 91*a^12*x^3*(b + c*x^3) - 364*a^11*x
^6*(b + c*x^3)^2 + 1001*a^10*x^9*(b + c*x^3)^3 - 2002*a^9*x^12*(b
 + c*x^3)^4 + 3003*a^8*x^15*(b + c*x^3)^5 - 3432*a^7*x^18*(b + c*
x^3)^6 + 3003*a^6*x^21*(b + c*x^3)^7 - 2002*a^5*x^24*(b + c*x^3)^
8 + 1001*a^4*x^27*(b + c*x^3)^9 - 364*a^3*x^30*(b + c*x^3)^10 + 9
1*a^2*x^33*(b + c*x^3)^11 - 14*a*x^36*(b + c*x^3)^12 + x^39*(b +
c*x^3)^13))/42
```

Maple [B] time = 0.004, size = 47688, normalized size = 2384.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 0.790738, size = 1677, normalized size = 83.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3 - a)^13*(2*c*x^3 + b)*x^2,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 1/6*(13*b^2*c^12 - 2*a*c^13)*x
^78 + 13/3*(2*b^3*c^11 - a*b*c^12)*x^75 + 13/6*(11*b^4*c^10 - 12*
a*b^2*c^11 + a^2*c^12)*x^72 + 13/3*(11*b^5*c^9 - 22*a*b^3*c^10 +
6*a^2*b*c^11)*x^69 + 13/6*(33*b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^
2*c^10 - 4*a^3*c^11)*x^66 + 143/21*(12*b^7*c^7 - 63*a*b^5*c^8 + 7
0*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^63 + 143/6*(3*b^8*c^6 - 24*a*b^6
*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^60 + 143/3*(
b^9*c^5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*
b*c^9)*x^57 + 143/6*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 1
20*a^3*b^4*c^7 + 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^54 + 13/3*(2*b^11*
c^3 - 55*a*b^9*c^4 + 396*a^2*b^7*c^5 - 924*a^3*b^5*c^6 + 660*a^4*
b^3*c^7 - 99*a^5*b*c^8)*x^51 + 13/6*(b^12*c^2 - 44*a*b^10*c^3 + 4
95*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*b^4*c^6 - 792*a^5*b^
```

```
2*c^7 + 33*a^6*c^8)*x^48 + 1/3*(b^13*c - 78*a*b^11*c^2 + 1430*a^2
*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^45 + 1/42*(b^14 - 182*a*b^12*c + 6006*a^2*
b^10*c^2 - 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^
4*c^5 + 84084*a^6*b^2*c^6 - 3432*a^7*c^7)*x^42 - 1/3*(a*b^13 - 78
*a^2*b^11*c + 1430*a^3*b^9*c^2 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5
*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^6)*x^39 + 13/6*(a^2*b^12
- 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c^3 + 2310*a^6*b
^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^36 - 13/3*(2*a^3*b^11 -
55*a^4*b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^
4 - 99*a^8*b*c^5)*x^33 + 143/6*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*
b^6*c^2 - 120*a^7*b^4*c^3 + 45*a^8*b^2*c^4 - 2*a^9*c^5)*x^30 - 14
3/3*(a^5*b^9 - 12*a^6*b^7*c + 36*a^7*b^5*c^2 - 30*a^8*b^3*c^3 + 5
*a^9*b*c^4)*x^27 + 143/6*(3*a^6*b^8 - 24*a^7*b^6*c + 45*a^8*b^4*c
^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^24 - 143/21*(12*a^7*b^7 - 63*a^
8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c^3)*x^21 + 13/6*(33*a^8*b^6
 - 110*a^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)*x^18 - 1/3*a^13*
b*x^3 - 13/3*(11*a^9*b^5 - 22*a^10*b^3*c + 6*a^11*b*c^2)*x^15 + 1
3/6*(11*a^10*b^4 - 12*a^11*b^2*c + a^12*c^2)*x^12 - 13/3*(2*a^11*
b^3 - a^12*b*c)*x^9 + 1/6*(13*a^12*b^2 - 2*a^13*c)*x^6
```

<u>Fricas</u> [A] time = 0.258933, size = 1, normalized size = 0.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6 + b*x^3 - a)^13*(2*c*x^3 + b)*x^2,x, algorithm="fricas")
```

```
[Out] 1/42*x^84*c^14 + 1/3*x^81*c^13*b + 13/6*x^78*c^12*b^2 - 1/3*x^78*
c^13*a + 26/3*x^75*c^11*b^3 - 13/3*x^75*c^12*b*a + 143/6*x^72*c^1
0*b^4 - 26*x^72*c^11*b^2*a + 13/6*x^72*c^12*a^2 + 143/3*x^69*c^9*
b^5 - 286/3*x^69*c^10*b^3*a + 26*x^69*c^11*b*a^2 + 143/2*x^66*c^8
*b^6 - 715/3*x^66*c^9*b^4*a + 143*x^66*c^10*b^2*a^2 - 26/3*x^66*c
^11*a^3 + 572/7*x^63*c^7*b^7 - 429*x^63*c^8*b^5*a + 1430/3*x^63*c
^9*b^3*a^2 - 286/3*x^63*c^10*b*a^3 + 143/2*x^60*c^6*b^8 - 572*x^6
0*c^7*b^6*a + 2145/2*x^60*c^8*b^4*a^2 - 1430/3*x^60*c^9*b^2*a^3 +
 143/6*x^60*c^10*a^4 + 143/3*x^57*c^5*b^9 - 572*x^57*c^6*b^7*a +
1716*x^57*c^7*b^5*a^2 - 1430*x^57*c^8*b^3*a^3 + 715/3*x^57*c^9*b*
a^4 + 143/6*x^54*c^4*b^10 - 429*x^54*c^5*b^8*a + 2002*x^54*c^6*b^
6*a^2 - 2860*x^54*c^7*b^4*a^3 + 2145/2*x^54*c^8*b^2*a^4 - 143/3*x
^54*c^9*a^5 + 26/3*x^51*c^3*b^11 - 715/3*x^51*c^4*b^9*a + 1716*x^
51*c^5*b^7*a^2 - 4004*x^51*c^6*b^5*a^3 + 2860*x^51*c^7*b^3*a^4 -
429*x^51*c^8*b*a^5 + 13/6*x^48*c^2*b^12 - 286/3*x^48*c^3*b^10*a +
 2145/2*x^48*c^4*b^8*a^2 - 4004*x^48*c^5*b^6*a^3 + 5005*x^48*c^6*
b^4*a^4 - 1716*x^48*c^7*b^2*a^5 + 143/2*x^48*c^8*a^6 + 1/3*x^45*c
*b^13 - 26*x^45*c^2*b^11*a + 1430/3*x^45*c^3*b^9*a^2 - 2860*x^45*
c^4*b^7*a^3 + 6006*x^45*c^5*b^5*a^4 - 4004*x^45*c^6*b^3*a^5 + 572
*x^45*c^7*b*a^6 + 1/42*x^42*b^14 - 13/3*x^42*c*b^12*a + 143*x^42*
c^2*b^10*a^2 - 1430*x^42*c^3*b^8*a^3 + 5005*x^42*c^4*b^6*a^4 - 60
```

```
06*x^42*c^5*b^4*a^5 + 2002*x^42*c^6*b^2*a^6 - 572/7*x^42*c^7*a^7
- 1/3*x^39*b^13*a + 26*x^39*c*b^11*a^2 - 1430/3*x^39*c^2*b^9*a^3
+ 2860*x^39*c^3*b^7*a^4 - 6006*x^39*c^4*b^5*a^5 + 4004*x^39*c^5*b
^3*a^6 - 572*x^39*c^6*b*a^7 + 13/6*x^36*b^12*a^2 - 286/3*x^36*c*b
^10*a^3 + 2145/2*x^36*c^2*b^8*a^4 - 4004*x^36*c^3*b^6*a^5 + 5005*
x^36*c^4*b^4*a^6 - 1716*x^36*c^5*b^2*a^7 + 143/2*x^36*c^6*a^8 - 2
6/3*x^33*b^11*a^3 + 715/3*x^33*c*b^9*a^4 - 1716*x^33*c^2*b^7*a^5
+ 4004*x^33*c^3*b^5*a^6 - 2860*x^33*c^4*b^3*a^7 + 429*x^33*c^5*b*
a^8 + 143/6*x^30*b^10*a^4 - 429*x^30*c*b^8*a^5 + 2002*x^30*c^2*b^
6*a^6 - 2860*x^30*c^3*b^4*a^7 + 2145/2*x^30*c^4*b^2*a^8 - 143/3*x
^30*c^5*a^9 - 143/3*x^27*b^9*a^5 + 572*x^27*c*b^7*a^6 - 1716*x^27
*c^2*b^5*a^7 + 1430*x^27*c^3*b^3*a^8 - 715/3*x^27*c^4*b*a^9 + 143
/2*x^24*b^8*a^6 - 572*x^24*c*b^6*a^7 + 2145/2*x^24*c^2*b^4*a^8 -
1430/3*x^24*c^3*b^2*a^9 + 143/6*x^24*c^4*a^10 - 572/7*x^21*b^7*a^
7 + 429*x^21*c*b^5*a^8 - 1430/3*x^21*c^2*b^3*a^9 + 286/3*x^21*c^3
*b*a^10 + 143/2*x^18*b^6*a^8 - 715/3*x^18*c*b^4*a^9 + 143*x^18*c^
2*b^2*a^10 - 26/3*x^18*c^3*a^11 - 143/3*x^15*b^5*a^9 + 286/3*x^15
*c*b^3*a^10 - 26*x^15*c^2*b*a^11 + 143/6*x^12*b^4*a^10 - 26*x^12*
c*b^2*a^11 + 13/6*x^12*c^2*a^12 - 26/3*x^9*b^3*a^11 + 13/3*x^9*c*
b*a^12 + 13/6*x^6*b^2*a^12 - 1/3*x^6*c*a^13 - 1/3*x^3*b*a^13
```

Sympy [A] time = 0.947638, size = 1394, normalized size = 69.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3-a)**13,x)
```

```
[Out] -a**13*b*x**3/3 + b*c**13*x**81/3 + c**14*x**84/42 + x**78*(-a*c*
*13/3 + 13*b**2*c**12/6) + x**75*(-13*a*b*c**12/3 + 26*b**3*c**11
/3) + x**72*(13*a**2*c**12/6 - 26*a*b**2*c**11 + 143*b**4*c**10/6
) + x**69*(26*a**2*b*c**11 - 286*a*b**3*c**10/3 + 143*b**5*c**9/3
) + x**66*(-26*a**3*c**11/3 + 143*a**2*b**2*c**10 - 715*a*b**4*c*
*9/3 + 143*b**6*c**8/2) + x**63*(-286*a**3*b*c**10/3 + 1430*a**2*
b**3*c**9/3 - 429*a*b**5*c**8 + 572*b**7*c**7/7) + x**60*(143*a**
4*c**10/6 - 1430*a**3*b**2*c**9/3 + 2145*a**2*b**4*c**8/2 - 572*a
*b**6*c**7 + 143*b**8*c**6/2) + x**57*(715*a**4*b*c**9/3 - 1430*a
**3*b**3*c**8 + 1716*a**2*b**5*c**7 - 572*a*b**7*c**6 + 143*b**9*
c**5/3) + x**54*(-143*a**5*c**9/3 + 2145*a**4*b**2*c**8/2 - 2860*
a**3*b**4*c**7 + 2002*a**2*b**6*c**6 - 429*a*b**8*c**5 + 143*b**1
0*c**4/6) + x**51*(-429*a**5*b*c**8 + 2860*a**4*b**3*c**7 - 4004*
a**3*b**5*c**6 + 1716*a**2*b**7*c**5 - 715*a*b**9*c**4/3 + 26*b**
11*c**3/3) + x**48*(143*a**6*c**8/2 - 1716*a**5*b**2*c**7 + 5005*
a**4*b**4*c**6 - 4004*a**3*b**6*c**5 + 2145*a**2*b**8*c**4/2 - 28
6*a*b**10*c**3/3 + 13*b**12*c**2/6) + x**45*(572*a**6*b*c**7 - 40
04*a**5*b**3*c**6 + 6006*a**4*b**5*c**5 - 2860*a**3*b**7*c**4 + 1
430*a**2*b**9*c**3/3 - 26*a*b**11*c**2 + b**13*c/3) + x**42*(-572
*a**7*c**7/7 + 2002*a**6*b**2*c**6 - 6006*a**5*b**4*c**5 + 5005*a
```

```
**4*b**6*c**4 - 1430*a**3*b**8*c**3 + 143*a**2*b**10*c**2 - 13*a*
b**12*c/3 + b**14/42) + x**39*(-572*a**7*b*c**6 + 4004*a**6*b**3*
c**5 - 6006*a**5*b**5*c**4 + 2860*a**4*b**7*c**3 - 1430*a**3*b**9
*c**2/3 + 26*a**2*b**11*c - a*b**13/3) + x**36*(143*a**8*c**6/2 -
 1716*a**7*b**2*c**5 + 5005*a**6*b**4*c**4 - 4004*a**5*b**6*c**3
+ 2145*a**4*b**8*c**2/2 - 286*a**3*b**10*c/3 + 13*a**2*b**12/6) +
 x**33*(429*a**8*b*c**5 - 2860*a**7*b**3*c**4 + 4004*a**6*b**5*c*
*3 - 1716*a**5*b**7*c**2 + 715*a**4*b**9*c/3 - 26*a**3*b**11/3) +
 x**30*(-143*a**9*c**5/3 + 2145*a**8*b**2*c**4/2 - 2860*a**7*b**4
*c**3 + 2002*a**6*b**6*c**2 - 429*a**5*b**8*c + 143*a**4*b**10/6)
 + x**27*(-715*a**9*b*c**4/3 + 1430*a**8*b**3*c**3 - 1716*a**7*b*
*5*c**2 + 572*a**6*b**7*c - 143*a**5*b**9/3) + x**24*(143*a**10*c
**4/6 - 1430*a**9*b**2*c**3/3 + 2145*a**8*b**4*c**2/2 - 572*a**7*
b**6*c + 143*a**6*b**8/2) + x**21*(286*a**10*b*c**3/3 - 1430*a**9
*b**3*c**2/3 + 429*a**8*b**5*c - 572*a**7*b**7/7) + x**18*(-26*a*
*11*c**3/3 + 143*a**10*b**2*c**2 - 715*a**9*b**4*c/3 + 143*a**8*b
**6/2) + x**15*(-26*a**11*b*c**2 + 286*a**10*b**3*c/3 - 143*a**9*
b**5/3) + x**12*(13*a**12*c**2/6 - 26*a**11*b**2*c + 143*a**10*b*
*4/6) + x**9*(13*a**12*b*c/3 - 26*a**11*b**3/3) + x**6*(-a**13*c/
3 + 13*a**12*b**2/6)
```

GIAC/XCAS [A] time = 0.274326, size = 1, normalized size = 0.05

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6 + b*x^3 - a)^13*(2*c*x^3 + b)*x^2,x, algorithm="giac")

[Out] Done
```

3.100 $\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx$

Optimal. Leaf size=25

$$\frac{\left(a-bx^n-cx^{2n}\right)^{14}}{14n}$$

```
[Out] (a - b*x^n - c*x^(2*n))^14/(14*n)
```

Rubi [A] time = 0.0722563, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\left(a-bx^n-cx^{2n}\right)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^13,x]

[Out] (a - b*x^n - c*x^(2*n))^14/(14*n)
```

Rubi in Sympy [A] time = 15.756, size = 17, normalized size = 0.68

$$\frac{\left(a-bx^n-cx^{2n}\right)^{14}}{14n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**(-1+n)*(b+2*c*x**n)*(-a+b*x**n+c*x**(2*n))**13,x)

[Out] (a - b*x**n - c*x**(2*n))**14/(14*n)
```

Mathematica [B] time = 0.660647, size = 260, normalized size = 10.4

$$\frac{x^n(b+cx^n)\left(-14a^{13}+91a^{12}x^n(b+cx^n)-364a^{11}x^{2n}(b+cx^n)^2+1001a^{10}x^{3n}(b+cx^n)^3-2002a^9x^{4n}(b+cx^n)^4+3003a^8x\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (x^n*(b + c*x^n)*(-14*a^13 + 91*a^12*x^n*(b + c*x^n) - 364*a^11*x
^(2*n)*(b + c*x^n)^2 + 1001*a^10*x^(3*n)*(b + c*x^n)^3 - 2002*a^9
*x^(4*n)*(b + c*x^n)^4 + 3003*a^8*x^(5*n)*(b + c*x^n)^5 - 3432*a^
7*x^(6*n)*(b + c*x^n)^6 + 3003*a^6*x^(7*n)*(b + c*x^n)^7 - 2002*a
^5*x^(8*n)*(b + c*x^n)^8 + 1001*a^4*x^(9*n)*(b + c*x^n)^9 - 364*a
^3*x^(10*n)*(b + c*x^n)^10 + 91*a^2*x^(11*n)*(b + c*x^n)^11 - 14*
a*x^(12*n)*(b + c*x^n)^12 + x^(13*n)*(b + c*x^n)^13))/(14*n)
```

Maple [B] time = 0.097, size = 2046, normalized size = 81.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x)
```

```
[Out] -143*b^5*a^9/n*(x^n)^5+b^13*c/n*(x^n)^15-26*b^11*a^3/n*(x^n)^11-2
6*a^11*b^3/n*(x^n)^3-143*a^5*b^9/n*(x^n)^9+143*b^5*c^9/n*(x^n)^23
+26*c^11*b^3/n*(x^n)^25-a*b^13/n*(x^n)^13-b*a^13/n*x^n+b*c^13/n*(
x^n)^27-143*c^9/n*(x^n)^18*a^5+143/2*c^4/n*(x^n)^18*b^10-26*c^11/
n*(x^n)^22*a^3+429/2*c^8/n*(x^n)^22*b^6+26*b^11*c^3/n*(x^n)^17-c^
13/n*(x^n)^26*a+13/2*c^12/n*(x^n)^26*b^2-1716/7*b^7*a^7/n*(x^n)^7
+143*c^5*b^9/n*(x^n)^19+1716/7*b^7*c^7/n*(x^n)^21+13/2*c^12/n*(x^
n)^24*a^2+143/2*c^10/n*(x^n)^24*b^4-143*a^9/n*(x^n)^10*c^5+143/2*
a^4/n*(x^n)^10*b^10-a^13/n*(x^n)^2*c+13/2*a^12/n*(x^n)^2*b^2+6006
/n*(x^n)^14*a^6*b^2*c^6-18018/n*(x^n)^14*a^5*b^4*c^5+15015/n*(x^n
)^14*b^6*a^4*c^4-4290/n*(x^n)^14*a^3*b^8*c^3+429/n*(x^n)^14*a^2*b
^10*c^2-13/n*(x^n)^14*a*b^12*c+1/14*c^14/n*(x^n)^28+13*a^12*b/n*(
x^n)^3*c-1287*a^5/n*(x^n)^10*b^8*c+286*b*a^10/n*(x^n)^7*c^3-1430*
b^3*a^9/n*(x^n)^7*c^2+1287*b^5*a^8/n*(x^n)^7*c+715*c^9*b/n*(x^n)^
19*a^4-4290*c^8*b^3/n*(x^n)^19*a^3+5148*c^7*b^5/n*(x^n)^19*a^2-17
16*c^6*b^7/n*(x^n)^19*a-78*c^11/n*(x^n)^24*a*b^2+6435/2*a^8/n*(x^
n)^10*b^2*c^4-8580*a^7/n*(x^n)^10*b^4*c^3+6006*a^6/n*(x^n)^10*b^6
*c^2+143/2*a^10/n*(x^n)^8*c^4+429/2*a^6/n*(x^n)^8*b^8-26*a^11/n*(
x^n)^6*c^3+429/2*a^8/n*(x^n)^6*b^6+143/2*c^10/n*(x^n)^20*a^4+429/
2*c^6/n*(x^n)^20*b^8+429/2*c^8/n*(x^n)^16*a^6+13/2*c^2/n*(x^n)^16
*b^12+429/2*a^8/n*(x^n)^12*c^6+13/2*a^2/n*(x^n)^12*b^12+13/2*a^12
/n*(x^n)^4*c^2+143/2*a^10/n*(x^n)^4*b^4-1716/7/n*(x^n)^14*a^7*c^7
-1430*c^9/n*(x^n)^20*a^3*b^2+6435/2*c^8/n*(x^n)^20*a^2*b^4-1716*c
^7/n*(x^n)^20*a*b^6-5148*c^7/n*(x^n)^16*a^5*b^2+15015*c^6/n*(x^n)
^16*a^4*b^4-12012*c^5/n*(x^n)^16*a^3*b^6+6435/2*c^4/n*(x^n)^16*a^
2*b^8-286*c^3/n*(x^n)^16*a*b^10+6435/2*c^8/n*(x^n)^18*b^2*a^4-858
0*c^7/n*(x^n)^18*b^4*a^3+6006*c^6/n*(x^n)^18*b^6*a^2-1287*c^5/n*(
x^n)^18*a*b^8+429*c^10/n*(x^n)^22*a^2*b^2-715*c^9/n*(x^n)^22*a*b^
4+1/14/n*(x^n)^14*b^14-13*c^12*b/n*(x^n)^25*a-1716*a^7*b/n*(x^n)^
13*c^6+12012*a^6*b^3/n*(x^n)^13*c^5-18018*a^5*b^5/n*(x^n)^13*c^4+
8580*a^4*b^7/n*(x^n)^13*c^3-1430*a^3*b^9/n*(x^n)^13*c^2+78*a^2*b^
```

```
11/n*(x^n)^13*c-1430*a^9/n*(x^n)^8*b^2*c^3+6435/2*a^8/n*(x^n)^8*b
^4*c^2-1716*a^7/n*(x^n)^8*b^6*c+429*a^10/n*(x^n)^6*b^2*c^2-715*a^
9/n*(x^n)^6*b^4*c-12012*b^5*c^6/n*(x^n)^17*a^3+5148*b^7*c^5/n*(x^
n)^17*a^2-715*b^9*c^4/n*(x^n)^17*a-5148*a^7/n*(x^n)^12*b^2*c^5+15
015*a^6/n*(x^n)^12*b^4*c^4-12012*a^5/n*(x^n)^12*b^6*c^3+6435/2*a^
4/n*(x^n)^12*b^8*c^2-286*a^3/n*(x^n)^12*b^10*c-78*a^11/n*(x^n)^4*
b^2*c-78*b*a^11/n*(x^n)^5*c^2+286*b^3*a^10/n*(x^n)^5*c+1716*b*c^7
/n*(x^n)^15*a^6-12012*b^3*c^6/n*(x^n)^15*a^5+18018*b^5*c^5/n*(x^n
)^15*a^4-8580*b^7*c^4/n*(x^n)^15*a^3+1430*b^9*c^3/n*(x^n)^15*a^2-
78*b^11*c^2/n*(x^n)^15*a+1287*b*a^8/n*(x^n)^11*c^5-8580*b^3*a^7/n
*(x^n)^11*c^4+12012*b^5*a^6/n*(x^n)^11*c^3-5148*b^7*a^5/n*(x^n)^1
1*c^2+715*b^9*a^4/n*(x^n)^11*c-715*a^9*b/n*(x^n)^9*c^4+4290*a^8*b
^3/n*(x^n)^9*c^3-5148*a^7*b^5/n*(x^n)^9*c^2+1716*a^6*b^7/n*(x^n)^
9*c+78*b*c^11/n*(x^n)^23*a^2-286*b^3*c^10/n*(x^n)^23*a-1287*b*c^8
/n*(x^n)^17*a^5+8580*b^3*c^7/n*(x^n)^17*a^4-286*b*c^10/n*(x^n)^21
*a^3+1430*b^3*c^9/n*(x^n)^21*a^2-1287*b^5*c^8/n*(x^n)^21*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + b*x^n - a)^13*(2*c*x^n + b)*x^(n - 1),x, algorithm="maxima

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.328511, size = 1754, normalized size = 70.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + b*x^n - a)^13*(2*c*x^n + b)*x^(n - 1),x, algorithm="fricas

[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) - 14*a^13*b*x^n + 7*(13*
b^2*c^12 - 2*a*c^13)*x^(26*n) + 182*(2*b^3*c^11 - a*b*c^12)*x^(25
*n) + 91*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c^12)*x^(24*n) + 182*
(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^(23*n) + 91*(33*b^6
*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 - 4*a^3*c^11)*x^(22*n) + 2
86*(12*b^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 14*a^3*b*c^10)*x
^(21*n) + 1001*(3*b^8*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 - 20*a^
3*b^2*c^9 + a^4*c^10)*x^(20*n) + 2002*(b^9*c^5 - 12*a*b^7*c^6 + 3
6*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9)*x^(19*n) + 1001*(b^
10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7 + 45*a^4
```

```
*b^2*c^8 - 2*a^5*c^9)*x^(18*n) + 182*(2*b^11*c^3 - 55*a*b^9*c^4 +
 396*a^2*b^7*c^5 - 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c
^8)*x^(17*n) + 91*(b^12*c^2 - 44*a*b^10*c^3 + 495*a^2*b^8*c^4 - 1
848*a^3*b^6*c^5 + 2310*a^4*b^4*c^6 - 792*a^5*b^2*c^7 + 33*a^6*c^8
)*x^(16*n) + 14*(b^13*c - 78*a*b^11*c^2 + 1430*a^2*b^9*c^3 - 8580
*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c^6 + 1716*a^6*b
*c^7)*x^(15*n) + (b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 - 60060
*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^
6*b^2*c^6 - 3432*a^7*c^7)*x^(14*n) - 14*(a*b^13 - 78*a^2*b^11*c +
 1430*a^3*b^9*c^2 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*
a^6*b^3*c^5 + 1716*a^7*b*c^6)*x^(13*n) + 91*(a^2*b^12 - 44*a^3*b^
10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c^3 + 2310*a^6*b^4*c^4 - 79
2*a^7*b^2*c^5 + 33*a^8*c^6)*x^(12*n) - 182*(2*a^3*b^11 - 55*a^4*b
^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 - 99*a
^8*b*c^5)*x^(11*n) + 1001*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*b^6*c
^2 - 120*a^7*b^4*c^3 + 45*a^8*b^2*c^4 - 2*a^9*c^5)*x^(10*n) - 200
2*(a^5*b^9 - 12*a^6*b^7*c + 36*a^7*b^5*c^2 - 30*a^8*b^3*c^3 + 5*a
^9*b*c^4)*x^(9*n) + 1001*(3*a^6*b^8 - 24*a^7*b^6*c + 45*a^8*b^4*c
^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^(8*n) - 286*(12*a^7*b^7 - 63*a^
8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c^3)*x^(7*n) + 91*(33*a^8*b^
6 - 110*a^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)*x^(6*n) - 182*(
11*a^9*b^5 - 22*a^10*b^3*c + 6*a^11*b*c^2)*x^(5*n) + 91*(11*a^10*
b^4 - 12*a^11*b^2*c + a^12*c^2)*x^(4*n) - 182*(2*a^11*b^3 - a^12*
b*c)*x^(3*n) + 7*(13*a^12*b^2 - 2*a^13*c)*x^(2*n))/n
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+n)*(b+2*c*x**n)*(-a+b*x**n+c*x**(2*n))**13,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.327072, size = 1, normalized size = 0.04

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + b*x^n - a)^13*(2*c*x^n + b)*x^(n - 1),x, algorithm="giac")

[Out] Done
```

3.101 $\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx$

Optimal. Leaf size=15

$$\frac{1}{14}\left(bx+cx^2\right)^{14}$$

```
[Out] (b*x + c*x^2)^14/14
```

Rubi [A] time = 0.0132339, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$\frac{1}{14}\left(bx+cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

```
[Out] (b*x + c*x^2)^14/14
```

Rubi in Sympy [A] time = 3.18902, size = 10, normalized size = 0.67

$$\frac{\left(bx+cx^2\right)^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*c*x+b)*(c*x**2+b*x)**13,x)
```

```
[Out] (b*x + c*x**2)**14/14
```

Mathematica [B] time = 0.00929359, size = 172, normalized size = 11.47

$$\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13}{2}b^{12}c^2x^{16}+26b^{11}c^3x^{17}+\frac{143}{2}b^{10}c^4x^{18}+143b^9c^5x^{19}+\frac{429}{2}b^8c^6x^{20}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^6c^8x^{22}+143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

```
[Out] (b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3
*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x
^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c
^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12
*x^26)/2 + b*c^13*x^27 + (c^14*x^28)/14
```

Maple [B] time = 0.004, size = 155, normalized size = 10.3

$$\frac{c^{14}x^{28}}{14}+bc^{13}x^{27}+\frac{13\,b^{2}c^{12}x^{26}}{2}+26\,b^{3}c^{11}x^{25}+\frac{143\,b^{4}c^{10}x^{24}}{2}+143\,b^{5}c^{9}x^{23}+\frac{429\,b^{6}c^{8}x^{22}}{2}+\frac{1716\,b^{7}c^{7}x^{21}}{7}+\frac{429\,b^{8}c^{6}x^{20}}{2}+143\,b^{9}c^{5}x^{19}+\frac{143\,b^{10}c^{4}x^{18}}{2}+26\,b^{11}c^{3}x^{17}+\frac{13\,b^{12}c^{2}x^{16}}{2}+b^{13}cx^{15}+\frac{b^{14}x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*c*x+b)*(c*x^2+b*x)^13,x)
```

```
[Out] 1/14*c^14*x^28+b*c^13*x^27+13/2*b^2*c^12*x^26+26*b^3*c^11*x^25+14
3/2*b^4*c^10*x^24+143*b^5*c^9*x^23+429/2*b^6*c^8*x^22+1716/7*b^7*
c^7*x^21+429/2*b^8*c^6*x^20+143*b^9*c^5*x^19+143/2*b^10*c^4*x^18+
26*b^11*c^3*x^17+13/2*b^12*c^2*x^16+b^13*c*x^15+1/14*b^14*x^14
```

Maxima [A] time = 0.771138, size = 18, normalized size = 1.2

$$\frac{1}{14}\left(cx^{2}+bx\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^13*(2*c*x + b),x, algorithm="maxima")
```

```
[Out] 1/14*(c*x^2 + b*x)^14
```

Fricas [A] time = 0.255404, size = 1, normalized size = 0.07

$$\frac{1}{14}x^{28}c^{14}+x^{27}c^{13}b+\frac{13}{2}x^{26}c^{12}b^{2}+26x^{25}c^{11}b^{3}+\frac{143}{2}x^{24}c^{10}b^{4}+143x^{23}c^{9}b^{5}+\frac{429}{2}x^{22}c^{8}b^{6}+\frac{1716}{7}x^{21}c^{7}b^{7}+\frac{429}{2}x^{20}c^{6}b^{8}+143x^{19}c^{5}b^{9}+\frac{143}{2}x^{18}c^{4}b^{10}+26x^{17}c^{3}b^{11}+\frac{13}{2}x^{16}c^{2}b^{12}+x^{15}cb^{13}+\frac{1}{14}x^{14}b^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^13*(2*c*x + b),x, algorithm="fricas")
```

```
[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 + 26*x^25*c^11*
b^3 + 143/2*x^24*c^10*b^4 + 143*x^23*c^9*b^5 + 429/2*x^22*c^8*b^6
 + 1716/7*x^21*c^7*b^7 + 429/2*x^20*c^6*b^8 + 143*x^19*c^5*b^9 +
143/2*x^18*c^4*b^10 + 26*x^17*c^3*b^11 + 13/2*x^16*c^2*b^12 + x^1
5*c*b^13 + 1/14*x^14*b^14
```

Sympy [A] time = 0.267342, size = 175, normalized size = 11.67

$$\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13b^{12}c^2x^{16}}{2}+26b^{11}c^3x^{17}+\frac{143b^{10}c^4x^{18}}{2}+143b^9c^5x^{19}+\frac{429b^8c^6x^{20}}{2}+\frac{1716b^7c^7x^{21}}{7}+\frac{429b^6c^8x^{22}}{2}+143b^5c^9x^{23}+\frac{143b^4c^{10}x^{24}}{2}+26b^3c^{11}x^{25}+\frac{13b^2c^{12}x^{26}}{2}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x+b)*(c*x**2+b*x)**13,x)
```

```
[Out] b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11
*c**3*x**17 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*
b**8*c**6*x**20/2 + 1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/
2 + 143*b**5*c**9*x**23 + 143*b**4*c**10*x**24/2 + 26*b**3*c**11*
x**25 + 13*b**2*c**12*x**26/2 + b*c**13*x**27 + c**14*x**28/14
```

GIAC/XCAS [A] time = 0.262101, size = 208, normalized size = 13.87

$$\frac{1}{14}c^{14}x^{28}+bc^{13}x^{27}+\frac{13}{2}b^2c^{12}x^{26}+26\,b^3c^{11}x^{25}+\frac{143}{2}b^4c^{10}x^{24}+143\,b^5c^9x^{23}+\frac{429}{2}b^6c^8x^{22}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^8c^6x^{20}+143\,b^9c^5x^{19}+\frac{143}{2}b^{10}c^4x^{18}+26\,b^{11}c^3x^{17}+\frac{13}{2}b^{12}c^2x^{16}+b^{13}cx^{15}+\frac{1}{14}b^{14}x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^13*(2*c*x + b),x, algorithm="giac")
```

```
[Out] 1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + 26*b^3*c^11*x
^25 + 143/2*b^4*c^10*x^24 + 143*b^5*c^9*x^23 + 429/2*b^6*c^8*x^22
 + 1716/7*b^7*c^7*x^21 + 429/2*b^8*c^6*x^20 + 143*b^9*c^5*x^19 +
```

143/2*b^10*c^4*x^18 + 26*b^11*c^3*x^17 + 13/2*b^12*c^2*x^16 + b^1
3*c*x^15 + 1/14*b^14*x^14

3.102 $\int x \left(b+2 c x^2\right) \left(b x^2+c x^4\right)^{13} \, dx$

Optimal. Leaf size=17

$$\frac{1}{28} \left(b x^2+c x^4\right)^{14}$$

```
[Out] (b*x^2 + c*x^4)^14/28
```

Rubi [A] time = 0.0196502, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{1}{28} \left(b x^2+c x^4\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13,x]

[Out] (b*x^2 + c*x^4)^14/28
```

Rubi in Sympy [A] time = 13.3007, size = 12, normalized size = 0.71

$$\frac{x^{28} \left(b + c x^{2}\right)^{14}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2)**13,x)

[Out] x**28*(b + c*x**2)**14/28
```

Mathematica [B] time = 0.00917135, size = 182, normalized size = 10.71

$$\frac{b^{14} x^{28}}{28}+\frac{1}{2} b^{13} c x^{30}+\frac{13}{4} b^{12} c^2 x^{32}+13 b^{11} c^3 x^{34}+\frac{143}{4} b^{10} c^4 x^{36}+\frac{143}{2} b^9 c^5 x^{38}+\frac{429}{4} b^8 c^6 x^{40}+\frac{858}{7} b^7 c^7 x^{42}+\frac{429}{4} b^6 c^8 x^{44}+\frac{143}{2} b^5 c^9 x^{46}+\frac{143}{4} b^4 c^{10} x^{48}+13 b^3 c^{11} x^{50}+\frac{13}{4} b^2 c^{12} x^{52}+\frac{1}{2} b c^{13} x^{54}+\frac{c^{14} x^{56}}{28}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13,x]
```

```
[Out] (b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11
*c^3*x^34 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b
^8*c^6*x^40)/4 + (858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (1
43*b^5*c^9*x^46)/2 + (143*b^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (
13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/2 + (c^14*x^56)/28
```

Maple [B] time = 0.004, size = 157, normalized size = 9.2

$$\frac{c^{14}x^{56}}{28}+\frac{bc^{13}x^{54}}{2}+\frac{13\,b^{2}c^{12}x^{52}}{4}+13\,b^{3}c^{11}x^{50}+\frac{143\,b^{4}c^{10}x^{48}}{4}+\frac{143\,b^{5}c^{9}x^{46}}{2}+\frac{429\,b^{6}c^{8}x^{44}}{4}+\frac{858\,b^{7}c^{7}x^{42}}{7}+\frac{429\,b^{8}c^{6}x^{40}}{4}+\frac{143\,b^{9}c^{5}x^{38}}{2}+\frac{143\,b^{10}c^{4}x^{36}}{4}+13\,b^{11}c^{3}x^{34}+\frac{13\,b^{12}c^{2}x^{32}}{4}+\frac{b^{13}cx^{30}}{2}+\frac{b^{14}x^{28}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x)
```

```
[Out] 1/28*c^14*x^56+1/2*b*c^13*x^54+13/4*b^2*c^12*x^52+13*b^3*c^11*x^5
0+143/4*b^4*c^10*x^48+143/2*b^5*c^9*x^46+429/4*b^6*c^8*x^44+858/7
*b^7*c^7*x^42+429/4*b^8*c^6*x^40+143/2*b^9*c^5*x^38+143/4*b^10*c^
4*x^36+13*b^11*c^3*x^34+13/4*b^12*c^2*x^32+1/2*b^13*c*x^30+1/28*b
^14*x^28
```

Maxima [A] time = 0.76493, size = 211, normalized size = 12.41

$$\frac{1}{28}\,c^{14}x^{56}+\frac{1}{2}\,bc^{13}x^{54}+\frac{13}{4}\,b^{2}c^{12}x^{52}+13\,b^{3}c^{11}x^{50}+\frac{143}{4}\,b^{4}c^{10}x^{48}+\frac{143}{2}\,b^{5}c^{9}x^{46}+\frac{429}{4}\,b^{6}c^{8}x^{44}+\frac{858}{7}\,b^{7}c^{7}x^{42}+\frac{429}{4}\,b^{8}c^{6}x^{40}+\frac{143}{2}\,b^{9}c^{5}x^{38}+\frac{143}{4}\,b^{10}c^{4}x^{36}+13\,b^{11}c^{3}x^{34}+\frac{13}{4}\,b^{12}c^{2}x^{32}+\frac{1}{2}\,b^{13}cx^{30}+\frac{1}{28}\,b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^13*(2*c*x^2 + b)*x,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^
11*x^50 + 143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^
8*x^44 + 858/7*b^7*c^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*
x^38 + 143/4*b^10*c^4*x^36 + 13*b^11*c^3*x^34 + 13/4*b^12*c^2*x^3
2 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Fricas [A] time = 0.253056, size = 1, normalized size = 0.06

$$\frac{1}{28}x^{56}c^{14}+\frac{1}{2}x^{54}c^{13}b+\frac{13}{4}x^{52}c^{12}b^2+13x^{50}c^{11}b^3+\frac{143}{4}x^{48}c^{10}b^4+\frac{143}{2}x^{46}c^9b^5+\frac{429}{4}x^{44}c^8b^6+\frac{858}{7}x^{42}c^7b^7+\frac{429}{4}x^{40}c^6b^8+\frac{143}{2}x^{38}c^5b^9+\frac{143}{4}x^{36}c^4b^{10}+13x^{34}c^3b^{11}+\frac{13}{4}x^{32}c^2b^{12}+\frac{1}{2}x^{30}cb^{13}+\frac{1}{28}x^{28}b^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^13*(2*c*x^2 + b)*x,x, algorithm="fricas")
```

```
[Out] 1/28*x^56*c^14 + 1/2*x^54*c^13*b + 13/4*x^52*c^12*b^2 + 13*x^50*c
^11*b^3 + 143/4*x^48*c^10*b^4 + 143/2*x^46*c^9*b^5 + 429/4*x^44*c
^8*b^6 + 858/7*x^42*c^7*b^7 + 429/4*x^40*c^6*b^8 + 143/2*x^38*c^5
*b^9 + 143/4*x^36*c^4*b^10 + 13*x^34*c^3*b^11 + 13/4*x^32*c^2*b^1
2 + 1/2*x^30*c*b^13 + 1/28*x^28*b^14
```

Sympy [A] time = 0.277862, size = 182, normalized size = 10.71

$$\frac{b^{14}x^{28}}{28}+\frac{b^{13}cx^{30}}{2}+\frac{13b^{12}c^2x^{32}}{4}+13b^{11}c^3x^{34}+\frac{143b^{10}c^4x^{36}}{4}+\frac{143b^9c^5x^{38}}{2}+\frac{429b^8c^6x^{40}}{4}+\frac{858b^7c^7x^{42}}{7}+\frac{429b^6c^8x^{44}}{4}+\frac{143b^5c^9x^{46}}{2}+\frac{143b^4c^{10}x^{48}}{4}+13b^3c^{11}x^{50}+\frac{13b^2c^{12}x^{52}}{4}+\frac{bc^{13}x^{54}}{2}+\frac{c^{14}x^{56}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2)**13,x)
```

```
[Out] b**14*x**28/28 + b**13*c*x**30/2 + 13*b**12*c**2*x**32/4 + 13*b**
11*c**3*x**34 + 143*b**10*c**4*x**36/4 + 143*b**9*c**5*x**38/2 +
429*b**8*c**6*x**40/4 + 858*b**7*c**7*x**42/7 + 429*b**6*c**8*x**
44/4 + 143*b**5*c**9*x**46/2 + 143*b**4*c**10*x**48/4 + 13*b**3*c
**11*x**50 + 13*b**2*c**12*x**52/4 + b*c**13*x**54/2 + c**14*x**5
6/28
```

GIAC/XCAS [A] time = 0.263396, size = 211, normalized size = 12.41

$$\frac{1}{28}\,c^{14}x^{56}+\frac{1}{2}\,bc^{13}x^{54}+\frac{13}{4}\,b^2c^{12}x^{52}+13\,b^3c^{11}x^{50}+\frac{143}{4}\,b^4c^{10}x^{48}+\frac{143}{2}\,b^5c^9x^{46}+\frac{429}{4}\,b^6c^8x^{44}+\frac{858}{7}\,b^7c^7x^{42}+\frac{429}{4}\,b^8c^6x^{40}+\frac{143}{2}\,b^9c^5x^{38}+\frac{143}{4}\,b^{10}c^4x^{36}+13\,b^{11}c^3x^{34}+\frac{13}{4}\,b^{12}c^2x^{32}+\frac{1}{2}\,b^{13}cx^{30}+\frac{1}{28}\,b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^13*(2*c*x^2 + b)*x,x, algorithm="giac")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^
11*x^50 + 143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^
8*x^44 + 858/7*b^7*c^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*
x^38 + 143/4*b^10*c^4*x^36 + 13*b^11*c^3*x^34 + 13/4*b^12*c^2*x^3
2 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

3.103 $\int x^2 \left(b+2cx^3\right) \left(bx^3+cx^6\right)^{13} \, dx$

Optimal. Leaf size=17

$$\frac{1}{42} \left(bx^3+cx^6\right)^{14}$$

```
[Out] (b*x^3 + c*x^6)^14/42
```

Rubi [A] time = 0.0212501, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\frac{1}{42} \left(bx^3+cx^6\right)^{14}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^13,x]
```

```
[Out] (b*x^3 + c*x^6)^14/42
```

Rubi in Sympy [A] time = 12.6838, size = 12, normalized size = 0.71

$$\frac{x^{42} \left(b+cx^3\right)^{14}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3)**13,x)
```

```
[Out] x**42*(b + c*x**3)**14/42
```

Mathematica [B] time = 0.00987148, size = 186, normalized size = 10.94

$$\frac{b^{14}x^{42}}{42}+\frac{1}{3}b^{13}cx^{45}+\frac{13}{6}b^{12}c^2x^{48}+\frac{26}{3}b^{11}c^3x^{51}+\frac{143}{6}b^{10}c^4x^{54}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{2}b^8c^6x^{60}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^6c^8x^{66}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{6}b^4c^{10}x^{72}+\frac{26}{3}b^3c^{11}x^{75}+\frac{13}{6}b^2c^{12}x^{78}+\frac{1}{3}bc^{13}x^{81}+\frac{c^{14}x^{84}}{42}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^13,x]
```

```
[Out] (b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^1
1*c^3*x^51)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (1
43*b^8*c^6*x^60)/2 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2
+ (143*b^5*c^9*x^69)/3 + (143*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^7
5)/3 + (13*b^2*c^12*x^78)/6 + (b*c^13*x^81)/3 + (c^14*x^84)/42
```

Maple [B] time = 0.004, size = 157, normalized size = 9.2

$$\frac{c^{14}x^{84}}{42}+\frac{bc^{13}x^{81}}{3}+\frac{13\,b^{2}c^{12}x^{78}}{6}+\frac{26\,b^{3}c^{11}x^{75}}{3}+\frac{143\,b^{4}c^{10}x^{72}}{6}+\frac{143\,b^{5}c^{9}x^{69}}{3}+\frac{143\,b^{6}c^{8}x^{66}}{2}+\frac{572\,b^{7}c^{7}x^{63}}{7}+\frac{143\,b^{8}c^{6}x^{60}}{2}+\frac{143\,b^{9}c^{5}x^{57}}{3}+\frac{143\,b^{10}c^{4}x^{54}}{6}+\frac{26\,b^{11}c^{3}x^{51}}{3}+\frac{13\,b^{12}c^{2}x^{48}}{6}+\frac{b^{13}cx^{45}}{3}+\frac{b^{14}x^{42}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x)
```

```
[Out] 1/42*c^14*x^84+1/3*b*c^13*x^81+13/6*b^2*c^12*x^78+26/3*b^3*c^11*x
^75+143/6*b^4*c^10*x^72+143/3*b^5*c^9*x^69+143/2*b^6*c^8*x^66+572
/7*b^7*c^7*x^63+143/2*b^8*c^6*x^60+143/3*b^9*c^5*x^57+143/6*b^10*
c^4*x^54+26/3*b^11*c^3*x^51+13/6*b^12*c^2*x^48+1/3*b^13*c*x^45+1/
42*b^14*x^42
```

Maxima [A] time = 0.771469, size = 211, normalized size = 12.41

$$\frac{1}{42}\,c^{14}x^{84}+\frac{1}{3}\,bc^{13}x^{81}+\frac{13}{6}\,b^{2}c^{12}x^{78}+\frac{26}{3}\,b^{3}c^{11}x^{75}+\frac{143}{6}\,b^{4}c^{10}x^{72}+\frac{143}{3}\,b^{5}c^{9}x^{69}+\frac{143}{2}\,b^{6}c^{8}x^{66}+\frac{572}{7}\,b^{7}c^{7}x^{63}+\frac{143}{2}\,b^{8}c^{6}x^{60}+\frac{143}{3}\,b^{9}c^{5}x^{57}+\frac{143}{6}\,b^{10}c^{4}x^{54}+\frac{26}{3}\,b^{11}c^{3}x^{51}+\frac{13}{6}\,b^{12}c^{2}x^{48}+\frac{1}{3}\,b^{13}cx^{45}+\frac{1}{42}\,b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6 + b*x^3)^13*(2*c*x^3 + b)*x^2,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*
c^11*x^75 + 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*
c^8*x^66 + 572/7*b^7*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^
5*x^57 + 143/6*b^10*c^4*x^54 + 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2
*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

Fricas [A] time = 0.269495, size = 1, normalized size = 0.06

$$\frac{1}{42}x^{84}c^{14}+\frac{1}{3}x^{81}c^{13}b+\frac{13}{6}x^{78}c^{12}b^{2}+\frac{26}{3}x^{75}c^{11}b^{3}+\frac{143}{6}x^{72}c^{10}b^{4}+\frac{143}{3}x^{69}c^{9}b^{5}+\frac{143}{2}x^{66}c^{8}b^{6}+\frac{572}{7}x^{63}c^{7}b^{7}+\frac{143}{2}x^{60}c^{6}b^{8}+\frac{143}{3}x^{57}c^{5}b^{9}+\frac{143}{6}x^{54}c^{4}b^{10}+\frac{26}{3}x^{51}c^{3}b^{11}+\frac{13}{6}x^{48}c^{2}b^{12}+\frac{1}{3}x^{45}cb^{13}+\frac{1}{42}x^{42}b^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6 + b*x^3)^13*(2*c*x^3 + b)*x^2,x, algorithm="fricas")
```

```
[Out] 1/42*x^84*c^14 + 1/3*x^81*c^13*b + 13/6*x^78*c^12*b^2 + 26/3*x^75
*c^11*b^3 + 143/6*x^72*c^10*b^4 + 143/3*x^69*c^9*b^5 + 143/2*x^66
*c^8*b^6 + 572/7*x^63*c^7*b^7 + 143/2*x^60*c^6*b^8 + 143/3*x^57*c
^5*b^9 + 143/6*x^54*c^4*b^10 + 26/3*x^51*c^3*b^11 + 13/6*x^48*c^2
*b^12 + 1/3*x^45*c*b^13 + 1/42*x^42*b^14
```

Sympy [A] time = 0.283172, size = 185, normalized size = 10.88

$$\frac{b^{14}x^{42}}{42}+\frac{b^{13}cx^{45}}{3}+\frac{13b^{12}c^{2}x^{48}}{6}+\frac{26b^{11}c^{3}x^{51}}{3}+\frac{143b^{10}c^{4}x^{54}}{6}+\frac{143b^{9}c^{5}x^{57}}{3}+\frac{143b^{8}c^{6}x^{60}}{2}+\frac{572b^{7}c^{7}x^{63}}{7}+\frac{143b^{6}c^{8}x^{66}}{2}+\frac{143b^{5}c^{9}x^{69}}{3}+\frac{143b^{4}c^{10}x^{72}}{6}+\frac{26b^{3}c^{11}x^{75}}{3}+\frac{13b^{2}c^{12}x^{78}}{6}+\frac{bc^{13}x^{81}}{3}+\frac{c^{14}x^{84}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3)**13,x)
```

```
[Out] b**14*x**42/42 + b**13*c*x**45/3 + 13*b**12*c**2*x**48/6 + 26*b**
11*c**3*x**51/3 + 143*b**10*c**4*x**54/6 + 143*b**9*c**5*x**57/3
+ 143*b**8*c**6*x**60/2 + 572*b**7*c**7*x**63/7 + 143*b**6*c**8*x
**66/2 + 143*b**5*c**9*x**69/3 + 143*b**4*c**10*x**72/6 + 26*b**3
*c**11*x**75/3 + 13*b**2*c**12*x**78/6 + b*c**13*x**81/3 + c**14*
x**84/42
```

GIAC/XCAS [A] time = 0.269077, size = 211, normalized size = 12.41

$$\frac{1}{42}\,c^{14}x^{84}+\frac{1}{3}\,bc^{13}x^{81}+\frac{13}{6}\,b^{2}c^{12}x^{78}+\frac{26}{3}\,b^{3}c^{11}x^{75}+\frac{143}{6}\,b^{4}c^{10}x^{72}+\frac{143}{3}\,b^{5}c^{9}x^{69}+\frac{143}{2}\,b^{6}c^{8}x^{66}+\frac{572}{7}\,b^{7}c^{7}x^{63}+\frac{143}{2}\,b^{8}c^{6}x^{60}+\frac{143}{3}\,b^{9}c^{5}x^{57}+\frac{143}{6}\,b^{10}c^{4}x^{54}+\frac{26}{3}\,b^{11}c^{3}x^{51}+\frac{13}{6}\,b^{12}c^{2}x^{48}+\frac{1}{3}\,b^{13}cx^{45}+\frac{1}{42}\,b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^6 + b*x^3)^13*(2*c*x^3 + b)*x^2,x, algorithm="giac")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*
c^11*x^75 + 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*
c^8*x^66 + 572/7*b^7*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^
5*x^57 + 143/6*b^10*c^4*x^54 + 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2
*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

3.104 $\int x^{-1+n}\left(b+2cx^n\right)\left(bx^n+cx^{2n}\right)^{13}\,dx$

Optimal. Leaf size=21

$$\frac{x^{14n}\left(b+cx^n\right)^{14}}{14n}$$

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Rubi [A] time = 0.0616745, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{x^{14n}\left(b+cx^n\right)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^13,x]

[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Rubi in Sympy [A] time = 12.011, size = 15, normalized size = 0.71

$$\frac{x^{14n}\left(b+cx^n\right)^{14}}{14n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+n)*(b+2*c*x**n)*(b*x**n+c*x**(2*n))**13,x)

[Out] x**(14*n)*(b + c*x**n)**14/(14*n)
```

Mathematica [A] time = 0.0541722, size = 21, normalized size = 1.

$$\frac{x^{14n}\left(b+cx^n\right)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Maple [B] time = 0.057, size = 230, normalized size = 11.

$$\frac{c^{14}(x^n)^{28}}{14n} + \frac{bc^{13}(x^n)^{27}}{n} + \frac{13c^{12}(x^n)^{26}b^2}{2n} + 26\frac{b^3c^{11}(x^n)^{25}}{n} + \frac{143c^{10}(x^n)^{24}b^4}{2n} + 143\frac{b^5c^9(x^n)^{23}}{n} + \frac{429c^8(x^n)^{22}b^6}{2n} + \frac{1716b^7c^7(x^n)^{21}}{7n} + \frac{429c^6(x^n)^{20}b^8}{2n} + 143\frac{b^9c^5(x^n)^{19}}{n} + \frac{143c^4(x^n)^{18}b^{10}}{2n} + 26\frac{b^{11}c^3(x^n)^{17}}{n} + \frac{13c^2(x^n)^{16}b^{12}}{2n} + \frac{b^{13}c(x^n)^{15}}{n} + \frac{(x^n)^{14}b^{14}}{14n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x)
```

```
[Out] 1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*c^12/n*(x^n)^26*b^2+2
6*c^11*b^3/n*(x^n)^25+143/2*c^10/n*(x^n)^24*b^4+143*b^5*c^9/n*(x^
n)^23+429/2*c^8/n*(x^n)^22*b^6+1716/7*b^7*c^7/n*(x^n)^21+429/2*c^
6/n*(x^n)^20*b^8+143*c^5*b^9/n*(x^n)^19+143/2*c^4/n*(x^n)^18*b^10
+26*b^11*c^3/n*(x^n)^17+13/2*c^2/n*(x^n)^16*b^12+b^13*c/n*(x^n)^1
5+1/14/n*(x^n)^14*b^14
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n)^13*(2*c*x^n + b)*x^(n - 1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.316339, size = 255, normalized size = 12.14

$$\frac{c^{14}x^{28n} + 14bc^{13}x^{27n} + 91b^2c^{12}x^{26n} + 364b^3c^{11}x^{25n} + 1001b^4c^{10}x^{24n} + 2002b^5c^9x^{23n} + 3003b^6c^8x^{22n} + 3432b^7c^7x^{21n} +}{14n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n)^13*(2*c*x^n + b)*x^(n - 1),x, algorithm="fricas")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) +
 364*b^3*c^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^
(23*n) + 3003*b^6*c^8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8
*c^6*x^(20*n) + 2002*b^9*c^5*x^(19*n) + 1001*b^10*c^4*x^(18*n) +
364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(16*n) + 14*b^13*c*x^(15*n)
 + b^14*x^(14*n))/n
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+n)*(b+2*c*x**n)*(b*x**n+c*x**(2*n))**13,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.270288, size = 255, normalized size = 12.14

$$\frac{c^{14}x^{28\,n} + 14\,bc^{13}x^{27\,n} + 91\,b^{2}c^{12}x^{26\,n} + 364\,b^{3}c^{11}x^{25\,n} + 1001\,b^{4}c^{10}x^{24\,n} + 2002\,b^{5}c^{9}x^{23\,n} + 3003\,b^{6}c^{8}x^{22\,n} + 3432\,b^{7}c^{7}x^{21\,n} + \cdots}{14\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n)^13*(2*c*x^n + b)*x^(n - 1),x, algorithm="giac")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) +
 364*b^3*c^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^
(23*n) + 3003*b^6*c^8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8
*c^6*x^(20*n) + 2002*b^9*c^5*x^(19*n) + 1001*b^10*c^4*x^(18*n) +
364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(16*n) + 14*b^13*c*x^(15*n)
 + b^14*x^(14*n))/n
```

3.105 $\int \frac{b+2cx}{a+bx+cx^2} \, dx$

Optimal. Leaf size=11

$$\log \left(a+bx+cx^2\right)$$

```
[Out] Log[a + b*x + c*x^2]
```

Rubi [A] time = 0.00832596, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\log \left(a+bx+cx^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 2*c*x)/(a + b*x + c*x^2),x]

[Out] Log[a + b*x + c*x^2]
```

Rubi in Sympy [A] time = 3.69863, size = 10, normalized size = 0.91

$$\log \left(a+bx+cx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*c*x+b)/(c*x**2+b*x+a),x)

[Out] log(a + b*x + c*x**2)
```

Mathematica [A] time = 0.00455208, size = 10, normalized size = 0.91

$$\log(a+x(b+cx))$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*c*x)/(a + b*x + c*x^2),x]

[Out] Log[a + x*(b + c*x)]
```

Maple [A] time = 0.002, size = 12, normalized size = 1.1

$$\ln \left(c x^2 + b x + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*c*x+b)/(c*x^2+b*x+a),x)
```

```
[Out] ln(c*x^2+b*x+a)
```

Maxima [A] time = 0.772864, size = 15, normalized size = 1.36

$$\log \left(c x^2 + b x + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x + a),x, algorithm="maxima")
```

```
[Out] log(c*x^2 + b*x + a)
```

Fricas [A] time = 0.2787, size = 15, normalized size = 1.36

$$\log \left(c x^2 + b x + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x + a),x, algorithm="fricas")
```

```
[Out] log(c*x^2 + b*x + a)
```

Sympy [A] time = 1.16436, size = 10, normalized size = 0.91

$$\log \left(a + b x + c x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x+b)/(c*x**2+b*x+a),x)

[Out] log(a + b*x + c*x**2)
```

GIAC/XCAS [A] time = 0.264829, size = 15, normalized size = 1.36

$$\ln\left(cx^2 + bx + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x + a),x, algorithm="giac")

[Out] ln(c*x^2 + b*x + a)
```

3.106 $\int \frac{x(b+2cx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}\log\left(a+bx^2+cx^4\right)$$

```
[Out] Log[a + b*x^2 + c*x^4]/2
```

Rubi [A] time = 0.0105358, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{1}{2}\log\left(a+bx^2+cx^4\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] Log[a + b*x^2 + c*x^4]/2
```

Rubi in Sympy [A] time = 4.94338, size = 14, normalized size = 0.82

$$\frac{\log\left(a+bx^2+cx^4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2+a),x)
```

```
[Out] log(a + b*x**2 + c*x**4)/2
```

Mathematica [A] time = 0.00970252, size = 17, normalized size = 1.

$$\frac{1}{2}\log\left(a+bx^2+cx^4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] Log[a + b*x^2 + c*x^4]/2
```

Maple [A] time = 0.002, size = 16, normalized size = 0.9

$$\frac{\ln\left(cx^4 + bx^2 + a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*ln(c*x^4+b*x^2+a)
```

Maxima [A] time = 0.768463, size = 20, normalized size = 1.18

$$\frac{1}{2}\log\left(cx^4 + bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2 + b)*x/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/2*log(c*x^4 + b*x^2 + a)
```

Fricas [A] time = 0.270551, size = 20, normalized size = 1.18

$$\frac{1}{2}\log\left(cx^4 + bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2 + b)*x/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/2*log(c*x^4 + b*x^2 + a)
```

Sympy [A] time = 1.59331, size = 14, normalized size = 0.82

$$\frac{\log\left(a + bx^2 + cx^4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2+a),x)
```

```
[Out] log(a + b*x**2 + c*x**4)/2
```

GIAC/XCAS [A] time = 0.290826, size = 20, normalized size = 1.18

$$\frac{1}{2}\ln\left(cx^4 + bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)*x/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/2*ln(c*x^4 + b*x^2 + a)
```

3.107 $\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=17

$$\frac{1}{3}\log\left(a+bx^3+cx^6\right)$$

```
[Out] Log[a + b*x^3 + c*x^6]/3
```

Rubi [A] time = 0.0109252, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{1}{3}\log\left(a+bx^3+cx^6\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6),x]

[Out] Log[a + b*x^3 + c*x^6]/3
```

Rubi in Sympy [A] time = 4.9955, size = 14, normalized size = 0.82

$$\frac{\log\left(a+bx^3+cx^6\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3+a),x)

[Out] log(a + b*x**3 + c*x**6)/3
```

Mathematica [A] time = 0.010969, size = 17, normalized size = 1.

$$\frac{1}{3}\log\left(a+bx^3+cx^6\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] Log[a + b*x^3 + c*x^6]/3
```

Maple [A] time = 0.002, size = 16, normalized size = 0.9

$$\frac{\ln\left(cx^6+bx^3+a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*ln(c*x^6+b*x^3+a)
```

Maxima [A] time = 0.772063, size = 20, normalized size = 1.18

$$\frac{1}{3}\log\left(cx^6+bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3 + b)*x^2/(c*x^6 + b*x^3 + a),x, algorithm="maxima")
```

```
[Out] 1/3*log(c*x^6 + b*x^3 + a)
```

Fricas [A] time = 0.282079, size = 20, normalized size = 1.18

$$\frac{1}{3}\log\left(cx^6+bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3 + b)*x^2/(c*x^6 + b*x^3 + a),x, algorithm="fricas")
```

```
[Out] 1/3*log(c*x^6 + b*x^3 + a)
```

Sympy [A] time = 1.98869, size = 14, normalized size = 0.82

$$\frac{\log\left(a+bx^3+cx^6\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3+a),x)
```

```
[Out] log(a + b*x**3 + c*x**6)/3
```

GIAC/XCAS [A] time = 0.265643, size = 20, normalized size = 1.18

$$\frac{1}{3}\ln\left(cx^{6}+bx^{3}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^3 + b)*x^2/(c*x^6 + b*x^3 + a),x, algorithm="giac")
```

```
[Out] 1/3*ln(c*x^6 + b*x^3 + a)
```

3.108 $\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=19

$$\frac{\log\left(a+bx^n+cx^{2n}\right)}{n}$$

```
[Out] Log[a + b*x^n + c*x^(2*n)]/n
```

Rubi [A] time = 0.0668016, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{\log\left(a+bx^n+cx^{2n}\right)}{n}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[a + b*x^n + c*x^(2*n)]/n
```

Rubi in Sympy [A] time = 12.1377, size = 15, normalized size = 0.79

$$\frac{\log\left(a+bx^n+cx^{2n}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+n)*(b+2*c*x**n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] log(a + b*x**n + c*x**(2*n))/n
```

Mathematica [A] time = 0.0489865, size = 26, normalized size = 1.37

$$\frac{\log\left(ax^{-2n}+bx^{-n}+c\right)}{n}+2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n)),x]

[Out] 2*Log[x] + Log[c + a/x^(2*n) + b/x^n]/n
```

Maple [A] time = 0.033, size = 24, normalized size = 1.3

$$\frac{\ln\left(a+b\mathrm{e}^{n\ln(x)}+c\left(\mathrm{e}^{n\ln(x)}\right)^{2}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x)

[Out] 1/n*ln(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)
```

Maxima [A] time = 0.825763, size = 31, normalized size = 1.63

$$\frac{\log\left(\frac{cx^{2n}+bx^{n}+a}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^n + b)*x^(n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")

[Out] log((c*x^(2*n) + b*x^n + a)/c)/n
```

Fricas [A] time = 0.297355, size = 26, normalized size = 1.37

$$\frac{\log\left(cx^{2n}+bx^{n}+a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^n + b)*x^(n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")

[Out] log(c*x^(2*n) + b*x^n + a)/n
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+n)*(b+2*c*x**n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.26569, size = 26, normalized size = 1.37

$$\frac{\ln\left(cx^{2n} + bx^n + a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^n + b)*x^(n - 1)/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")
```

```
[Out] ln(c*x^(2*n) + b*x^n + a)/n
```

3.109 $\int \frac{b+2cx}{(a+bx+cx^2)^8}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{7\,(a+bx+cx^2)^7}$$

```
[Out] -1/(7*(a + b*x + c*x^2)^7)
```

Rubi [A] time = 0.00910288, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$-\frac{1}{7\,(a+bx+cx^2)^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 2*c*x)/(a + b*x + c*x^2)^8,x]

[Out] -1/(7*(a + b*x + c*x^2)^7)
```

Rubi in Sympy [A] time = 3.66627, size = 15, normalized size = 0.94

$$-\frac{1}{7\,(a+bx+cx^2)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*c*x+b)/(c*x**2+b*x+a)**8,x)

[Out] -1/(7*(a + b*x + c*x**2)**7)
```

Mathematica [A] time = 0.0177459, size = 15, normalized size = 0.94

$$-\frac{1}{7(a+x(b+cx))^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*c*x)/(a + b*x + c*x^2)^8,x]
```

```
[Out] -1/(7*(a + x*(b + c*x))^7)
```

Maple [A] time = 0.001, size = 15, normalized size = 0.9

$$-\frac{1}{7\left(cx^{2}+bx+a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*c*x+b)/(c*x^2+b*x+a)^8,x)
```

```
[Out] -1/7/(c*x^2+b*x+a)^7
```

Maxima [A] time = 0.763245, size = 19, normalized size = 1.19

$$-\frac{1}{7\left(cx^{2}+bx+a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x + a)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c*x^2 + b*x + a)^7
```

Fricas [A] time = 0.327656, size = 473, normalized size = 29.56

$$-\frac{}{7\left(c^{7}x^{14}+7\,bc^{6}x^{13}+7\left(3\,b^{2}c^{5}+ac^{6}\right)x^{12}+7\left(5\,b^{3}c^{4}+6\,abc^{5}\right)x^{11}+7\left(5\,b^{4}c^{3}+15\,ab^{2}c^{4}+3\,a^{2}c^{5}\right)x^{10}+7\left(3\,b^{5}c^{2}+20\,ab^{3}c^{3}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x + a)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 7*(3*b^2*c^5 + a*c^6)*x^12 + 7*(5
*b^3*c^4 + 6*a*b*c^5)*x^11 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*
c^5)*x^10 + 7*(3*b^5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^9 + 7*(
b^6*c + 15*a*b^4*c^2 + 30*a^2*b^2*c^3 + 5*a^3*c^4)*x^8 + 7*a^6*b*
```

```
x + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3*b*c^3)*x^7 + a^
7 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^6 + 7
*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^5 + 7*(5*a^3*b^4 + 1
5*a^4*b^2*c + 3*a^5*c^2)*x^4 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^3 + 7*
(3*a^5*b^2 + a^6*c)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x+b)/(c*x**2+b*x+a)**8,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.274759, size = 19, normalized size = 1.19

$$-\frac{1}{7\,(cx^2 + bx + a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x + a)^8,x, algorithm="giac")

[Out] -1/7/(c*x^2 + b*x + a)^7
```

3.110 $\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8} dx$

Optimal. Leaf size=18

$$-\frac{1}{14(a+bx^2+cx^4)^7}$$

```
[Out] -1/(14*(a + b*x^2 + c*x^4)^7)
```

Rubi [A] time = 0.0120266, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$-\frac{1}{14(a+bx^2+cx^4)^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8,x]

[Out] -1/(14*(a + b*x^2 + c*x^4)^7)
```

Rubi in Sympy [A] time = 4.9566, size = 17, normalized size = 0.94

$$-\frac{1}{14(a+bx^2+cx^4)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2+a)**8,x)

[Out] -1/(14*(a + b*x**2 + c*x**4)**7)
```

Mathematica [A] time = 0.0204354, size = 18, normalized size = 1.

$$-\frac{1}{14(a+bx^2+cx^4)^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/(14*(a + b*x^2 + c*x^4)^7)
```

Maple [A] time = 0.002, size = 17, normalized size = 0.9

$$-\frac{1}{14\,(cx^4+bx^2+a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x)
```

```
[Out] -1/14/(c*x^4+b*x^2+a)^7
```

Maxima [A] time = 0.858208, size = 475, normalized size = 26.39

$$-\frac{}{14\,(c^7x^{28}+7\,bc^6x^{26}+7\,(3\,b^2c^5+ac^6)x^{24}+7\,(5\,b^3c^4+6\,abc^5)x^{22}+7\,(5\,b^4c^3+15\,ab^2c^4+3\,a^2c^5)x^{20}+7\,(3\,b^5c^2+20\,ab^3c^3+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2 + b)*x/(c*x^4 + b*x^2 + a)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 + a*c^6)*x^24 + 7*(
5*b^3*c^4 + 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2
*c^5)*x^20 + 7*(3*b^5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7
*(b^6*c + 15*a*b^4*c^2 + 30*a^2*b^2*c^3 + 5*a^3*c^4)*x^16 + (b^7
+ 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3*b*c^3)*x^14 + 7*(a*b^6 +
 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^12 + 7*(3*a^2*b^5 +
 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 + 7*(5*a^3*b^4 +
 15*a^4*b^2*c + 3*a^5*c^2)*x^8 + a^7 + 7*(5*a^4*b^3 + 6*a^5*b*c)*
x^6 + 7*(3*a^5*b^2 + a^6*c)*x^4)
```

Fricas [A] time = 0.371777, size = 475, normalized size = 26.39

$$-\frac{}{14\,(c^7x^{28}+7\,bc^6x^{26}+7\,(3\,b^2c^5+ac^6)x^{24}+7\,(5\,b^3c^4+6\,abc^5)x^{22}+7\,(5\,b^4c^3+15\,ab^2c^4+3\,a^2c^5)x^{20}+7\,(3\,b^5c^2+20\,ab^3c^3+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)*x/(c*x^4 + b*x^2 + a)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 + a*c^6)*x^24 + 7*(
5*b^3*c^4 + 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2
*c^5)*x^20 + 7*(3*b^5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7
*(b^6*c + 15*a*b^4*c^2 + 30*a^2*b^2*c^3 + 5*a^3*c^4)*x^16 + (b^7
+ 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3*b*c^3)*x^14 + 7*(a*b^6 +
 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^12 + 7*(3*a^2*b^5 +
 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 + 7*(5*a^3*b^4 +
 15*a^4*b^2*c + 3*a^5*c^2)*x^8 + a^7 + 7*(5*a^4*b^3 + 6*a^5*b*c)*
x^6 + 7*(3*a^5*b^2 + a^6*c)*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2+a)**8,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)*x/(c*x^4 + b*x^2 + a)^8,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.111 $\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{21\left(a+bx^3+cx^6\right)^7}$$

```
[Out] -1/(21*(a + b*x^3 + c*x^6)^7)
```

Rubi [A] time = 0.013359, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$-\frac{1}{21\left(a+bx^3+cx^6\right)^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6)^8,x]

[Out] -1/(21*(a + b*x^3 + c*x^6)^7)
```

Rubi in Sympy [A] time = 5.00563, size = 17, normalized size = 0.94

$$-\frac{1}{21\left(a+bx^3+cx^6\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3+a)**8,x)

[Out] -1/(21*(a + b*x**3 + c*x**6)**7)
```

Mathematica [A] time = 0.0209314, size = 18, normalized size = 1.

$$-\frac{1}{21\left(a+bx^3+cx^6\right)^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/(21*(a + b*x^3 + c*x^6)^7)
```

Maple [A] time = 0.001, size = 17, normalized size = 0.9

$$-\frac{1}{21\,(cx^6+bx^3+a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x)
```

```
[Out] -1/21/(c*x^6+b*x^3+a)^7
```

Maxima [A] time = 0.877058, size = 475, normalized size = 26.39

$$-\frac{}{21\,(c^7x^{42}+7\,bc^6x^{39}+7\,(3\,b^2c^5+ac^6)x^{36}+7\,(5\,b^3c^4+6\,abc^5)x^{33}+7\,(5\,b^4c^3+15\,ab^2c^4+3\,a^2c^5)x^{30}+7\,(3\,b^5c^2+20\,ab^3c^3+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3 + b)*x^2/(c*x^6 + b*x^3 + a)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 + a*c^6)*x^36 + 7*(
5*b^3*c^4 + 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2
*c^5)*x^30 + 7*(3*b^5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7
*(b^6*c + 15*a*b^4*c^2 + 30*a^2*b^2*c^3 + 5*a^3*c^4)*x^24 + (b^7
+ 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3*b*c^3)*x^21 + 7*(a*b^6 +
 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^18 + 7*(3*a^2*b^5 +
 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 + 7*(5*a^3*b^4 + 15*a^4*b^2*c
+ 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^9 +
 a^7 + 7*(3*a^5*b^2 + a^6*c)*x^6)
```

Fricas [A] time = 0.363255, size = 475, normalized size = 26.39

$$-\frac{}{21\,(c^7x^{42}+7\,bc^6x^{39}+7\,(3\,b^2c^5+ac^6)x^{36}+7\,(5\,b^3c^4+6\,abc^5)x^{33}+7\,(5\,b^4c^3+15\,ab^2c^4+3\,a^2c^5)x^{30}+7\,(3\,b^5c^2+20\,ab^3c^3+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3 + b)*x^2/(c*x^6 + b*x^3 + a)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 + a*c^6)*x^36 + 7*(
5*b^3*c^4 + 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2
*c^5)*x^30 + 7*(3*b^5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7
*(b^6*c + 15*a*b^4*c^2 + 30*a^2*b^2*c^3 + 5*a^3*c^4)*x^24 + (b^7
+ 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3*b*c^3)*x^21 + 7*(a*b^6 +
 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^18 + 7*(3*a^2*b^5 +
 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 + 7*(5*a^3*b^4 + 15*a^4*b^2*c
+ 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^9 +
 a^7 + 7*(3*a^5*b^2 + a^6*c)*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3+a)**8,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.359095, size = 22, normalized size = 1.22

$$-\frac{1}{21\,(cx^6 + bx^3 + a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3 + b)*x^2/(c*x^6 + b*x^3 + a)^8,x, algorithm="giac")
```

```
[Out] -1/21/(c*x^6 + b*x^3 + a)^7
```

3.112 $\int \frac{x^{-1+n}(b+2cx^n)}{(a+bx^n+cx^{2n})^8}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{7n\left(a+bx^n+cx^{2n}\right)^7}$$

```
[Out] -1/(7*n*(a + b*x^n + c*x^(2*n))^7)
```

Rubi [A] time = 0.0721155, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$-\frac{1}{7n\left(a+bx^n+cx^{2n}\right)^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n))^8,x]

[Out] -1/(7*n*(a + b*x^n + c*x^(2*n))^7)
```

Rubi in Sympy [A] time = 12.2668, size = 20, normalized size = 0.87

$$-\frac{1}{7n\left(a+bx^n+cx^{2n}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+n)*(b+2*c*x**n)/(a+b*x**n+c*x**(2*n))**8,x)

[Out] -1/(7*n*(a + b*x**n + c*x**(2*n))**7)
```

Mathematica [A] time = 0.0818763, size = 23, normalized size = 1.

$$-\frac{1}{7n\left(a+bx^n+cx^{2n}\right)^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n))^8,x]
```

```
[Out] -1/(7*n*(a + b*x^n + c*x^(2*n))^7)
```

Maple [A] time = 0.095, size = 22, normalized size = 1.

$$-\frac{1}{7\,n\left(a+bx^{n}+c\,(x^{n})^{2}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x)
```

```
[Out] -1/7/n/(a+b*x^n+c*(x^n)^2)^7
```

Maxima [A] time = 1.21423, size = 562, normalized size = 24.43

$$-\frac{}{7\,(c^{7}nx^{14\,n}+7\,bc^{6}nx^{13\,n}+7\,a^{6}bnx^{n}+a^{7}n+7\,(3\,b^{2}c^{5}n+ac^{6}n)x^{12\,n}+7\,(5\,b^{3}c^{4}n+6\,abc^{5}n)x^{11\,n}+7\,(5\,b^{4}c^{3}n+15\,ab^{2}c^{4}n+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^n + b)*x^(n - 1)/(c*x^(2*n) + b*x^n + a)^8,x, algorithm="maxima"
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n + a^7*n
 + 7*(3*b^2*c^5*n + a*c^6*n)*x^(12*n) + 7*(5*b^3*c^4*n + 6*a*b*c^
5*n)*x^(11*n) + 7*(5*b^4*c^3*n + 15*a*b^2*c^4*n + 3*a^2*c^5*n)*x^
(10*n) + 7*(3*b^5*c^2*n + 20*a*b^3*c^3*n + 15*a^2*b*c^4*n)*x^(9*n
) + 7*(b^6*c*n + 15*a*b^4*c^2*n + 30*a^2*b^2*c^3*n + 5*a^3*c^4*n)
*x^(8*n) + (b^7*n + 42*a*b^5*c*n + 210*a^2*b^3*c^2*n + 140*a^3*b*
c^3*n)*x^(7*n) + 7*(a*b^6*n + 15*a^2*b^4*c*n + 30*a^3*b^2*c^2*n +
 5*a^4*c^3*n)*x^(6*n) + 7*(3*a^2*b^5*n + 20*a^3*b^3*c*n + 15*a^4*
b*c^2*n)*x^(5*n) + 7*(5*a^3*b^4*n + 15*a^4*b^2*c*n + 3*a^5*c^2*n)
*x^(4*n) + 7*(5*a^4*b^3*n + 6*a^5*b*c*n)*x^(3*n) + 7*(3*a^5*b^2*n
 + a^6*c*n)*x^(2*n))
```

Fricas [A] time = 0.362501, size = 532, normalized size = 23.13

$$-\frac{}{7\,(c^{7}nx^{14\,n}+7\,bc^{6}nx^{13\,n}+7\,a^{6}bnx^{n}+a^{7}n+7\,(3\,b^{2}c^{5}+ac^{6})nx^{12\,n}+7\,(5\,b^{3}c^{4}+6\,abc^{5})nx^{11\,n}+7\,(5\,b^{4}c^{3}+15\,ab^{2}c^{4}+3\,a^{2}c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^n + b)*x^(n - 1)/(c*x^(2*n) + b*x^n + a)^8,x, algorithm="fricas"
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n + a^7*n
 + 7*(3*b^2*c^5 + a*c^6)*n*x^(12*n) + 7*(5*b^3*c^4 + 6*a*b*c^5)*n
*x^(11*n) + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*n*x^(10*n) +
 7*(3*b^5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*n*x^(9*n) + 7*(b^6*c
 + 15*a*b^4*c^2 + 30*a^2*b^2*c^3 + 5*a^3*c^4)*n*x^(8*n) + (b^7 +
42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3*b*c^3)*n*x^(7*n) + 7*(a*b^
6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*n*x^(6*n) + 7*(3*a
^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*n*x^(5*n) + 7*(5*a^3*b^4 +
15*a^4*b^2*c + 3*a^5*c^2)*n*x^(4*n) + 7*(5*a^4*b^3 + 6*a^5*b*c)*n
*x^(3*n) + 7*(3*a^5*b^2 + a^6*c)*n*x^(2*n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(a+b*x**n+c*x**(2*n))**8,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.281919, size = 28, normalized size = 1.22

$$-\frac{1}{7\,{\left(cx^{2\,n}+bx^{n}+a\right)}^{7}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^n + b)*x^(n - 1)/(c*x^(2*n) + b*x^n + a)^8,x, algorithm="giac")
```

```
[Out] -1/7/((c*x^(2*n) + b*x^n + a)^7*n)
```

3.113 $\int \frac{b+2cx}{-a+bx+cx^2}\,dx$

Optimal. Leaf size=13

$$\log\left(a-bx-cx^2\right)$$

```
[Out] Log[a - b*x - c*x^2]
```

Rubi [A] time = 0.00850707, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\log\left(a-bx-cx^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 2*c*x)/(-a + b*x + c*x^2),x]

[Out] Log[a - b*x - c*x^2]
```

Rubi in Sympy [A] time = 4.01225, size = 10, normalized size = 0.77

$$\log\left(-a+bx+cx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*c*x+b)/(c*x**2+b*x-a),x)

[Out] log(-a + b*x + c*x**2)
```

Mathematica [A] time = 0.00761912, size = 12, normalized size = 0.92

$$\log(x(b+cx)-a)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*c*x)/(-a + b*x + c*x^2),x]

[Out] Log[-a + x*(b + c*x)]
```

Maple [A] time = 0.002, size = 14, normalized size = 1.1

$$\ln \left(c x^{2}+b x-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*c*x+b)/(c*x^2+b*x-a),x)
```

```
[Out] ln(c*x^2+b*x-a)
```

Maxima [A] time = 0.750719, size = 18, normalized size = 1.38

$$\log \left(c x^{2}+b x-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x - a),x, algorithm="maxima")
```

```
[Out] log(c*x^2 + b*x - a)
```

Fricas [A] time = 0.274267, size = 18, normalized size = 1.38

$$\log \left(c x^{2}+b x-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x - a),x, algorithm="fricas")
```

```
[Out] log(c*x^2 + b*x - a)
```

Sympy [A] time = 1.18518, size = 10, normalized size = 0.77

$$\log \left(-a+b x+c x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x+b)/(c*x**2+b*x-a),x)
```

```
[Out] log(-a + b*x + c*x**2)
```

GIAC/XCAS [A] time = 0.26508, size = 18, normalized size = 1.38

$$\ln\left(cx^2 + bx - a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x - a),x, algorithm="giac")
```

```
[Out] ln(c*x^2 + b*x - a)
```

3.114 $\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4}\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}\log\left(a-bx^2-cx^4\right)$$

```
[Out] Log[a - b*x^2 - c*x^4]/2
```

Rubi [A] time = 0.0103489, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$

$$\frac{1}{2}\log\left(a-bx^2-cx^4\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4),x]

[Out] Log[a - b*x^2 - c*x^4]/2
```

Rubi in Sympy [A] time = 5.40042, size = 14, normalized size = 0.74

$$\frac{\log\left(-a+bx^2+cx^4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2-a),x)

[Out] log(-a + b*x**2 + c*x**4)/2
```

Mathematica [A] time = 0.0104468, size = 19, normalized size = 1.

$$\frac{1}{2}\log\left(-a+bx^2+cx^4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4),x]
```

```
[Out] Log[-a + b*x^2 + c*x^4]/2
```

Maple [A] time = 0.001, size = 18, normalized size = 1.

$$\frac{\ln\left(cx^4+bx^2-a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x)
```

```
[Out] 1/2*ln(c*x^4+b*x^2-a)
```

Maxima [A] time = 0.74384, size = 23, normalized size = 1.21

$$\frac{1}{2}\log\left(cx^4+bx^2-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)*x/(c*x^4 + b*x^2 - a),x, algorithm="maxima")
```

```
[Out] 1/2*log(c*x^4 + b*x^2 - a)
```

Fricas [A] time = 0.2538, size = 23, normalized size = 1.21

$$\frac{1}{2}\log\left(cx^4+bx^2-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)*x/(c*x^4 + b*x^2 - a),x, algorithm="fricas")
```

```
[Out] 1/2*log(c*x^4 + b*x^2 - a)
```

Sympy [A] time = 1.59674, size = 14, normalized size = 0.74

$$\frac{\log\left(-a+bx^2+cx^4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2-a),x)
```

```
[Out] log(-a + b*x**2 + c*x**4)/2
```

GIAC/XCAS [A] time = 0.290812, size = 23, normalized size = 1.21

$$\frac{1}{2}\ln\left(cx^4 + bx^2 - a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)*x/(c*x^4 + b*x^2 - a),x, algorithm="giac")
```

```
[Out] 1/2*ln(c*x^4 + b*x^2 - a)
```

3.115 $\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6}\,dx$

Optimal. Leaf size=19

$$\frac{1}{3}\log\left(a-bx^3-cx^6\right)$$

```
[Out] Log[a - b*x^3 - c*x^6]/3
```

Rubi [A] time = 0.010457, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$\frac{1}{3}\log\left(a-bx^3-cx^6\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6),x]
```

```
[Out] Log[a - b*x^3 - c*x^6]/3
```

Rubi in Sympy [A] time = 5.4267, size = 14, normalized size = 0.74

$$\frac{\log\left(-a+bx^3+cx^6\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3-a),x)
```

```
[Out] log(-a + b*x**3 + c*x**6)/3
```

Mathematica [A] time = 0.0114698, size = 19, normalized size = 1.

$$\frac{1}{3}\log\left(-a+bx^3+cx^6\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6),x]
```

```
[Out] Log[-a + b*x^3 + c*x^6]/3
```

Maple [A] time = 0.002, size = 18, normalized size = 1.

$$\frac{\ln\left(cx^6 + bx^3 - a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x)
```

```
[Out] 1/3*ln(c*x^6+b*x^3-a)
```

Maxima [A] time = 0.759191, size = 23, normalized size = 1.21

$$\frac{1}{3}\log\left(cx^6 + bx^3 - a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3 + b)*x^2/(c*x^6 + b*x^3 - a),x, algorithm="maxima")
```

```
[Out] 1/3*log(c*x^6 + b*x^3 - a)
```

Fricas [A] time = 0.256263, size = 23, normalized size = 1.21

$$\frac{1}{3}\log\left(cx^6 + bx^3 - a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3 + b)*x^2/(c*x^6 + b*x^3 - a),x, algorithm="fricas")
```

```
[Out] 1/3*log(c*x^6 + b*x^3 - a)
```

Sympy [A] time = 1.93796, size = 14, normalized size = 0.74

$$\frac{\log\left(-a + bx^3 + cx^6\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3-a),x)
```

```
[Out] log(-a + b*x**3 + c*x**6)/3
```

GIAC/XCAS [A] time = 0.266664, size = 23, normalized size = 1.21

$$\frac{1}{3}\ln\left(cx^6 + bx^3 - a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^3 + b)*x^2/(c*x^6 + b*x^3 - a),x, algorithm="giac")
```

```
[Out] 1/3*ln(c*x^6 + b*x^3 - a)
```

3.116 $\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=21

$$\frac{\log\left(a-bx^n-cx^{2n}\right)}{n}$$

```
[Out] Log[a - b*x^n - c*x^(2*n)]/n
```

Rubi [A] time = 0.0732985, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\log\left(a-bx^n-cx^{2n}\right)}{n}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n)),x]

[Out] Log[a - b*x^n - c*x^(2*n)]/n
```

Rubi in Sympy [A] time = 12.9037, size = 15, normalized size = 0.71

$$\frac{\log\left(-a+bx^n+cx^{2n}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+n)*(b+2*c*x**n)/(-a+b*x**n+c*x**(2*n)),x)

[Out] log(-a + b*x**n + c*x**(2*n))/n
```

Mathematica [A] time = 0.0479568, size = 29, normalized size = 1.38

$$\frac{\log\left(ax^{-2n}-bx^{-n}-c\right)}{n}+2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n)),x]
```

```
[Out] 2*Log[x] + Log[-c + a/x^(2*n) - b/x^n]/n
```

Maple [A] time = 0.033, size = 26, normalized size = 1.2

$$\frac{\ln\left(-c\left(e^{n\ln(x)}\right)^{2}-b e^{n\ln(x)}+a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x)
```

```
[Out] 1/n*ln(-c*exp(n*ln(x))^2-b*exp(n*ln(x))+a)
```

Maxima [A] time = 0.852579, size = 34, normalized size = 1.62

$$\frac{\log\left(\frac{cx^{2n}+bx^{n}-a}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^n + b)*x^(n - 1)/(c*x^(2*n) + b*x^n - a),x, algorithm="maxima")
```

```
[Out] log((c*x^(2*n) + b*x^n - a)/c)/n
```

Fricas [A] time = 0.311909, size = 28, normalized size = 1.33

$$\frac{\log\left(cx^{2n}+bx^{n}-a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^n + b)*x^(n - 1)/(c*x^(2*n) + b*x^n - a),x, algorithm="fricas")
```

```
[Out] log(c*x^(2*n) + b*x^n - a)/n
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+n)*(b+2*c*x**n)/(-a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.26687, size = 28, normalized size = 1.33

$$\frac{\ln\left(cx^{2\,n} + bx^{n} - a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^n + b)*x^(n - 1)/(c*x^(2*n) + b*x^n - a),x, algorithm="giac")
```

```
[Out] ln(c*x^(2*n) + b*x^n - a)/n
```

3.117 $\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx$

Optimal. Leaf size=18

$$\frac{1}{7\,(a-bx-cx^2)^7}$$

```
[Out] 1/(7*(a - b*x - c*x^2)^7)
```

Rubi [A] time = 0.010642, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{1}{7\,(a-bx-cx^2)^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 2*c*x)/(-a + b*x + c*x^2)^8,x]
```

```
[Out] 1/(7*(a - b*x - c*x^2)^7)
```

Rubi in Sympy [A] time = 4.02, size = 15, normalized size = 0.83

$$-\frac{1}{7\,(-a+bx+cx^2)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*c*x+b)/(c*x**2+b*x-a)**8,x)
```

```
[Out] -1/(7*(-a + b*x + c*x**2)**7)
```

Mathematica [A] time = 0.021195, size = 16, normalized size = 0.89

$$\frac{1}{7(a-x(b+cx))^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*c*x)/(-a + b*x + c*x^2)^8,x]

[Out] 1/(7*(a - x*(b + c*x))^7)
```

Maple [A] time = 0.001, size = 17, normalized size = 0.9

$$-\frac{1}{7\,\left(cx^{2}+bx-a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*c*x+b)/(c*x^2+b*x-a)^8,x)

[Out] -1/7/(c*x^2+b*x-a)^7
```

Maxima [A] time = 0.757407, size = 22, normalized size = 1.22

$$-\frac{1}{7\,(cx^{2}+bx-a)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x - a)^8,x, algorithm="maxima")

[Out] -1/7/(c*x^2 + b*x - a)^7
```

Fricas [A] time = 0.321819, size = 478, normalized size = 26.56

$$-\frac{}{7\,(c^{7}x^{14}+7\,bc^{6}x^{13}+7\,(3\,b^{2}c^{5}-ac^{6})x^{12}+7\,(5\,b^{3}c^{4}-6\,abc^{5})x^{11}+7\,(5\,b^{4}c^{3}-15\,ab^{2}c^{4}+3\,a^{2}c^{5})x^{10}+7\,(3\,b^{5}c^{2}-20\,ab^{3}c^{3}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x - a)^8,x, algorithm="fricas")

[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 7*(3*b^2*c^5 - a*c^6)*x^12 + 7*(5
*b^3*c^4 - 6*a*b*c^5)*x^11 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*
c^5)*x^10 + 7*(3*b^5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^9 + 7*(
b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 5*a^3*c^4)*x^8 + 7*a^6*b*
```

```
x + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3*b*c^3)*x^7 - a^
7 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^6 + 7
*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^5 - 7*(5*a^3*b^4 - 1
5*a^4*b^2*c + 3*a^5*c^2)*x^4 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^3 - 7*
(3*a^5*b^2 - a^6*c)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x+b)/(c*x**2+b*x-a)**8,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.27453, size = 22, normalized size = 1.22

$$-\frac{1}{7\,(cx^2+bx-a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x - a)^8,x, algorithm="giac")

[Out] -1/7/(c*x^2 + b*x - a)^7
```

3.118 $\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8} dx$

Optimal. Leaf size=20

$$\frac{1}{14(a-bx^2-cx^4)^7}$$

```
[Out] 1/(14*(a - b*x^2 - c*x^4)^7)
```

Rubi [A] time = 0.0123891, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{1}{14(a-bx^2-cx^4)^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4)^8,x]

[Out] 1/(14*(a - b*x^2 - c*x^4)^7)
```

Rubi in Sympy [A] time = 5.42602, size = 15, normalized size = 0.75

$$\frac{1}{14(a-bx^2-cx^4)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2-a)**8,x)

[Out] 1/(14*(a - b*x**2 - c*x**4)**7)
```

Mathematica [A] time = 0.02658, size = 20, normalized size = 1.

$$-\frac{1}{14(-a+bx^2+cx^4)^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/(14*(-a + b*x^2 + c*x^4)^7)
```

Maple [A] time = 0.001, size = 19, normalized size = 1.

$$-\frac{1}{14\,(cx^4 + bx^2 - a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x)
```

```
[Out] -1/14/(c*x^4+b*x^2-a)^7
```

Maxima [A] time = 0.839686, size = 481, normalized size = 24.05

$$-\frac{}{14\,(c^7x^{28} + 7\,bc^6x^{26} + 7\,(3\,b^2c^5 - ac^6)x^{24} + 7\,(5\,b^3c^4 - 6\,abc^5)x^{22} + 7\,(5\,b^4c^3 - 15\,ab^2c^4 + 3\,a^2c^5)x^{20} + 7\,(3\,b^5c^2 - 20\,ab^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2 + b)*x/(c*x^4 + b*x^2 - a)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 - a*c^6)*x^24 + 7*(
5*b^3*c^4 - 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2
*c^5)*x^20 + 7*(3*b^5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7
*(b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 5*a^3*c^4)*x^16 + (b^7
- 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3*b*c^3)*x^14 - 7*(a*b^6 -
 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^12 + 7*(3*a^2*b^5 -
 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 - 7*(5*a^3*b^4 -
 15*a^4*b^2*c + 3*a^5*c^2)*x^8 - a^7 + 7*(5*a^4*b^3 - 6*a^5*b*c)*
x^6 - 7*(3*a^5*b^2 - a^6*c)*x^4)
```

Fricas [A] time = 0.334348, size = 481, normalized size = 24.05

$$-\frac{}{14\,(c^7x^{28} + 7\,bc^6x^{26} + 7\,(3\,b^2c^5 - ac^6)x^{24} + 7\,(5\,b^3c^4 - 6\,abc^5)x^{22} + 7\,(5\,b^4c^3 - 15\,ab^2c^4 + 3\,a^2c^5)x^{20} + 7\,(3\,b^5c^2 - 20\,ab^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)*x/(c*x^4 + b*x^2 - a)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 - a*c^6)*x^24 + 7*(
5*b^3*c^4 - 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2
*c^5)*x^20 + 7*(3*b^5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7
*(b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 5*a^3*c^4)*x^16 + (b^7
- 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3*b*c^3)*x^14 - 7*(a*b^6 -
 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^12 + 7*(3*a^2*b^5 -
 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 - 7*(5*a^3*b^4 -
 15*a^4*b^2*c + 3*a^5*c^2)*x^8 - a^7 + 7*(5*a^4*b^3 - 6*a^5*b*c)*
x^6 - 7*(3*a^5*b^2 - a^6*c)*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2-a)**8,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)*x/(c*x^4 + b*x^2 - a)^8,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.119 $\int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8}\,dx$

Optimal. Leaf size=20

$$\frac{1}{21\left(a-bx^3-cx^6\right)^7}$$

```
[Out] 1/(21*(a - b*x^3 - c*x^6)^7)
```

Rubi [A] time = 0.0136783, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$\frac{1}{21\left(a-bx^3-cx^6\right)^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6)^8,x]
```

```
[Out] 1/(21*(a - b*x^3 - c*x^6)^7)
```

Rubi in Sympy [A] time = 5.48165, size = 15, normalized size = 0.75

$$\frac{1}{21\left(a-bx^3-cx^6\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3-a)**8,x)
```

```
[Out] 1/(21*(a - b*x**3 - c*x**6)**7)
```

Mathematica [A] time = 0.0275861, size = 20, normalized size = 1.

$$-\frac{1}{21\left(-a+bx^3+cx^6\right)^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/(21*(-a + b*x^3 + c*x^6)^7)
```

Maple [A] time = 0.001, size = 19, normalized size = 1.

$$-\frac{1}{21\,\left(cx^{6}+bx^{3}-a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x)
```

```
[Out] -1/21/(c*x^6+b*x^3-a)^7
```

Maxima [A] time = 0.874977, size = 481, normalized size = 24.05

$$-\frac{}{21\,(c^{7}x^{42}+7\,bc^{6}x^{39}+7\,(3\,b^{2}c^{5}-ac^{6})x^{36}+7\,(5\,b^{3}c^{4}-6\,abc^{5})x^{33}+7\,(5\,b^{4}c^{3}-15\,ab^{2}c^{4}+3\,a^{2}c^{5})x^{30}+7\,(3\,b^{5}c^{2}-20\,ab^{3}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3 + b)*x^2/(c*x^6 + b*x^3 - a)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 - a*c^6)*x^36 + 7*(
5*b^3*c^4 - 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2
*c^5)*x^30 + 7*(3*b^5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7
*(b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 5*a^3*c^4)*x^24 + (b^7
- 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3*b*c^3)*x^21 - 7*(a*b^6 -
 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^18 + 7*(3*a^2*b^5 -
 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 - 7*(5*a^3*b^4 - 15*a^4*b^2*c
+ 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^9 -
 a^7 - 7*(3*a^5*b^2 - a^6*c)*x^6)
```

Fricas [A] time = 0.334645, size = 481, normalized size = 24.05

$$-\frac{}{21\,(c^{7}x^{42}+7\,bc^{6}x^{39}+7\,(3\,b^{2}c^{5}-ac^{6})x^{36}+7\,(5\,b^{3}c^{4}-6\,abc^{5})x^{33}+7\,(5\,b^{4}c^{3}-15\,ab^{2}c^{4}+3\,a^{2}c^{5})x^{30}+7\,(3\,b^{5}c^{2}-20\,ab^{3}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3 + b)*x^2/(c*x^6 + b*x^3 - a)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 - a*c^6)*x^36 + 7*(
5*b^3*c^4 - 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2
*c^5)*x^30 + 7*(3*b^5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7
*(b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 5*a^3*c^4)*x^24 + (b^7
- 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3*b*c^3)*x^21 - 7*(a*b^6 -
 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^18 + 7*(3*a^2*b^5 -
 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 - 7*(5*a^3*b^4 - 15*a^4*b^2*c
+ 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^9 -
 a^7 - 7*(3*a^5*b^2 - a^6*c)*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3-a)**8,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.36293, size = 24, normalized size = 1.2

$$-\frac{1}{21\,(cx^6+bx^3-a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3 + b)*x^2/(c*x^6 + b*x^3 - a)^8,x, algorithm="giac")
```

```
[Out] -1/21/(c^7*x^6 + b*x^3 - a)^7
```

3.120 $\int \frac{x^{-1+n}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8}\,dx$

Optimal. Leaf size=25

$$\frac{1}{7n\,(a-bx^n-cx^{2n})^7}$$

```
[Out] 1/(7*n*(a - b*x^n - c*x^(2*n))^7)
```

Rubi [A] time = 0.0764887, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{1}{7n\,(a-bx^n-cx^{2n})^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n))^8,x]

[Out] 1/(7*n*(a - b*x^n - c*x^(2*n))^7)
```

Rubi in Sympy [A] time = 13.2993, size = 19, normalized size = 0.76

$$\frac{1}{7n\,(a-bx^n-cx^{2n})^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+n)*(b+2*c*x**n)/(-a+b*x**n+c*x**(2*n))**8,x)

[Out] 1/(7*n*(a - b*x**n - c*x**(2*n))**7)
```

Mathematica [A] time = 0.0910688, size = 25, normalized size = 1.

$$-\frac{1}{7n\,(-a+bx^n+cx^{2n})^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n))^8,x]

[Out] -1/(7*n*(-a + b*x^n + c*x^(2*n))^7)
```

Maple [A] time = 0.096, size = 24, normalized size = 1.

$$\frac{1}{7\,n\left(-c\,(x^n)^2-bx^n+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x)

[Out] 1/7/n/(-c*(x^n)^2-b*x^n+a)^7
```

Maxima [A] time = 1.22849, size = 566, normalized size = 22.64

$$-\frac{}{7\,(c^7nx^{14\,n}+7\,bc^6nx^{13\,n}+7\,a^6bnx^n-a^7n+7\,(3\,b^2c^5n-ac^6n)x^{12\,n}+7\,(5\,b^3c^4n-6\,abc^5n)x^{11\,n}+7\,(5\,b^4c^3n-15\,ab^2c^4n+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^n + b)*x^(n - 1)/(c*x^(2*n) + b*x^n - a)^8,x, algorithm="maxima"

[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n - a^7*n
 + 7*(3*b^2*c^5*n - a*c^6*n)*x^(12*n) + 7*(5*b^3*c^4*n - 6*a*b*c^
5*n)*x^(11*n) + 7*(5*b^4*c^3*n - 15*a*b^2*c^4*n + 3*a^2*c^5*n)*x^
(10*n) + 7*(3*b^5*c^2*n - 20*a*b^3*c^3*n + 15*a^2*b*c^4*n)*x^(9*n
) + 7*(b^6*c*n - 15*a*b^4*c^2*n + 30*a^2*b^2*c^3*n - 5*a^3*c^4*n)
*x^(8*n) + (b^7*n - 42*a*b^5*c*n + 210*a^2*b^3*c^2*n - 140*a^3*b*
c^3*n)*x^(7*n) - 7*(a*b^6*n - 15*a^2*b^4*c*n + 30*a^3*b^2*c^2*n -
 5*a^4*c^3*n)*x^(6*n) + 7*(3*a^2*b^5*n - 20*a^3*b^3*c*n + 15*a^4*
b*c^2*n)*x^(5*n) - 7*(5*a^3*b^4*n - 15*a^4*b^2*c*n + 3*a^5*c^2*n)
*x^(4*n) + 7*(5*a^4*b^3*n - 6*a^5*b*c*n)*x^(3*n) - 7*(3*a^5*b^2*n
 - a^6*c*n)*x^(2*n))
```

Fricas [A] time = 0.348273, size = 536, normalized size = 21.44

$$-\frac{}{7\,(c^7nx^{14\,n}+7\,bc^6nx^{13\,n}+7\,a^6bnx^n-a^7n+7\,(3\,b^2c^5-ac^6)nx^{12\,n}+7\,(5\,b^3c^4-6\,abc^5)nx^{11\,n}+7\,(5\,b^4c^3-15\,ab^2c^4+3\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^n + b)*x^(n - 1)/(c*x^(2*n) + b*x^n - a)^8,x, algorithm="fricas"
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n - a^7*n
 + 7*(3*b^2*c^5 - a*c^6)*n*x^(12*n) + 7*(5*b^3*c^4 - 6*a*b*c^5)*n
*x^(11*n) + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*n*x^(10*n) +
 7*(3*b^5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*n*x^(9*n) + 7*(b^6*c
 - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 5*a^3*c^4)*n*x^(8*n) + (b^7 -
42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3*b*c^3)*n*x^(7*n) - 7*(a*b^
6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*n*x^(6*n) + 7*(3*a
^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*n*x^(5*n) - 7*(5*a^3*b^4 -
15*a^4*b^2*c + 3*a^5*c^2)*n*x^(4*n) + 7*(5*a^4*b^3 - 6*a^5*b*c)*n
*x^(3*n) - 7*(3*a^5*b^2 - a^6*c)*n*x^(2*n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+n)*(b+2*c*x**n)/(-a+b*x**n+c*x**(2*n))**8,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.281261, size = 31, normalized size = 1.24

$$-\frac{1}{7\,(cx^{2\,n}+bx^{n}-a)^{7}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^n + b)*x^(n - 1)/(c*x^(2*n) + b*x^n - a)^8,x, algorithm="giac")
```

```
[Out] -1/7/((c*x^(2*n) + b*x^n - a)^7*n)
```

3.121 $\int \frac{b+2cx}{bx+cx^2}\,dx$

Optimal. Leaf size=10

$$\log\left(bx + cx^2\right)$$

```
[Out] Log[b*x + c*x^2]
```

Rubi [A] time = 0.00826196, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$\log\left(bx + cx^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 2*c*x)/(b*x + c*x^2),x]

[Out] Log[b*x + c*x^2]
```

Rubi in Sympy [A] time = 3.17163, size = 8, normalized size = 0.8

$$\log\left(bx + cx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*c*x+b)/(c*x**2+b*x),x)

[Out] log(b*x + c*x**2)
```

Mathematica [A] time = 0.00614655, size = 9, normalized size = 0.9

$$\log(b + cx) + \log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*c*x)/(b*x + c*x^2),x]

[Out] Log[x] + Log[b + c*x]
```

Maple [A] time = 0.002, size = 9, normalized size = 0.9

$$\ln(x(cx+b))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*c*x+b)/(c*x^2+b*x),x)
```

```
[Out] ln(x*(c*x+b))
```

Maxima [A] time = 0.740983, size = 14, normalized size = 1.4

$$\log\left(cx^2+bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x),x, algorithm="maxima")
```

```
[Out] log(c*x^2 + b*x)
```

Fricas [A] time = 0.256372, size = 14, normalized size = 1.4

$$\log\left(cx^2+bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x),x, algorithm="fricas")
```

```
[Out] log(c*x^2 + b*x)
```

Sympy [A] time = 1.07065, size = 8, normalized size = 0.8

$$\log\left(bx+cx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x+b)/(c*x**2+b*x),x)

[Out] log(b*x + c*x**2)
```

GIAC/XCAS [A] time = 0.262621, size = 15, normalized size = 1.5

$$\ln(|cx+b|)+\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x),x, algorithm="giac")

[Out] ln(abs(c*x + b)) + ln(abs(x))
```

3.122 $\int \frac{x(b+2cx^2)}{bx^2+cx^4} \, dx$

Optimal. Leaf size=16

$$\frac{1}{2}\log\left(bx^2+cx^4\right)$$

```
[Out] Log[b*x^2 + c*x^4]/2
```

Rubi [A] time = 0.0089944, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{1}{2}\log\left(bx^2+cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4),x]

[Out] Log[b*x^2 + c*x^4]/2
```

Rubi in Sympy [A] time = 11.5813, size = 15, normalized size = 0.94

$$\frac{\log\left(x^2\right)}{2}+\frac{\log\left(b+cx^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2),x)

[Out] log(x**2)/2 + log(b + c*x**2)/2
```

Mathematica [A] time = 0.0100363, size = 15, normalized size = 0.94

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] Log[x] + Log[b + c*x^2]/2
```

Maple [A] time = 0.007, size = 14, normalized size = 0.9

$$\frac{\ln\left(cx^2+b\right)}{2}+\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(2*c*x^2+b)/(c*x^4+b*x^2),x)
```

```
[Out] 1/2*ln(c*x^2+b)+ln(x)
```

Maxima [A] time = 0.750682, size = 23, normalized size = 1.44

$$\frac{1}{2}\log\left(cx^2+b\right)+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)*x/(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(c*x^2 + b) + 1/2*log(x^2)
```

Fricas [A] time = 0.282258, size = 18, normalized size = 1.12

$$\frac{1}{2}\log\left(cx^2+b\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)*x/(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*log(c*x^2 + b) + log(x)
```

Sympy [A] time = 1.23381, size = 12, normalized size = 0.75

$$\log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2),x)
```

```
[Out] log(x) + log(b/c + x**2)/2
```

GIAC/XCAS [A] time = 0.263608, size = 20, normalized size = 1.25

$$\frac{1}{2}\ln\left(\left|cx^2+b\right|\right)+\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)*x/(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(c*x^2 + b)) + ln(abs(x))
```

3.123 $\int \frac{x^2(b+2cx^3)}{bx^3+cx^6}\,dx$

Optimal. Leaf size=16

$$\frac{1}{3}\log\left(bx^3+cx^6\right)$$

```
[Out] Log[b*x^3 + c*x^6]/3
```

Rubi [A] time = 0.00993579, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04

$$\frac{1}{3}\log\left(bx^3+cx^6\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6),x]

[Out] Log[b*x^3 + c*x^6]/3
```

Rubi in Sympy [A] time = 10.8673, size = 15, normalized size = 0.94

$$\frac{\log\left(x^3\right)}{3}+\frac{\log\left(b+cx^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3),x)

[Out] log(x**3)/3 + log(b + c*x**3)/3
```

Mathematica [A] time = 0.0114407, size = 15, normalized size = 0.94

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Maple [A] time = 0.007, size = 14, normalized size = 0.9

$$\frac{\ln(cx^3+b)}{3}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x)
```

```
[Out] 1/3*ln(c*x^3+b)+ln(x)
```

Maxima [A] time = 0.745737, size = 23, normalized size = 1.44

$$\frac{1}{3}\log(cx^3+b)+\frac{1}{3}\log(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3 + b)*x^2/(c*x^6 + b*x^3),x, algorithm="maxima")
```

```
[Out] 1/3*log(c*x^3 + b) + 1/3*log(x^3)
```

Fricas [A] time = 0.259117, size = 18, normalized size = 1.12

$$\frac{1}{3}\log(cx^3+b)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3 + b)*x^2/(c*x^6 + b*x^3),x, algorithm="fricas")
```

```
[Out] 1/3*log(c*x^3 + b) + log(x)
```

Sympy [A] time = 1.29646, size = 12, normalized size = 0.75

$$\log(x)+\frac{\log\left(\frac{b}{c}+x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3),x)
```

```
[Out] log(x) + log(b/c + x**3)/3
```

GIAC/XCAS [A] time = 0.265025, size = 20, normalized size = 1.25

$$\frac{1}{3}\ln\left(\left|cx^3+b\right|\right)+\ln\left(\left|x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^3 + b)*x^2/(c*x^6 + b*x^3),x, algorithm="giac")
```

```
[Out] 1/3*ln(abs(c*x^3 + b)) + ln(abs(x))
```

3.124 $\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=15

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

```
[Out] Log[x] + Log[b + c*x^n]/n
```

Rubi [A] time = 0.0711117, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n)),x]

[Out] Log[x] + Log[b + c*x^n]/n
```

Rubi in Sympy [A] time = 13.2241, size = 36, normalized size = 2.4

$$\frac{x^{-n+1}x^{n-1}\log(x^n)}{n}+\frac{x^{-n+1}x^{n-1}\log(b+cx^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+n)*(b+2*c*x**n)/(b*x**n+c*x**(2*n)),x)

[Out] x**(-n + 1)*x**(n - 1)*log(x**n)/n + x**(-n + 1)*x**(n - 1)*log(b
 + c*x**n)/n
```

Mathematica [A] time = 0.0175578, size = 15, normalized size = 1.

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n)),x]

[Out] Log[x] + Log[b + c*x^n]/n
```

Maple [A] time = 0.03, size = 18, normalized size = 1.2

$$\ln (x)+\frac{\ln \left(c\,{\mathrm e}^{n \ln (x)}+b\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x)

[Out] ln(x)+1/n*ln(c*exp(n*ln(x))+b)
```

Maxima [A] time = 0.765521, size = 63, normalized size = 4.2

$$b\left(\frac{\log (x)}{b}-\frac{\log \left(\frac{cx^{n}+b}{c}\right)}{bn}\right)+\frac{2\,\log \left(\frac{cx^{n}+b}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^n + b)*x^(n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="maxima")

[Out] b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n
```

Fricas [A] time = 0.295318, size = 23, normalized size = 1.53

$$\frac{n\log (x)+\log \left(cx^{n}+b\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^n + b)*x^(n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="fricas")

[Out] (n*log(x) + log(c*x^n + b))/n
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+n)*(b+2*c*x**n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.26528, size = 23, normalized size = 1.53

$$\frac{\ln\left(|cx^n + b|\right)}{n} + \ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^n + b)*x^(n - 1)/(c*x^(2*n) + b*x^n),x, algorithm="giac")
```

```
[Out] ln(abs(c*x^n + b))/n + ln(abs(x))
```

3.125 $\int \frac{b+2cx}{(bx+cx^2)^8} \, dx$

Optimal. Leaf size=15

$$-\frac{1}{7\left(bx+cx^2\right)^7}$$

```
[Out] -1/(7*(b*x + c*x^2)^7)
```

Rubi [A] time = 0.00843283, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$-\frac{1}{7\left(bx+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 2*c*x)/(b*x + c*x^2)^8,x]

[Out] -1/(7*(b*x + c*x^2)^7)
```

Rubi in Sympy [A] time = 3.18446, size = 14, normalized size = 0.93

$$-\frac{1}{7\left(bx+cx^2\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*c*x+b)/(c*x**2+b*x)**8,x)

[Out] -1/(7*(b*x + c*x**2)**7)
```

Mathematica [A] time = 0.033947, size = 14, normalized size = 0.93

$$-\frac{1}{7x^7(b+cx)^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

```
[Out] -1/(7*x^7*(b + c*x)^7)
```

Maple [B] time = 0.028, size = 177, normalized size = 11.8

$$-\frac{1}{7\,b^{7}x^{7}}-132\,\frac{c^{6}}{b^{13}x}+66\,\frac{c^{5}}{b^{12}x^{2}}-30\,\frac{c^{4}}{b^{11}x^{3}}+12\,\frac{c^{3}}{b^{10}x^{4}}-4\,\frac{c^{2}}{b^{9}x^{5}}+\frac{c}{b^{8}x^{6}}+132\,\frac{c^{7}}{b^{13}\left(cx+b\right)}+66\,\frac{c^{7}}{b^{12}\left(cx+b\right)^{2}}+30\,\frac{c^{7}}{b^{11}\left(cx+b\right)^{3}}+12\,\frac{c^{7}}{b^{10}\left(cx+b\right)^{4}}+4\,\frac{c^{7}}{b^{9}\left(cx+b\right)^{5}}+\frac{c^{7}}{b^{8}\left(cx+b\right)^{6}}+\frac{c^{7}}{7\,b^{7}\left(cx+b\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*c*x+b)/(c*x^2+b*x)^8,x)
```

```
[Out] -1/7/b^7/x^7-132/b^13*c^6/x+66/b^12*c^5/x^2-30/b^11*c^4/x^3+12/b^
10*c^3/x^4-4/b^9*c^2/x^5+1/b^8*c/x^6+132*c^7/b^13/(c*x+b)+66*c^7/
b^12/(c*x+b)^2+30/b^11*c^7/(c*x+b)^3+12*c^7/b^10/(c*x+b)^4+4/b^9*
c^7/(c*x+b)^5+c^7/b^8/(c*x+b)^6+1/7*c^7/b^7/(c*x+b)^7
```

Maxima [A] time = 0.745359, size = 18, normalized size = 1.2

$$-\frac{1}{7\,\left(cx^{2}+bx\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

Fricas [A] time = 0.259154, size = 109, normalized size = 7.27

$$-\frac{1}{7\,\left(c^{7}x^{14}+7\,bc^{6}x^{13}+21\,b^{2}c^{5}x^{12}+35\,b^{3}c^{4}x^{11}+35\,b^{4}c^{3}x^{10}+21\,b^{5}c^{2}x^{9}+7\,b^{6}cx^{8}+b^{7}x^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11
 + 35*b^4*c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

Sympy [A] time = 17.6276, size = 87, normalized size = 5.8

$$-\frac{1}{7b^7x^7 + 49b^6cx^8 + 147b^5c^2x^9 + 245b^4c^3x^{10} + 245b^3c^4x^{11} + 147b^2c^5x^{12} + 49bc^6x^{13} + 7c^7x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x+b)/(c*x**2+b*x)**8,x)
```

```
[Out] -1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*
c**3*x**10 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**
6*x**13 + 7*c**7*x**14)
```

GIAC/XCAS [A] time = 0.267178, size = 18, normalized size = 1.2

$$-\frac{1}{7\left(cx^2 + bx\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)/(c*x^2 + b*x)^8,x, algorithm="giac")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

3.126 $\int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8} dx$

Optimal. Leaf size=17

$$-\frac{1}{14\,(bx^2+cx^4)^7}$$

```
[Out] -1/(14*(b*x^2 + c*x^4)^7)
```

Rubi [A] time = 0.0108445, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$-\frac{1}{14\,(bx^2+cx^4)^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8,x]

[Out] -1/(14*(b*x^2 + c*x^4)^7)
```

Rubi in Sympy [A] time = 13.0289, size = 15, normalized size = 0.88

$$-\frac{1}{14x^{14}\,(b+cx^2)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2)**8,x)

[Out] -1/(14*x**14*(b + c*x**2)**7)
```

Mathematica [A] time = 0.0491235, size = 16, normalized size = 0.94

$$-\frac{1}{14x^{14}\,(b+cx^2)^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

Maple [B] time = 0.028, size = 197, normalized size = 11.6

$$-\frac{c^8}{2\,b^{13}}\left(-\frac{b^6}{7\,c\,(cx^2+b)^7}-66\,\frac{b}{c\,(cx^2+b)^2}-\frac{b^5}{c\,(cx^2+b)^6}-30\,\frac{b^2}{c\,(cx^2+b)^3}-12\,\frac{b^3}{c\,(cx^2+b)^4}-4\,\frac{b^4}{c\,(cx^2+b)^5}-132\,\frac{1}{(cx^2+b)\,c}\right.$$
$$-\frac{1}{14\,b^7x^{14}}-66\,\frac{c^6}{b^{13}x^2}+33\,\frac{c^5}{b^{12}x^4}-15\,\frac{c^4}{b^{11}x^6}+6\,\frac{c^3}{b^{10}x^8}-2\,\frac{c^2}{b^9x^{10}}+\frac{c}{2\,b^8x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x)
```

```
[Out] -1/2*c^8/b^13*(-1/7*b^6/c/(c*x^2+b)^7-66*b/c/(c*x^2+b)^2-b^5/c/(c
*x^2+b)^6-30*b^2/c/(c*x^2+b)^3-12*b^3/c/(c*x^2+b)^4-4*b^4/c/(c*x^
2+b)^5-132/(c*x^2+b)/c)-1/14/b^7/x^14-66/b^13*c^6/x^2+33/b^12*c^5
/x^4-15/b^11*c^4/x^6+6/b^10*c^3/x^8-2/b^9*c^2/x^10+1/2/b^8*c/x^12
```

Maxima [A] time = 0.763783, size = 109, normalized size = 6.41

$$-\frac{1}{14\,(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)*x/(c*x^4 + b*x^2)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^2
2 + 35*b^4*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Fricas [A] time = 0.27915, size = 109, normalized size = 6.41

$$-\frac{1}{14\,(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)*x/(c*x^4 + b*x^2)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^2
2 + 35*b^4*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2)**8,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.267844, size = 20, normalized size = 1.18

$$-\frac{1}{14\,(cx^4+bx^2)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)*x/(c*x^4 + b*x^2)^8,x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2)^7
```

3.127 $\int \frac{x^2(b+2cx^3)}{(bx^3+cx^6)^8}\,dx$

Optimal. Leaf size=17

$$-\frac{1}{21\,(bx^3+cx^6)^7}$$

```
[Out] -1/(21*(b*x^3 + c*x^6)^7)
```

Rubi [A] time = 0.0114013, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04

$$-\frac{1}{21\,(bx^3+cx^6)^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/(21*(b*x^3 + c*x^6)^7)
```

Rubi in Sympy [A] time = 13.1662, size = 15, normalized size = 0.88

$$-\frac{1}{21x^{21}\,(b+cx^3)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3)**8,x)
```

```
[Out] -1/(21*x**21*(b + c*x**3)**7)
```

Mathematica [A] time = 0.0589748, size = 16, normalized size = 0.94

$$-\frac{1}{21x^{21}\,(b+cx^3)^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

Maple [B] time = 0.021, size = 197, normalized size = 11.6

$$-\frac{c^8}{3\,b^{13}}\left(-\frac{b^6}{7\,c\,(cx^3+b)^7}-66\,\frac{b}{c\,(cx^3+b)^2}-\frac{b^5}{c\,(cx^3+b)^6}-30\,\frac{b^2}{c\,(cx^3+b)^3}-12\,\frac{b^3}{c\,(cx^3+b)^4}-4\,\frac{b^4}{c\,(cx^3+b)^5}-132\,\frac{1}{c\,(cx^3+b)}\right.$$
$$-\frac{1}{21\,b^7x^{21}}-44\,\frac{c^6}{b^{13}x^3}+22\,\frac{c^5}{b^{12}x^6}-10\,\frac{c^4}{b^{11}x^9}+4\,\frac{c^3}{b^{10}x^{12}}-\frac{4\,c^2}{3\,b^9x^{15}}+\frac{c}{3\,b^8x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x)
```

```
[Out] -1/3*c^8/b^13*(-1/7*b^6/c/(c*x^3+b)^7-66*b/c/(c*x^3+b)^2-b^5/c/(c
*x^3+b)^6-30*b^2/c/(c*x^3+b)^3-12*b^3/c/(c*x^3+b)^4-4*b^4/c/(c*x^
3+b)^5-132/c/(c*x^3+b))-1/21/b^7/x^21-44/b^13*c^6/x^3+22/b^12*c^5
/x^6-10/b^11*c^4/x^9+4/b^10*c^3/x^12-4/3/b^9*c^2/x^15+1/3/b^8*c/x
^18
```

Maxima [A] time = 0.767351, size = 109, normalized size = 6.41

$$-\frac{1}{21\,(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^3 + b)*x^2/(c*x^6 + b*x^3)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^3
3 + 35*b^4*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Fricas [A] time = 0.274183, size = 109, normalized size = 6.41

$$-\frac{1}{21\,(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3 + b)*x^2/(c*x^6 + b*x^3)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^3
3 + 35*b^4*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3)**8,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.268839, size = 20, normalized size = 1.18

$$-\frac{1}{21\,(cx^6 + bx^3)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3 + b)*x^2/(c*x^6 + b*x^3)^8,x, algorithm="giac")
```

```
[Out] -1/21/(c*x^6 + b*x^3)^7
```

3.128 $\int \frac{x^{-1+n}(b+2cx^n)}{(bx^n+cx^{2n})^8}\,dx$

Optimal. Leaf size=21

$$-\frac{x^{-7n}}{7n\left(b+cx^n\right)^7}$$

```
[Out] -1/(7*n*x^(7*n)*(b + c*x^n)^7)
```

Rubi [A] time = 0.0627039, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103

$$-\frac{x^{-7n}}{7n\left(b+cx^n\right)^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n))^8,x]

[Out] -1/(7*n*x^(7*n)*(b + c*x^n)^7)
```

Rubi in Sympy [A] time = 11.0252, size = 19, normalized size = 0.9

$$-\frac{x^{-7n}}{7n\left(b+cx^n\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+n)*(b+2*c*x**n)/(b*x**n+c*x**(2*n))**8,x)

[Out] -x**(-7*n)/(7*n*(b + c*x**n)**7)
```

Mathematica [B] time = 0.0788099, size = 127, normalized size = 6.05

$$-\frac{x^{-7n}\left(b^{14}+1716b^7c^7x^{7n}+12012b^6c^8x^{8n}+36036b^5c^9x^{9n}+60060b^4c^{10}x^{10n}+60060b^3c^{11}x^{11n}+36036b^2c^{12}x^{12n}+12012bc\right.}{7b^{14}n\left(b+cx^n\right)^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n))^8,x]
```

```
[Out] -(b^14 + 1716*b^7*c^7*x^(7*n) + 12012*b^6*c^8*x^(8*n) + 36036*b^5
*c^9*x^(9*n) + 60060*b^4*c^10*x^(10*n) + 60060*b^3*c^11*x^(11*n)
+ 36036*b^2*c^12*x^(12*n) + 12012*b*c^13*x^(13*n) + 1716*c^14*x^(
14*n))/(7*b^14*n*x^(7*n)*(b + c*x^n)^7)
```

Maple [B] time = 0.086, size = 203, normalized size = 9.7

$$-132\,\frac{c^6}{b^{13}nx^n}+66\,\frac{c^5}{b^{12}n\,(x^n)^2}-30\,\frac{c^4}{b^{11}n\,(x^n)^3}+12\,\frac{c^3}{b^{10}n\,(x^n)^4}-4\,\frac{c^2}{b^9n\,(x^n)^5}+\frac{c}{b^8n\,(x^n)^6}-\frac{1}{7\,b^7n\,(x^n)^7}+\frac{c^7\left(924\,(x^n)^6c^6+6006\,bc^5\,(x^n)^5+16380\,b^2c^4\,(x^n)^4+24024\,b^3c^3\,(x^n)^3+20020\,b^4c^2\,(x^n)^2+9009\,b^5cx^n+1716\,b^6\right)}{7\,b^{13}n\,(b+cx^n)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x)
```

```
[Out] -132/b^13*c^6/n/(x^n)+66/b^12*c^5/n/(x^n)^2-30/b^11*c^4/n/(x^n)^3
+12/b^10*c^3/n/(x^n)^4-4/b^9*c^2/n/(x^n)^5+1/b^8*c/n/(x^n)^6-1/7/
b^7/n/(x^n)^7+1/7*c^7*(924*(x^n)^6*c^6+6006*b*c^5*(x^n)^5+16380*b
^2*c^4*(x^n)^4+24024*b^3*c^3*(x^n)^3+20020*b^4*c^2*(x^n)^2+9009*b
^5*c*x^n+1716*b^6)/b^13/n/(b+c*x^n)^7
```

Maxima [A] time = 0.798319, size = 826, normalized size = 39.33

$$-\frac{1}{105}\,b\left(\frac{360360\,c^{13}x^{13\,n}+2342340\,bc^{12}x^{12\,n}+6426420\,b^2c^{11}x^{11\,n}+9579570\,b^3c^{10}x^{10\,n}+8270262\,b^4c^9x^{9\,n}+4018014\,b^5c^8x^{8}}{b^{14}c^7nx^{14\,n}+7\,b^{15}c^6nx^{13\,n}+21\,b^{16}c^5nx^{12\,n}+35\,b^{17}c^4nx}\right.$$
$$+\frac{1}{105}\,c\left(\frac{360360\,c^{12}x^{12\,n}+2342340\,bc^{11}x^{11\,n}+6426420\,b^2c^{10}x^{10\,n}+9579570\,b^3c^9x^{9\,n}+8270262\,b^4c^8x^{8\,n}+4018014\,b^5c^7x^{7\,n}}{b^{13}c^7nx^{13\,n}+7\,b^{14}c^6nx^{12\,n}+21\,b^{15}c^5nx^{11\,n}+35\,b^{16}c^4nx^{10\,n}+3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^n + b)*x^(n - 1)/(c*x^(2*n) + b*x^n)^8,x, algorithm="maxima")
```

```
[Out] -1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 64264
20*b^2*c^11*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^
9*x^(9*n) + 4018014*b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45
045*b^7*c^6*x^(6*n) - 5005*b^8*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n)
```

```
 - 273*b^10*c^3*x^(3*n) + 91*b^11*c^2*x^(2*n) - 35*b^12*c*x^n + 1
5*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x^(13*n) + 21*b^16*c^
5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*x^(10*n) +
21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) + 36
0360*c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n)) +
1/105*c*((360360*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 642642
0*b^2*c^10*x^(10*n) + 9579570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x
^(8*n) + 4018014*b^5*c^7*x^(7*n) + 934362*b^6*c^6*x^(6*n) + 45045
*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^(4*n) + 1001*b^9*c^3*x^(3*n) -
273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b^12)/(b^13*c^7*n*x^(13
*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n) + 35*b^16*c^
4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n) + 7*
b^19*c*n*x^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360
360*c^6*log((c*x^n + b)/c)/(b^14*n))
```

Fricas [A] time = 0.353914, size = 142, normalized size = 6.76

$$-\frac{1}{7\left(c^{7} n x^{14 n} + 7 b c^{6} n x^{13 n} + 21 b^{2} c^{5} n x^{12 n} + 35 b^{3} c^{4} n x^{11 n} + 35 b^{4} c^{3} n x^{10 n} + 21 b^{5} c^{2} n x^{9 n} + 7 b^{6} c n x^{8 n} + b^{7} n x^{7 n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^n + b)*x^(n - 1)/(c*x^(2*n) + b*x^n)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 21*b^2*c^5*n*x^(12*n)
 + 35*b^3*c^4*n*x^(11*n) + 35*b^4*c^3*n*x^(10*n) + 21*b^5*c^2*n*x
^(9*n) + 7*b^6*c*n*x^(8*n) + b^7*n*x^(7*n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+n)*(b+2*c*x**n)/(b*x**n+c*x**(2*n))**8,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.272335, size = 27, normalized size = 1.29

$$-\frac{1}{7\left(c x^{2 n} + b x^{n}\right)^{7} n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^n + b)*x^(n - 1)/(c*x^(2*n) + b*x^n)^8,x, algorithm="giac")
```

```
[Out] -1/7/((c*x^(2*n) + b*x^n)^7*n)
```

3.129 $\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx$

Optimal. Leaf size=20

$$\frac{\left(a+bx+cx^2\right)^{p+1}}{p+1}$$

```
[Out] (a + b*x + c*x^2)^(1 + p)/(1 + p)
```

Rubi [A] time = 0.0132601, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{\left(a+bx+cx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 2*c*x)*(a + b*x + c*x^2)^p,x]

[Out] (a + b*x + c*x^2)^(1 + p)/(1 + p)
```

Rubi in Sympy [A] time = 3.9911, size = 15, normalized size = 0.75

$$\frac{\left(a+bx+cx^2\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*c*x+b)*(c*x**2+b*x+a)**p,x)

[Out] (a + b*x + c*x**2)**(p + 1)/(p + 1)
```

Mathematica [A] time = 0.0302989, size = 19, normalized size = 0.95

$$\frac{(a+x(b+cx))^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*c*x)*(a + b*x + c*x^2)^p,x]

[Out] (a + x*(b + c*x))^(1 + p)/(1 + p)
```

Maple [A] time = 0.004, size = 21, normalized size = 1.1

$$\frac{\left(cx^2 + bx + a\right)^{1+p}}{1 + p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*c*x+b)*(c*x^2+b*x+a)^p,x)

[Out] (c*x^2+b*x+a)^(1+p)/(1+p)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)*(c*x^2 + b*x + a)^p,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.300159, size = 38, normalized size = 1.9

$$\frac{\left(cx^2 + bx + a\right)\left(cx^2 + bx + a\right)^p}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)*(c*x^2 + b*x + a)^p,x, algorithm="fricas")

[Out] (c*x^2 + b*x + a)*(c*x^2 + b*x + a)^p/(p + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x+b)*(c*x**2+b*x+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.269229, size = 80, normalized size = 4.

$$\frac{cx^{2}e^{\left(p\ln\left(cx^{2}+bx+a\right)\right)}+bxe^{\left(p\ln\left(cx^{2}+bx+a\right)\right)}+ae^{\left(p\ln\left(cx^{2}+bx+a\right)\right)}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)*(c*x^2 + b*x + a)^p,x, algorithm="giac")
```

```
[Out] (c*x^2*e^(p*ln(c*x^2 + b*x + a)) + b*x*e^(p*ln(c*x^2 + b*x + a))
+ a*e^(p*ln(c*x^2 + b*x + a)))/(p + 1)
```

3.130 $\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^p\,dx$

Optimal. Leaf size=25

$$\frac{\left(a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

```
[Out] (a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rubi [A] time = 0.0138469, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{\left(a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p,x]

[Out] (a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rubi in Sympy [A] time = 5.55645, size = 19, normalized size = 0.76

$$\frac{\left(a+bx^2+cx^4\right)^{p+1}}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2+a)**p,x)

[Out] (a + b*x**2 + c*x**4)**(p + 1)/(2*(p + 1))
```

Mathematica [A] time = 0.0336139, size = 24, normalized size = 0.96

$$\frac{\left(a+bx^2+cx^4\right)^{p+1}}{2p+2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p,x]

[Out] (a + b*x^2 + c*x^4)^(1 + p)/(2 + 2*p)
```

Maple [A] time = 0.005, size = 24, normalized size = 1.

$$\frac{\left(cx^4 + bx^2 + a\right)^{1+p}}{2 + 2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x)

[Out] 1/2*(c*x^4+b*x^2+a)^(1+p)/(1+p)
```

Maxima [A] time = 0.856237, size = 45, normalized size = 1.8

$$\frac{\left(cx^4 + bx^2 + a\right)\left(cx^4 + bx^2 + a\right)^p}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2 + b)*(c*x^4 + b*x^2 + a)^p*x,x, algorithm="maxima")

[Out] 1/2*(c*x^4 + b*x^2 + a)*(c*x^4 + b*x^2 + a)^p/(p + 1)
```

Fricas [A] time = 0.278075, size = 45, normalized size = 1.8

$$\frac{\left(cx^4 + bx^2 + a\right)\left(cx^4 + bx^2 + a\right)^p}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2 + b)*(c*x^4 + b*x^2 + a)^p*x,x, algorithm="fricas")

[Out] 1/2*(c*x^4 + b*x^2 + a)*(c*x^4 + b*x^2 + a)^p/(p + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.272383, size = 92, normalized size = 3.68

$$\frac{cx^4e^{\left(p\ln\left(cx^4+bx^2+a\right)\right)} + bx^2e^{\left(p\ln\left(cx^4+bx^2+a\right)\right)} + ae^{\left(p\ln\left(cx^4+bx^2+a\right)\right)}}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)*(c*x^4 + b*x^2 + a)^p*x,x, algorithm="giac")
```

```
[Out] 1/2*(c*x^4*e^(p*ln(c*x^4 + b*x^2 + a)) + b*x^2*e^(p*ln(c*x^4 + b*
x^2 + a)) + a*e^(p*ln(c*x^4 + b*x^2 + a)))/(p + 1)
```

3.131 $\int x^2 \left(b + 2cx^3\right) \left(a + bx^3 + cx^6\right)^p \, dx$

Optimal. Leaf size=25

$$\frac{\left(a + bx^3 + cx^6\right)^{p+1}}{3(p + 1)}$$

```
[Out] (a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Rubi [A] time = 0.0144127, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$

$$\frac{\left(a + bx^3 + cx^6\right)^{p+1}}{3(p + 1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p,x]

[Out] (a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Rubi in Sympy [A] time = 5.62997, size = 19, normalized size = 0.76

$$\frac{\left(a + bx^3 + cx^6\right)^{p+1}}{3\left(p + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3+a)**p,x)

[Out] (a + b*x**3 + c*x**6)**(p + 1)/(3*(p + 1))
```

Mathematica [A] time = 0.0325372, size = 24, normalized size = 0.96

$$\frac{\left(a + bx^3 + cx^6\right)^{p+1}}{3p + 3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (a + b*x^3 + c*x^6)^(1 + p)/(3 + 3*p)
```

Maple [A] time = 0.008, size = 24, normalized size = 1.

$$\frac{\left(c x^6+b x^3+a\right)^{1+p}}{3+3 p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] 1/3*(c*x^6+b*x^3+a)^(1+p)/(1+p)
```

Maxima [A] time = 0.849011, size = 45, normalized size = 1.8

$$\frac{\left(c x^6+b x^3+a\right)\left(c x^6+b x^3+a\right)^p}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3 + b)*(c*x^6 + b*x^3 + a)^p*x^2,x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^6 + b*x^3 + a)*(c*x^6 + b*x^3 + a)^p/(p + 1)
```

Fricas [A] time = 0.276342, size = 45, normalized size = 1.8

$$\frac{\left(c x^6+b x^3+a\right)\left(c x^6+b x^3+a\right)^p}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3 + b)*(c*x^6 + b*x^3 + a)^p*x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(c*x^6 + b*x^3 + a)*(c*x^6 + b*x^3 + a)^p/(p + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.280736, size = 92, normalized size = 3.68

$$\frac{cx^6 e^{\left(p \ln\left(cx^6+bx^3+a\right)\right)} + bx^3 e^{\left(p \ln\left(cx^6+bx^3+a\right)\right)} + a e^{\left(p \ln\left(cx^6+bx^3+a\right)\right)}}{3\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^3 + b)*(c*x^6 + b*x^3 + a)^p*x^2,x, algorithm="giac")
```

```
[Out] 1/3*(c*x^6*e^(p*ln(c*x^6 + b*x^3 + a)) + b*x^3*e^(p*ln(c*x^6 + b*
x^3 + a)) + a*e^(p*ln(c*x^6 + b*x^3 + a)))/(p + 1)
```

3.132 $\int x^{-1+n} (b + 2cx^n) (a + bx^n + cx^{2n})^p \, dx$

Optimal. Leaf size=27

$$\frac{(a + bx^n + cx^{2n})^{p+1}}{n(p + 1)}$$

```
[Out] (a + b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rubi [A] time = 0.0685957, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{(a + bx^n + cx^{2n})^{p+1}}{n(p + 1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^p,x]

[Out] (a + b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rubi in Sympy [A] time = 12.312, size = 20, normalized size = 0.74

$$\frac{(a + bx^n + cx^{2n})^{p+1}}{n (p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+n)*(b+2*c*x**n)*(a+b*x**n+c*x**(2*n))**p,x)

[Out] (a + b*x**n + c*x**(2*n))**(p + 1)/(n*(p + 1))
```

Mathematica [A] time = 0.0868031, size = 26, normalized size = 0.96

$$\frac{(a + bx^n + cx^{2n})^{p+1}}{np + n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^p,x]

[Out] (a + b*x^n + c*x^(2*n))^(1 + p)/(n + n*p)
```

Maple [A] time = 0.089, size = 40, normalized size = 1.5

$$\frac{\left(a+bx^{n}+c\left(x^{n}\right)^{2}\right)\left(a+bx^{n}+c\left(x^{n}\right)^{2}\right)^{p}}{n\left(1+p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^p,x)

[Out] (a+b*x^n+c*(x^n)^2)/n/(1+p)*(a+b*x^n+c*(x^n)^2)^p
```

Maxima [A] time = 1.03521, size = 53, normalized size = 1.96

$$\frac{\left(cx^{2n}+bx^{n}+a\right)\left(cx^{2n}+bx^{n}+a\right)^{p}}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^n + b)*(c*x^(2*n) + b*x^n + a)^p*x^(n - 1),x, algorithm="maxima"

[Out] (c*x^(2*n) + b*x^n + a)*(c*x^(2*n) + b*x^n + a)^p/(n*(p + 1))
```

Fricas [A] time = 0.285231, size = 51, normalized size = 1.89

$$\frac{\left(cx^{2n}+bx^{n}+a\right)\left(cx^{2n}+bx^{n}+a\right)^{p}}{np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^n + b)*(c*x^(2*n) + b*x^n + a)^p*x^(n - 1),x, algorithm="fricas"

[Out] (c*x^(2*n) + b*x^n + a)*(c*x^(2*n) + b*x^n + a)^p/(n*p + n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+n)*(b+2*c*x**n)*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.288136, size = 36, normalized size = 1.33

$$\frac{\left(cx^{2\,n} + bx^{n} + a\right)^{p+1}}{n(p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^n + b)*(c*x^(2*n) + b*x^n + a)^p*x^(n - 1),x, algorithm="giac")
```

```
[Out] (c*x^(2*n) + b*x^n + a)^(p + 1)/(n*(p + 1))
```

3.133 $\int (b+2cx)\left(-a+bx+cx^2\right)^p \, dx$

Optimal. Leaf size=22

$$\frac{\left(-a+bx+cx^2\right)^{p+1}}{p+1}$$

```
[Out] (-a + b*x + c*x^2)^(1 + p)/(1 + p)
```

Rubi [A] time = 0.0108548, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{\left(-a+bx+cx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 2*c*x)*(-a + b*x + c*x^2)^p,x]
```

```
[Out] (-a + b*x + c*x^2)^(1 + p)/(1 + p)
```

Rubi in Sympy [A] time = 4.38538, size = 15, normalized size = 0.68

$$\frac{\left(-a+bx+cx^2\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*c*x+b)*(c*x**2+b*x-a)**p,x)
```

```
[Out] (-a + b*x + c*x**2)**(p + 1)/(p + 1)
```

Mathematica [A] time = 0.0310867, size = 21, normalized size = 0.95

$$\frac{(x(b+cx)-a)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*c*x)*(-a + b*x + c*x^2)^p,x]

[Out] (-a + x*(b + c*x))^(1 + p)/(1 + p)
```

Maple [A] time = 0.004, size = 23, normalized size = 1.1

$$\frac{\left(cx^2+bx-a\right)^{1+p}}{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*c*x+b)*(c*x^2+b*x-a)^p,x)

[Out] (c*x^2+b*x-a)^(1+p)/(1+p)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)*(c*x^2 + b*x - a)^p,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.281159, size = 43, normalized size = 1.95

$$\frac{\left(cx^2+bx-a\right)\left(cx^2+bx-a\right)^{p}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)*(c*x^2 + b*x - a)^p,x, algorithm="fricas")

[Out] (c*x^2 + b*x - a)*(c*x^2 + b*x - a)^p/(p + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x+b)*(c*x**2+b*x-a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.266364, size = 89, normalized size = 4.05

$$\frac{cx^2e^{\left(p\ln\left(cx^2+bx-a\right)\right)} + bxe^{\left(p\ln\left(cx^2+bx-a\right)\right)} - ae^{\left(p\ln\left(cx^2+bx-a\right)\right)}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)*(c*x^2 + b*x - a)^p,x, algorithm="giac")
```

```
[Out] (c*x^2*e^(p*ln(c*x^2 + b*x - a)) + b*x*e^(p*ln(c*x^2 + b*x - a))
- a*e^(p*ln(c*x^2 + b*x - a)))/(p + 1)
```

3.134 $\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^p\,dx$

Optimal. Leaf size=27

$$\frac{\left(-a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

```
[Out] (-a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rubi [A] time = 0.0133516, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{\left(-a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (-a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rubi in Sympy [A] time = 6.06874, size = 19, normalized size = 0.7

$$\frac{\left(-a+bx^2+cx^4\right)^{p+1}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2-a)**p,x)
```

```
[Out] (-a + b*x**2 + c*x**4)**(p + 1)/(2*(p + 1))
```

Mathematica [A] time = 0.0381282, size = 26, normalized size = 0.96

$$\frac{\left(-a+bx^2+cx^4\right)^{p+1}}{2p+2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^p,x]

[Out] (-a + b*x^2 + c*x^4)^(1 + p)/(2 + 2*p)
```

Maple [A] time = 0.005, size = 26, normalized size = 1.

$$\frac{\left(cx^4 + bx^2 - a\right)^{1+p}}{2 + 2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x)

[Out] 1/2*(c*x^4+b*x^2-a)^(1+p)/(1+p)
```

Maxima [A] time = 0.831103, size = 50, normalized size = 1.85

$$\frac{\left(cx^4 + bx^2 - a\right)\left(cx^4 + bx^2 - a\right)^p}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)*(c*x^4 + b*x^2 - a)^p*x,x, algorithm="maxima")

[Out] 1/2*(c*x^4 + b*x^2 - a)*(c*x^4 + b*x^2 - a)^p/(p + 1)
```

Fricas [A] time = 0.287448, size = 50, normalized size = 1.85

$$\frac{\left(cx^4 + bx^2 - a\right)\left(cx^4 + bx^2 - a\right)^p}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)*(c*x^4 + b*x^2 - a)^p*x,x, algorithm="fricas")

[Out] 1/2*(c*x^4 + b*x^2 - a)*(c*x^4 + b*x^2 - a)^p/(p + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2-a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.2727, size = 101, normalized size = 3.74

$$\frac{cx^4e^{\left(p\ln\left(cx^4+bx^2-a\right)\right)} + bx^2e^{\left(p\ln\left(cx^4+bx^2-a\right)\right)} - ae^{\left(p\ln\left(cx^4+bx^2-a\right)\right)}}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)*(c*x^4 + b*x^2 - a)^p*x,x, algorithm="giac")
```

```
[Out] 1/2*(c*x^4*e^(p*ln(c*x^4 + b*x^2 - a)) + b*x^2*e^(p*ln(c*x^4 + b*
x^2 - a)) - a*e^(p*ln(c*x^4 + b*x^2 - a)))/(p + 1)
```

3.135 $\int x^2 \left(b+2cx^3\right) \left(-a+bx^3+cx^6\right)^p \, dx$

Optimal. Leaf size=27

$$\frac{\left(-a+bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

```
[Out] (-a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Rubi [A] time = 0.0136882, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$

$$\frac{\left(-a+bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^p,x]

[Out] (-a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Rubi in Sympy [A] time = 6.13775, size = 19, normalized size = 0.7

$$\frac{\left(-a+bx^3+cx^6\right)^{p+1}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3-a)**p,x)

[Out] (-a + b*x**3 + c*x**6)**(p + 1)/(3*(p + 1))
```

Mathematica [A] time = 0.0375807, size = 26, normalized size = 0.96

$$\frac{\left(-a+bx^3+cx^6\right)^{p+1}}{3p+3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^p,x]

[Out] (-a + b*x^3 + c*x^6)^(1 + p)/(3 + 3*p)
```

Maple [A] time = 0.007, size = 26, normalized size = 1.

$$\frac{(cx^6 + bx^3 - a)^{1+p}}{3 + 3p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x)

[Out] 1/3*(c*x^6+b*x^3-a)^(1+p)/(1+p)
```

Maxima [A] time = 0.844018, size = 50, normalized size = 1.85

$$\frac{(cx^6 + bx^3 - a)(cx^6 + bx^3 - a)^p}{3(p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3 + b)*(c*x^6 + b*x^3 - a)^p*x^2,x, algorithm="maxima")

[Out] 1/3*(c*x^6 + b*x^3 - a)*(c*x^6 + b*x^3 - a)^p/(p + 1)
```

Fricas [A] time = 0.278871, size = 50, normalized size = 1.85

$$\frac{(cx^6 + bx^3 - a)(cx^6 + bx^3 - a)^p}{3(p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3 + b)*(c*x^6 + b*x^3 - a)^p*x^2,x, algorithm="fricas")

[Out] 1/3*(c*x^6 + b*x^3 - a)*(c*x^6 + b*x^3 - a)^p/(p + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3-a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.275319, size = 101, normalized size = 3.74

$$\frac{cx^6e^{\left(p\ln\left(cx^6+bx^3-a\right)\right)}+bx^3e^{\left(p\ln\left(cx^6+bx^3-a\right)\right)}-ae^{\left(p\ln\left(cx^6+bx^3-a\right)\right)}}{3\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^3 + b)*(c*x^6 + b*x^3 - a)^p*x^2,x, algorithm="giac")
```

```
[Out] 1/3*(c*x^6*e^(p*ln(c*x^6 + b*x^3 - a)) + b*x^3*e^(p*ln(c*x^6 + b*
x^3 - a)) - a*e^(p*ln(c*x^6 + b*x^3 - a)))/(p + 1)
```

3.136 $\int x^{-1+n}\left(b+2cx^n\right)\left(-a+bx^n+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=29

$$\frac{\left(-a+bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

```
[Out] (-a + b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rubi [A] time = 0.0692443, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\left(-a+bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (-a + b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rubi in Sympy [A] time = 13.2978, size = 20, normalized size = 0.69

$$\frac{\left(-a+bx^n+cx^{2n}\right)^{p+1}}{n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+n)*(b+2*c*x**n)*(-a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] (-a + b*x**n + c*x**(2*n))**(p + 1)/(n*(p + 1))
```

Mathematica [A] time = 0.0875493, size = 28, normalized size = 0.97

$$\frac{\left(-a+bx^n+cx^{2n}\right)^{p+1}}{np+n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^p,x]

[Out] (-a + b*x^n + c*x^(2*n))^(1 + p)/(n + n*p)
```

Maple [A] time = 0.094, size = 45, normalized size = 1.6

$$-\frac{\left(-c\left(x^{n}\right)^{2}-bx^{n}+a\right)\left(-a+bx^{n}+c\left(x^{n}\right)^{2}\right)^{p}}{n\left(1+p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x)

[Out] -(-c*(x^n)^2-b*x^n+a)/n/(1+p)*(-a+b*x^n+c*(x^n)^2)^p
```

Maxima [A] time = 1.04987, size = 58, normalized size = 2.

$$\frac{\left(cx^{2\,n}+bx^{n}-a\right)\left(cx^{2\,n}+bx^{n}-a\right)^{p}}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^n + b)*(c*x^(2*n) + b*x^n - a)^p*x^(n - 1),x, algorithm="maxima"

[Out] (c*x^(2*n) + b*x^n - a)*(c*x^(2*n) + b*x^n - a)^p/(n*(p + 1))
```

Fricas [A] time = 0.307078, size = 57, normalized size = 1.97

$$\frac{\left(cx^{2\,n}+bx^{n}-a\right)\left(cx^{2\,n}+bx^{n}-a\right)^{p}}{np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^n + b)*(c*x^(2*n) + b*x^n - a)^p*x^(n - 1),x, algorithm="fricas"

[Out] (c*x^(2*n) + b*x^n - a)*(c*x^(2*n) + b*x^n - a)^p/(n*p + n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+n)*(b+2*c*x**n)*(-a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.286956, size = 39, normalized size = 1.34

$$\frac{\left(cx^{2n} + bx^{n} - a\right)^{p+1}}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^n + b)*(c*x^(2*n) + b*x^n - a)^p*x^(n - 1),x, algorithm="giac")
```

```
[Out] (c*x^(2*n) + b*x^n - a)^(p + 1)/(n*(p + 1))
```

3.137 $\int (b+2cx)\left(bx+cx^2\right)^p\,dx$

Optimal. Leaf size=19

$$\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

```
[Out] (b*x + c*x^2)^(1 + p)/(1 + p)
```

Rubi [A] time = 0.0102823, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 2*c*x)*(b*x + c*x^2)^p,x]

[Out] (b*x + c*x^2)^(1 + p)/(1 + p)
```

Rubi in Sympy [A] time = 3.51661, size = 14, normalized size = 0.74

$$\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*c*x+b)*(c*x**2+b*x)**p,x)

[Out] (b*x + c*x**2)**(p + 1)/(p + 1)
```

Mathematica [A] time = 0.03231, size = 17, normalized size = 0.89

$$\frac{(x(b+cx))^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*c*x)*(b*x + c*x^2)^p,x]

[Out] (x*(b + c*x))^(1 + p)/(1 + p)
```

Maple [A] time = 0.006, size = 24, normalized size = 1.3

$$\frac{x(cx+b)\left(cx^2+bx\right)^p}{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*c*x+b)*(c*x^2+b*x)^p,x)

[Out] x*(c*x+b)/(1+p)*(c*x^2+b*x)^p
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)*(c*x^2 + b*x)^p,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.298969, size = 35, normalized size = 1.84

$$\frac{\left(cx^2+bx\right)\left(cx^2+bx\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)*(c*x^2 + b*x)^p,x, algorithm="fricas")

[Out] (c*x^2 + b*x)*(c*x^2 + b*x)^p/(p + 1)
```

Sympy [A] time = 1.72258, size = 46, normalized size = 2.42

$$\begin{cases} \frac{bx(bx+cx^2)^p}{p+1} + \frac{cx^2(bx+cx^2)^p}{p+1} & \text{for } p \neq -1 \\ \log(x) + \log\left(\frac{b}{c} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x+b)*(c*x**2+b*x)**p,x)
```

```
[Out] Piecewise((b*x*(b*x + c*x**2)**p/(p + 1) + c*x**2*(b*x + c*x**2)*
*p/(p + 1), Ne(p, -1)), (log(x) + log(b/c + x), True))
```

GIAC/XCAS [A] time = 0.264448, size = 55, normalized size = 2.89

$$\frac{cx^2e^{\left(p\ln\left(cx^2+bx\right)\right)} + bxe^{\left(p\ln\left(cx^2+bx\right)\right)}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x + b)*(c*x^2 + b*x)^p,x, algorithm="giac")
```

```
[Out] (c*x^2*e^(p*ln(c*x^2 + b*x)) + b*x*e^(p*ln(c*x^2 + b*x)))/(p + 1)
```

3.138 $\int x \left(b+2 c x^2\right) \left(b x^2+c x^4\right)^p \, dx$

Optimal. Leaf size=24

$$\frac{\left(bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

```
[Out] (b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rubi [A] time = 0.0117802, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{\left(bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p,x]

[Out] (b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rubi in Sympy [A] time = 4.97208, size = 17, normalized size = 0.71

$$\frac{\left(bx^2+cx^4\right)^{p+1}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2)**p,x)

[Out] (b*x**2 + c*x**4)**(p + 1)/(2*(p + 1))
```

Mathematica [A] time = 0.0441769, size = 24, normalized size = 1.

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p,x]

[Out] (x^2*(b + c*x^2))^(1 + p)/(2*(1 + p))
```

Maple [A] time = 0.005, size = 31, normalized size = 1.3

$$\frac{x^2\left(cx^2+b\right)\left(cx^4+bx^2\right)^p}{2+2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x)

[Out] 1/2*(c*x^2+b)*x^2/(1+p)*(c*x^4+b*x^2)^p
```

Maxima [A] time = 0.906413, size = 47, normalized size = 1.96

$$\frac{\left(cx^4+bx^2\right)e^{\left(p\log\left(cx^2+b\right)+2\,p\log\left(x\right)\right)}}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)*(c*x^4 + b*x^2)^p*x,x, algorithm="maxima")

[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

Fricas [A] time = 0.280106, size = 42, normalized size = 1.75

$$\frac{\left(cx^4+bx^2\right)\left(cx^4+bx^2\right)^p}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)*(c*x^4 + b*x^2)^p*x,x, algorithm="fricas")

[Out] 1/2*(c*x^4 + b*x^2)*(c*x^4 + b*x^2)^p/(p + 1)
```

Sympy [A] time = 61.0815, size = 85, normalized size = 3.54

$$\begin{cases} \frac{bx^2(bx^2+cx^4)^p}{2p+2} + \frac{cx^4(bx^2+cx^4)^p}{2p+2} & \text{for } p \neq -1 \\ \log(x) + \frac{\log\left(-i\sqrt{b}\sqrt{\frac{1}{c}}+x\right)}{2} + \frac{\log\left(i\sqrt{b}\sqrt{\frac{1}{c}}+x\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2)**p,x)
```

```
[Out] Piecewise((b*x**2*(b*x**2 + c*x**4)**p/(2*p + 2) + c*x**4*(b*x**2
 + c*x**4)**p/(2*p + 2), Ne(p, -1)), (log(x) + log(-I*sqrt(b)*sqr
t(1/c) + x)/2 + log(I*sqrt(b)*sqrt(1/c) + x)/2, True))
```

GIAC/XCAS [A] time = 0.268355, size = 65, normalized size = 2.71

$$\frac{cx^4e^{\left(p\ln\left(cx^4+bx^2\right)\right)} + bx^2e^{\left(p\ln\left(cx^4+bx^2\right)\right)}}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^2 + b)*(c*x^4 + b*x^2)^p*x,x, algorithm="giac")
```

```
[Out] 1/2*(c*x^4*e^(p*ln(c*x^4 + b*x^2)) + b*x^2*e^(p*ln(c*x^4 + b*x^2)
))/(p + 1)
```

3.139 $\int x^2 \left(b+2cx^3\right) \left(bx^3+cx^6\right)^p \, dx$

Optimal. Leaf size=24

$$\frac{\left(bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

```
[Out] (b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Rubi [A] time = 0.0126921, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\frac{\left(bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p,x]
```

```
[Out] (b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Rubi in Sympy [A] time = 5.01885, size = 17, normalized size = 0.71

$$\frac{\left(bx^3+cx^6\right)^{p+1}}{3\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3)**p,x)
```

```
[Out] (b*x**3 + c*x**6)**(p + 1)/(3*(p + 1))
```

Mathematica [A] time = 0.0454302, size = 24, normalized size = 1.

$$\frac{\left(x^3\left(b+cx^3\right)\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p,x]

[Out] (x^3*(b + c*x^3))^(1 + p)/(3*(1 + p))
```

Maple [A] time = 0.005, size = 31, normalized size = 1.3

$$\frac{x^3 \left(cx^3 + b\right) \left(cx^6 + bx^3\right)^p}{3 + 3p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x)

[Out] 1/3*(c*x^3+b)*x^3/(1+p)*(c*x^6+b*x^3)^p
```

Maxima [A] time = 0.922656, size = 47, normalized size = 1.96

$$\frac{\left(cx^6 + bx^3\right) e^{\left(p \log\left(cx^3+b\right) + 3p \log(x)\right)}}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^3 + b)*(c*x^6 + b*x^3)^p*x^2,x, algorithm="maxima")

[Out] 1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)
```

Fricas [A] time = 0.280533, size = 42, normalized size = 1.75

$$\frac{\left(cx^6 + bx^3\right) \left(cx^6 + bx^3\right)^p}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^3 + b)*(c*x^6 + b*x^3)^p*x^2,x, algorithm="fricas")

[Out] 1/3*(c*x^6 + b*x^3)*(c*x^6 + b*x^3)^p/(p + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.270885, size = 65, normalized size = 2.71

$$\frac{cx^6e^{\left(p\ln\left(cx^6+bx^3\right)\right)} + bx^3e^{\left(p\ln\left(cx^6+bx^3\right)\right)}}{3\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^3 + b)*(c*x^6 + b*x^3)^p*x^2,x, algorithm="giac")
```

```
[Out] 1/3*(c*x^6*e^(p*ln(c*x^6 + b*x^3)) + b*x^3*e^(p*ln(c*x^6 + b*x^3)
))/(p + 1)
```

3.140 $\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=26

$$\frac{\left(bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

```
[Out] (b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rubi [A] time = 0.126021, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069

$$\frac{\left(bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^p,x]

[Out] (b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rubi in Sympy [A] time = 15.9455, size = 19, normalized size = 0.73

$$\frac{\left(bx^n+cx^{2n}\right)^{p+1}}{n\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+n)*(b+2*c*x**n)*(b*x**n+c*x**(2*n))**p,x)

[Out] (b*x**n + c*x**(2*n))**(p + 1)/(n*(p + 1))
```

Mathematica [A] time = 0.0843165, size = 24, normalized size = 0.92

$$\frac{(x^n\,(b+cx^n))^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x^n*(b + c*x^n))^(1 + p)/(n*(1 + p))
```

Maple [C] time = 0.14, size = 155, normalized size = 6.

$$\frac{x^n(b+cx^n)}{n(1+p)}\mathrm{e}^{-\frac{p(i\pi(\mathrm{csgn}(ix^n(b+cx^n)))^3-i\pi(\mathrm{csgn}(ix^n(b+cx^n)))^2\mathrm{csgn}(ix^n)-i\pi(\mathrm{csgn}(ix^n(b+cx^n)))^2\mathrm{csgn}(i(b+cx^n))+i\pi\,\mathrm{csgn}(ix^n(b+cx^n))\mathrm{csgn}(ix^n)\mathrm{csgn}(i(b+cx^n))-2\ln(x}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x)
```

```
[Out] x^n*(b+c*x^n)/n/(1+p)*exp(-1/2*p*(I*Pi*csgn(I*x^n*(b+c*x^n))^3-I*
Pi*csgn(I*x^n*(b+c*x^n))^2*csgn(I*x^n)-I*Pi*csgn(I*x^n*(b+c*x^n))
^2*csgn(I*(b+c*x^n))+I*Pi*csgn(I*x^n*(b+c*x^n))*csgn(I*x^n)*csgn(
I*(b+c*x^n))-2*ln(x^n)-2*ln(b+c*x^n)))
```

Maxima [A] time = 1.10739, size = 54, normalized size = 2.08

$$\frac{\left(cx^{2\,n}+bx^n\right)e^{\left(p\log(cx^n+b)+p\log(x^n)\right)}}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^n + b)*(c*x^(2*n) + b*x^n)^p*x^(n - 1),x, algorithm="maxima")
```

```
[Out] (c*x^(2*n) + b*x^n)*e^(p*log(c*x^n + b) + p*log(x^n))/(n*(p + 1))
```

Fricas [A] time = 0.288179, size = 49, normalized size = 1.88

$$\frac{\left(cx^{2\,n}+bx^n\right)\left(cx^{2\,n}+bx^n\right)^p}{np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*c*x^n + b)*(c*x^(2*n) + b*x^n)^p*x^(n - 1),x, algorithm="fricas")
```

```
[Out] (c*x^(2*n) + b*x^n)*(c*x^(2*n) + b*x^n)^p/(n*p + n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.281465, size = 35, normalized size = 1.35

$$\frac{\left(cx^{2\,n}+bx^{n}\right)^{p+1}}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^n + b)*(c*x^(2*n) + b*x^n)^p*x^(n - 1),x, algorithm="giac")
```

```
[Out] (c*x^(2*n) + b*x^n)^(p + 1)/(n*(p + 1))
```

3.141 $\int \frac{(fx)^m(d+ex^n)}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=196

$$\frac{(fx)^{m+1}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\,_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{f(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(fx)^{m+1}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\,_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometr
ic2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a
*c])])/((b - Sqrt[b^2 - 4*a*c])*f*(1 + m)) + ((e - (2*c*d - b*e)/
Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n,
(1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b + Sqrt[b^
2 - 4*a*c])*f*(1 + m))
```

Rubi [A] time = 0.548562, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069

$$\frac{(fx)^{m+1}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\,_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{f(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(fx)^{m+1}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\,_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometr
ic2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a
*c])])/((b - Sqrt[b^2 - 4*a*c])*f*(1 + m)) + ((e - (2*c*d - b*e)/
Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n,
(1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b + Sqrt[b^
2 - 4*a*c])*f*(1 + m))
```

Rubi in Sympy [A] time = 50.0562, size = 182, normalized size = 0.93

$$\frac{(fx)^{m+1}\left(be-2cd+e\sqrt{-4ac+b^2}\right){}_2F_1\left(\begin{matrix}1,\frac{m+1}{n}\\ \frac{m+n+1}{n}\end{matrix}\middle|-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{f\left(b+\sqrt{-4ac+b^2}\right)(m+1)\sqrt{-4ac+b^2}} - \frac{(fx)^{m+1}\left(be-2cd-e\sqrt{-4ac+b^2}\right){}_2F_1\left(\begin{matrix}1,\frac{m+1}{n}\\ \frac{m+n+1}{n}\end{matrix}\middle|-\frac{2cx^n}{b-\sqrt{-4ac+b^2}}\right)}{f\left(b-\sqrt{-4ac+b^2}\right)(m+1)\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] (f*x)**(m + 1)*(b*e - 2*c*d + e*sqrt(-4*a*c + b**2))*hyper((1, (m
 + 1)/n), ((m + n + 1)/n,), -2*c*x**n/(b + sqrt(-4*a*c + b**2)))/
(f*(b + sqrt(-4*a*c + b**2))*(m + 1)*sqrt(-4*a*c + b**2)) - (f*x)
**(m + 1)*(b*e - 2*c*d - e*sqrt(-4*a*c + b**2))*hyper((1, (m + 1)
/n), ((m + n + 1)/n,), -2*c*x**n/(b - sqrt(-4*a*c + b**2)))/(f*(b
 - sqrt(-4*a*c + b**2))*(m + 1)*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 1.53813, size = 318, normalized size = 1.62

$$\frac{x2^{-\frac{m+n+1}{n}}(fx)^m\left(-\left(d\sqrt{b^2-4ac}-2ae+bd\right)\left(\frac{cx^n}{-\sqrt{b^2-4ac}+b+2cx^n}\right)^{-\frac{m+1}{n}}{}_2F_1\left(-\frac{m+1}{n},-\frac{m+1}{n};1-\frac{m+1}{n};\frac{b-\sqrt{b^2-4ac}}{2cx^n+b-\sqrt{b^2-4ac}}\right)-\left(d\sqrt{b^2\right.\right.}{a(m+1)\sqrt{b^2-4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (x*(f*x)^m*(2^((1 + m + n)/n)*Sqrt[b^2 - 4*a*c]*d - ((b*d + Sqrt[
b^2 - 4*a*c]*d - 2*a*e)*Hypergeometric2F1[-((1 + m)/n), -((1 + m)
/n), 1 - (1 + m)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^n)])/((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((1 +
m)/n) - ((-(b*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*Hypergeometric2F1
[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/n, (b + Sqrt[b^2 - 4*a*c
])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^n))^((1 + m)/n)))/(2^((1 + m + n)/n)*a*Sqrt[b^2 - 4
*a*c]*(1 + m))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (d + ex^n)}{a + bx^n + cx^{2n}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)(fx)^m}{cx^{2n} + bx^n + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^n + d)(fx)^m}{cx^{2n} + bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)(fx)^m}{cx^{2n} + bx^n + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

3.142 $\int \frac{(fx)^m(d+ex^n)}{(a+bx^n+cx^{2n})^2}\,dx$

Optimal. Leaf size=374

$$-\frac{c(fx)^{m+1}\left((m-n+1)(bd-2ae)-\frac{2aben+4acd(m-2n+1)+b^2(-d)(m-n+1)}{\sqrt{b^2-4ac}}\right)\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{af(m+1)n\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(fx)^{m+1}\left(\frac{2aben+4acd(m-2n+1)+b^2(-d)(m-n+1)}{\sqrt{b^2-4ac}}+(m-n+1)(bd-2ae)\right)\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{af(m+1)n\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}+b\right)}+\frac{(fx)^{m+1}\left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{afn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

```
[Out] ((f*x)^(1 + m)*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(
a*(b^2 - 4*a*c)*f*n*(a + b*x^n + c*x^(2*n))) - (c*((b*d - 2*a*e)*
(1 + m - n) - (4*a*c*d*(1 + m - 2*n) - b^2*d*(1 + m - n) + 2*a*b*
e*n)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m
)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2
- 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*f*(1 + m)*n) - (c*((b*d - 2*a*e)
*(1 + m - n) + (4*a*c*d*(1 + m - 2*n) - b^2*d*(1 + m - n) + 2*a*b
*e*n)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 +
m)/n, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2
 - 4*a*c)*(b + Sqrt[b^2 - 4*a*c])*f*(1 + m)*n)
```

Rubi [A] time = 2.72955, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103

$$-\frac{c(fx)^{m+1}\left((m-n+1)(bd-2ae)-\frac{2aben+4acd(m-2n+1)+b^2(-d)(m-n+1)}{\sqrt{b^2-4ac}}\right)\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{af(m+1)n\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(fx)^{m+1}\left(\frac{2aben+4acd(m-2n+1)+b^2(-d)(m-n+1)}{\sqrt{b^2-4ac}}+(m-n+1)(bd-2ae)\right)\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{af(m+1)n\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}+b\right)}+\frac{(fx)^{m+1}\left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{afn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] ((f*x)^(1 + m)*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(
a*(b^2 - 4*a*c)*f*n*(a + b*x^n + c*x^(2*n))) - (c*((b*d - 2*a*e)*
(1 + m - n) - (4*a*c*d*(1 + m - 2*n) - b^2*d*(1 + m - n) + 2*a*b*
```

```
e*n)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m
)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2
- 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*f*(1 + m)*n) - (c*((b*d - 2*a*e)
*(1 + m - n) + (4*a*c*d*(1 + m - 2*n) - b^2*d*(1 + m - n) + 2*a*b
*e*n)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 +
m)/n, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2
 - 4*a*c)*(b + Sqrt[b^2 - 4*a*c])*f*(1 + m)*n)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

Mathematica [B] time = 6.59179, size = 5363, normalized size = 14.34

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (d + ex^n)}{(a + bx^n + cx^{2n})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
```

```
[Out] int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(b^2dfm - (2\,cdf^m + bef^m)a\right)xx^m + (bcdf^m - 2\,acef^m)xe^{(m\log(x)+n\log(x))}}{a^2b^2n - 4\,a^3cn + (ab^2cn - 4\,a^2c^2n)x^{2\,n} + (ab^3n - 4\,a^2bcn)x^n} - \int \frac{\left(b^2df^m(m-n+1) - (2\,cdf^m(m-2\,n+1) + bef^m(m+1))a\right)x^m + (bcdf^m(m-n+1) - 2\,acef^m(m-n+1))e^{(m\log(x)+}}{a^2b^2n - 4\,a^3cn + (ab^2cn - 4\,a^2c^2n)x^{2\,n} + (ab^3n - 4\,a^2bcn)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a)^2,x, algorithm="maxima")
```

```
[Out] ((b^2*d*f^m - (2*c*d*f^m + b*e*f^m)*a)*x*x^m + (b*c*d*f^m - 2*a*c
*e*f^m)*x*e^(m*log(x) + n*log(x)))/(a^2*b^2*n - 4*a^3*c*n + (a*b^
2*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n) - int
egrate(((b^2*d*f^m*(m - n + 1) - (2*c*d*f^m*(m - 2*n + 1) + b*e*f
^m*(m + 1))*a)*x^m + (b*c*d*f^m*(m - n + 1) - 2*a*c*e*f^m*(m - n
+ 1))*e^(m*log(x) + n*log(x)))/(a^2*b^2*n - 4*a^3*c*n + (a*b^2*c*
n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^n + d)(fx)^m}{c^2x^{4\,n} + 2\,abx^n + a^2 + (2\,bcx^n + b^2 + 2\,ac)x^{2\,n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)*(f*x)^m/(c^2*x^(4*n) + 2*a*b*x^n + a^2 + (2*
b*c*x^n + b^2 + 2*a*c)*x^(2*n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)(fx)^m}{(cx^{2n} + bx^n + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a)^2,x, algorithm="giac")

[Out] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a)^2, x)
```

3.143 $\int \frac{(fx)^m(d+ex^n)}{(a+bx^n+cx^{2n})^3}\,dx$

Optimal. Leaf size=816

$$-\frac{c\left(\left(-d(m-2n+1)b^3+ae(m+1)b^2+2acd(2m-7n+2)b-4a^2ce(m-3n+1)\right)(m-n+1)+\frac{-d\left(m^2+(2-3n)m+2n^2-3n+1\right)b^4+a}{}\right.}{2a^2\left(b^2-4ac\right.}$$

$$-\frac{c\left(\left(-d(m-2n+1)b^3+ae(m+1)b^2+2acd(2m-7n+2)b-4a^2ce(m-3n+1)\right)(m-n+1)-\frac{-d\left(m^2+(2-3n)m+2n^2-3n+1\right)b^4+c}{}\right.}{2a^2\left(b^2-4a\right.}$$

$$+\frac{\left(c\left(-d(m-2n+1)b^3+ae(m+1)b^2+2acd(2m-7n+2)b-4a^2ce(m-3n+1)\right)x^n+\left(b^2-2ac\right)\left(-d(m-2n+1)b^2+ae(n\right.\right.}{2a^2\left(b^2-4ac\right)^2fn^2\left(bx^n+cx^{2n}+a\right)}$$

$$+\frac{\left(c(bd-2ae)x^n+b^2d-2acd-abe\right)(fx)^{m+1}}{2a\left(b^2-4ac\right)fn\left(bx^n+cx^{2n}+a\right)^2}$$

```
[Out] ((f*x)^(1 + m)*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(
2*a*(b^2 - 4*a*c)*f*n*(a + b*x^n + c*x^(2*n))^2) + ((f*x)^(1 + m)
*((b^2 - 2*a*c)*(a*b*e*(1 + m) + 2*a*c*d*(1 + m - 4*n) - b^2*d*(1
 + m - 2*n)) + a*b*c*(b*d - 2*a*e)*(1 + m - 3*n) + c*(a*b^2*e*(1
+ m) + 2*a*b*c*d*(2 + 2*m - 7*n) - 4*a^2*c*e*(1 + m - 3*n) - b^3*
d*(1 + m - 2*n))*x^n))/(2*a^2*(b^2 - 4*a*c)^2*f*n^2*(a + b*x^n +
c*x^(2*n))) - (c*((a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m - 7*n) -
4*a^2*c*e*(1 + m - 3*n) - b^3*d*(1 + m - 2*n))*(1 + m - n) + (a*b
^3*e*(1 + m)*(1 + m - n) - 4*a^2*b*c*e*(1 + m^2 + m*(2 - n) - n -
 3*n^2) - b^4*d*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*c
*d*(1 + m^2 + m*(2 - 4*n) - 4*n + 3*n^2) - 8*a^2*c^2*d*(1 + m^2 +
 m*(2 - 6*n) - 6*n + 8*n^2))/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hyp
ergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[
b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b - Sqrt[b^2 - 4*a*c])*f*
(1 + m)*n^2) - (c*((a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m - 7*n) -
 4*a^2*c*e*(1 + m - 3*n) - b^3*d*(1 + m - 2*n))*(1 + m - n) - (a*
b^3*e*(1 + m)*(1 + m - n) - 4*a^2*b*c*e*(1 + m^2 + m*(2 - n) - n
- 3*n^2) - b^4*d*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*
c*d*(1 + m^2 + m*(2 - 4*n) - 4*n + 3*n^2) - 8*a^2*c^2*d*(1 + m^2
+ m*(2 - 6*n) - 6*n + 8*n^2))/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hy
pergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b + Sqrt[b^2 - 4*a*c])*f
*(1 + m)*n^2)
```

Rubi [A] time = 9.34195, antiderivative size = 816, normalized size of antiderivative = 1., number of

steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$-\frac{c\left(\left(-d(m-2n+1)b^3+ae(m+1)b^2+2acd(2m-7n+2)b-4a^2ce(m-3n+1)\right)(m-n+1)+\frac{-d\left(m^2+(2-3n)m+2n^2-3n+1\right)b^4+a}{\cdots}\right.}{2a^2\left(b^2-4ac\cdots\right.}$$

$$-\frac{c\left(\left(-d(m-2n+1)b^3+ae(m+1)b^2+2acd(2m-7n+2)b-4a^2ce(m-3n+1)\right)(m-n+1)-\frac{-d\left(m^2+(2-3n)m+2n^2-3n+1\right)b^4+a}{\cdots}\right.}{2a^2\left(b^2-4a\cdots\right.}$$

$$+\frac{\left(c\left(-d(m-2n+1)b^3+ae(m+1)b^2+2acd(2m-7n+2)b-4a^2ce(m-3n+1)\right)x^n+\left(b^2-2ac\right)\left(-d(m-2n+1)b^2+ae(\cdots\right.\right.}{2a^2\left(b^2-4ac\right)^2fn^2\left(bx^n+cx^{2n}+a\right)}$$

$$+\frac{\left(c(bd-2ae)x^n+b^2d-2acd-abe\right)(fx)^{m+1}}{2a\left(b^2-4ac\right)fn\left(bx^n+cx^{2n}+a\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] ((f*x)^(1 + m)*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(
2*a*(b^2 - 4*a*c)*f*n*(a + b*x^n + c*x^(2*n))^2) + ((f*x)^(1 + m)
*((b^2 - 2*a*c)*(a*b*e*(1 + m) + 2*a*c*d*(1 + m - 4*n) - b^2*d*(1
 + m - 2*n)) + a*b*c*(b*d - 2*a*e)*(1 + m - 3*n) + c*(a*b^2*e*(1
+ m) + 2*a*b*c*d*(2 + 2*m - 7*n) - 4*a^2*c*e*(1 + m - 3*n) - b^3*
d*(1 + m - 2*n))*x^n))/(2*a^2*(b^2 - 4*a*c)^2*f*n^2*(a + b*x^n +
c*x^(2*n))) - (c*((a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m - 7*n) -
4*a^2*c*e*(1 + m - 3*n) - b^3*d*(1 + m - 2*n))*(1 + m - n) + (a*b
^3*e*(1 + m)*(1 + m - n) - 4*a^2*b*c*e*(1 + m^2 + m*(2 - n) - n -
 3*n^2) - b^4*d*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*c
*d*(1 + m^2 + m*(2 - 4*n) - 4*n + 3*n^2) - 8*a^2*c^2*d*(1 + m^2 +
 m*(2 - 6*n) - 6*n + 8*n^2))/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hyp
ergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[
b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b - Sqrt[b^2 - 4*a*c])*f*
(1 + m)*n^2) - (c*((a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m - 7*n) -
 4*a^2*c*e*(1 + m - 3*n) - b^3*d*(1 + m - 2*n))*(1 + m - n) - (a*
b^3*e*(1 + m)*(1 + m - n) - 4*a^2*b*c*e*(1 + m^2 + m*(2 - n) - n
- 3*n^2) - b^4*d*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*
c*d*(1 + m^2 + m*(2 - 4*n) - 4*n + 3*n^2) - 8*a^2*c^2*d*(1 + m^2
+ m*(2 - 6*n) - 6*n + 8*n^2))/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hy
pergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b + Sqrt[b^2 - 4*a*c])*f
*(1 + m)*n^2)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

Mathematica [B] time = 7.91735, size = 20515, normalized size = 25.14

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (d + ex^n)}{(a + bx^n + cx^{2n})^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
```

```
[Out] int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*((a*b^4*d*f^m*(m - 3*n + 1) + 2*(b*c*e*f^m*(2*m - 5*n + 2) +
 2*c^2*d*f^m*(m - 6*n + 1))*a^3 - (b^2*c*d*f^m*(5*m - 21*n + 5) +
 b^3*e*f^m*(m - n + 1))*a^2)*x*x^m + (b^3*c^2*d*f^m*(m - 2*n + 1)
 + 4*a^2*c^3*e*f^m*(m - 3*n + 1) - (2*b*c^3*d*f^m*(2*m - 7*n + 2)
 + b^2*c^2*e*f^m*(m + 1))*a)*x*e^(m*log(x) + 3*n*log(x)) + (2*b^4
*c*d*f^m*(m - 2*n + 1) + 2*(b*c^2*e*f^m*(4*m - 9*n + 4) + 2*c^3*d
```

```
*f^m*(m - 4*n + 1))*a^2 - (b^2*c^2*d*f^m*(9*m - 29*n + 9) + 2*b^3
*c*e*f^m*(m + 1))*a)*x*e^(m*log(x) + 2*n*log(x)) + (b^5*d*f^m*(m
- 2*n + 1) + 4*a^3*c^2*e*f^m*(m - 5*n + 1) + (b^2*c*e*f^m*(3*m -
4*n + 3) + 2*b*c^2*d*f^m*n)*a^2 - (4*b^3*c*d*f^m*(m - 3*n + 1) +
b^4*e*f^m*(m + 1))*a)*x*e^(m*log(x) + n*log(x)))/(a^4*b^4*n^2 - 8
*a^5*b^2*c*n^2 + 16*a^6*c^2*n^2 + (a^2*b^4*c^2*n^2 - 8*a^3*b^2*c^
3*n^2 + 16*a^4*c^4*n^2)*x^(4*n) + 2*(a^2*b^5*c*n^2 - 8*a^3*b^3*c^
2*n^2 + 16*a^4*b*c^3*n^2)*x^(3*n) + (a^2*b^6*n^2 - 6*a^3*b^4*c*n^
2 + 32*a^5*c^3*n^2)*x^(2*n) + 2*(a^3*b^5*n^2 - 8*a^4*b^3*c*n^2 +
16*a^5*b*c^2*n^2)*x^n) + integrate(1/2*(((m^2 - m*(3*n - 2) + 2*n
^2 - 3*n + 1)*b^4*d*f^m + 2*(2*(m^2 - 2*m*(3*n - 1) + 8*n^2 - 6*n
 + 1)*c^2*d*f^m + (2*m^2 - m*(5*n - 4) - 5*n + 2)*b*c*e*f^m)*a^2
- ((5*m^2 - m*(21*n - 10) + 16*n^2 - 21*n + 5)*b^2*c*d*f^m + (m^2
 - m*(n - 2) - n + 1)*b^3*e*f^m)*a)*x^m + ((m^2 - m*(3*n - 2) + 2
*n^2 - 3*n + 1)*b^3*c*d*f^m + 4*(m^2 - 2*m*(2*n - 1) + 3*n^2 - 4*
n + 1)*a^2*c^2*e*f^m - (2*(2*m^2 - m*(9*n - 4) + 7*n^2 - 9*n + 2)
*b*c^2*d*f^m + (m^2 - m*(n - 2) - n + 1)*b^2*c*e*f^m)*a)*e^(m*log
(x) + n*log(x)))/(a^3*b^4*n^2 - 8*a^4*b^2*c*n^2 + 16*a^5*c^2*n^2
+ (a^2*b^4*c*n^2 - 8*a^3*b^2*c^2*n^2 + 16*a^4*c^3*n^2)*x^(2*n) +
(a^2*b^5*n^2 - 8*a^3*b^3*c*n^2 + 16*a^4*b*c^2*n^2)*x^n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^n+d)(fx)^m}{c^3x^{6n}+b^3x^{3n}+3a^2bx^n+a^3+3(bc^2x^n+b^2c+ac^2)x^{4n}+3(2abcx^n+ab^2+a^2c)x^{2n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a)^3,x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)*(f*x)^m/(c^3*x^(6*n) + b^3*x^(3*n) + 3*a^2*b
*x^n + a^3 + 3*(b*c^2*x^n + b^2*c + a*c^2)*x^(4*n) + 3*(2*a*b*c*x
^n + a*b^2 + a^2*c)*x^(2*n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)(fx)^m}{(cx^{2n} + bx^n + a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a)^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a)^3, x)
```

3.144 $\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}}\,dx$

Optimal. Leaf size=47

$$-\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}d^{2/3}}$$

```
[Out] (-3*Log[c^(2/3) - c^(1/3)*d^(1/3)*x^(1/3) + d^(2/3)*x^(2/3)])/(c^
(1/3)*d^(2/3))
```

Rubi [A] time = 0.1022, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051

$$-\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c^(1/3) - 2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3) - c^(2/3)*d^(2/3)*x + c^(1/
```

```
[Out] (-3*Log[c^(2/3) - c^(1/3)*d^(1/3)*x^(1/3) + d^(2/3)*x^(2/3)])/(c^
(1/3)*d^(2/3))
```

Rubi in Sympy [A] time = 30.678, size = 48, normalized size = 1.02

$$-\frac{3\log\left(-c^{\frac{2}{3}}d^{\frac{2}{3}}\sqrt[3]{x}+\sqrt[3]{c}dx^{\frac{2}{3}}+c\sqrt[3]{d}\right)}{\sqrt[3]{c}d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c**(1/3)-2*d**(1/3)*x**(1/3))/(c*d**(1/3)*x**(2/3)-
c**(2/3)*d**(2/3)*x+c**(1/3)*d*x**(4/3)),x)
```

```
[Out] -3*log(-c**(2/3)*d**(2/3)*x**(1/3) + c**(1/3)*d*x**(2/3) + c*d**(
1/3))/(c**(1/3)*d**(2/3))
```

Mathematica [A] time = 0.0400513, size = 47, normalized size = 1.

$$-\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c^(1/3) - 2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3) - c^(2/3)*d^(2/3)*x +
```

```
[Out] (-3*Log[c^(2/3) - c^(1/3)*d^(1/3)*x^(1/3) + d^(2/3)*x^(2/3)])/(c^
(1/3)*d^(2/3))
```

Maple [A] time = 0.005, size = 36, normalized size = 0.8

$$-3\frac{\ln\left(c^{2/3}d^{2/3}\sqrt[3]{x}-\sqrt[3]{c}x^{2/3}d-c\sqrt[3]{d}\right)}{d^{2/3}\sqrt[3]{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+c^(1/3)*d
```

```
[Out] -3/d^(2/3)/c^(1/3)*ln(c^(2/3)*d^(2/3)*x^(1/3)-c^(1/3)*x^(2/3)*d-c
*d^(1/3))
```

Maxima [A] time = 0.767896, size = 46, normalized size = 0.98

$$-\frac{3\,\log\left(c^{\frac{1}{3}}dx^{\frac{2}{3}}-c^{\frac{2}{3}}d^{\frac{2}{3}}x^{\frac{1}{3}}+cd^{\frac{1}{3}}\right)}{c^{\frac{1}{3}}d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*d^(1/3)*x^(1/3) - c^(1/3))/(c^(1/3)*d*x^(4/3) - c^(2/3)*d^(2/3)*x
```

```
[Out] -3*log(c^(1/3)*d*x^(2/3) - c^(2/3)*d^(2/3)*x^(1/3) + c*d^(1/3))/(
c^(1/3)*d^(2/3))
```

Fricas [A] time = 0.273919, size = 46, normalized size = 0.98

$$-\frac{3 \log\left(c^{\frac{1}{3}} d x^{\frac{2}{3}} - c^{\frac{2}{3}} d^{\frac{2}{3}} x^{\frac{1}{3}} + c d^{\frac{1}{3}}\right)}{c^{\frac{1}{3}} d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*d^(1/3)*x^(1/3) - c^(1/3))/(c^(1/3)*d*x^(4/3) - c^(2/3)*d^(2/3)*x
```

```
[Out] -3*log(c^(1/3)*d*x^(2/3) - c^(2/3)*d^(2/3)*x^(1/3) + c*d^(1/3))/(
c^(1/3)*d^(2/3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt[3]{c}}{c^{\frac{2}{3}} d^{\frac{2}{3}} x - \sqrt[3]{c} d x^{\frac{4}{3}} - c \sqrt[3]{d} x^{\frac{2}{3}}}\, dx - \int \left(-\frac{2 \sqrt[3]{d} \sqrt[3]{x}}{c^{\frac{2}{3}} d^{\frac{2}{3}} x - \sqrt[3]{c} d x^{\frac{4}{3}} - c \sqrt[3]{d} x^{\frac{2}{3}}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**(1/3)-2*d**(1/3)*x**(1/3))/(c*d**(1/3)*x**(2/3)-c**(2/3)*d**(2/3
```

```
[Out] -Integral(c**(1/3)/(c**(2/3)*d**(2/3)*x - c**(1/3)*d*x**(4/3) - c
*d**(1/3)*x**(2/3)), x) - Integral(-2*d**(1/3)*x**(1/3)/(c**(2/3)
*d**(2/3)*x - c**(1/3)*d*x**(4/3) - c*d**(1/3)*x**(2/3)), x)
```

GIAC/XCAS [A] time = 0.297063, size = 46, normalized size = 0.98

$$-\frac{3 \ln\left(c^{\frac{1}{3}} d x^{\frac{2}{3}} - c^{\frac{2}{3}} d^{\frac{2}{3}} x^{\frac{1}{3}} + c d^{\frac{1}{3}}\right)}{c^{\frac{1}{3}} d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(2*d^(1/3)*x^(1/3) - c^(1/3))/(c^(1/3)*d*x^(4/3) - c^(2/3)*d^(2/3)*x
```

```
[Out] -3*ln(c^(1/3)*d*x^(2/3) - c^(2/3)*d^(2/3)*x^(1/3) + c*d^(1/3))/(c
^(1/3)*d^(2/3))
```

3.145 $\int \frac{(fx)^m(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=245

$$\frac{2c(fx)^{m+1}(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{m+1}{n};1,-q;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)} - \frac{2c(fx)^{m+1}(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{m+1}{n};1,-q;\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] (2*c*(f*x)^(1 + m)*(d + e*x^n)^q*AppellF1[(1 + m)/n, 1, -q, (1 +
m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/(Sqr
t[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*f*(1 + m)*(1 + (e*x^n)/d)^
q) - (2*c*(f*x)^(1 + m)*(d + e*x^n)^q*AppellF1[(1 + m)/n, 1, -q,
(1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])
/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*f*(1 + m)*(1 + (e*x^n
)/d)^q)
```

Rubi [A] time = 1.14957, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$\frac{2c(fx)^{m+1}(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{m+1}{n};1,-q;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)} - \frac{2c(fx)^{m+1}(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{m+1}{n};1,-q;\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*c*(f*x)^(1 + m)*(d + e*x^n)^q*AppellF1[(1 + m)/n, 1, -q, (1 +
m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/(Sqr
t[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*f*(1 + m)*(1 + (e*x^n)/d)^
q) - (2*c*(f*x)^(1 + m)*(d + e*x^n)^q*AppellF1[(1 + m)/n, 1, -q,
(1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])
/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*f*(1 + m)*(1 + (e*x^n
)/d)^q)
```

Rubi in Sympy [A] time = 110.144, size = 199, normalized size = 0.81

$$-\frac{2c(fx)^{m+1}\left(1+\frac{ex^n}{d}\right)^{-q}(d+ex^n)^q\,\mathrm{appellf}_1\left(\frac{m+1}{n},1,-q,\frac{m+n+1}{n},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}},-\frac{ex^n}{d}\right)}{f\left(b+\sqrt{-4ac+b^2}\right)(m+1)\sqrt{-4ac+b^2}}+\frac{2c(fx)^{m+1}\left(1+\frac{ex^n}{d}\right)^{-q}(d+ex^n)^q\,\mathrm{appellf}_1\left(\frac{m+1}{n},1,-q,\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{ex^n}{d}\right)}{f\left(b-\sqrt{-4ac+b^2}\right)(m+1)\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] -2*c*(f*x)**(m + 1)*(1 + e*x**n/d)**(-q)*(d + e*x**n)**q*appellf1
((m + 1)/n, 1, -q, (m + n + 1)/n, -2*c*x**n/(b + sqrt(-4*a*c + b*
*2)), -e*x**n/d)/(f*(b + sqrt(-4*a*c + b**2))*(m + 1)*sqrt(-4*a*c
 + b**2)) + 2*c*(f*x)**(m + 1)*(1 + e*x**n/d)**(-q)*(d + e*x**n)*
*q*appellf1((m + 1)/n, 1, -q, (m + n + 1)/n, -2*c*x**n/(b - sqrt(
-4*a*c + b**2)), -e*x**n/d)/(f*(b - sqrt(-4*a*c + b**2))*(m + 1)*
sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.111884, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Integrate[((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]
```

Maple [F] time = 0.11, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (d+ex^n)^q}{a+bx^n+cx^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^q (fx)^m}{cx^{2n} + bx^n + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^q*(f*x)^m/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^q*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^n + d)^q (fx)^m}{cx^{2n} + bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^q*(f*x)^m/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)^q*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^q (fx)^m}{cx^{2n} + bx^n + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^q*(f*x)^m/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^q*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

3.146 $\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=210

$$-\frac{2cx^3(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{3}{n};1,-q;\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}-\frac{2cx^3(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{3}{n};1,-q;\frac{n+3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[Out] (-2*c*x^3*(d + e*x^n)^q*AppellF1[3/n, 1, -q, (3 + n)/n, (-2*c*x^n
)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/(3*(b^2 - 4*a*c - b*Sqr
t[b^2 - 4*a*c])*(1 + (e*x^n)/d)^q) - (2*c*x^3*(d + e*x^n)^q*Appel
lF1[3/n, 1, -q, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -(
(e*x^n)/d)])/(3*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(1 + (e*x^n)/
d)^q)
```

Rubi [A] time = 1.15095, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$-\frac{2cx^3(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{3}{n};1,-q;\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}-\frac{2cx^3(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{3}{n};1,-q;\frac{n+3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```


```
[Out] (-2*c*x^3*(d + e*x^n)^q*AppellF1[3/n, 1, -q, (3 + n)/n, (-2*c*x^n
)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/(3*(b^2 - 4*a*c - b*Sqr
t[b^2 - 4*a*c])*(1 + (e*x^n)/d)^q) - (2*c*x^3*(d + e*x^n)^q*Appel
lF1[3/n, 1, -q, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -(
(e*x^n)/d)])/(3*(b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(1 + (e*x^n)/
d)^q)
```

Rubi in Sympy [A] time = 91.3635, size = 173, normalized size = 0.82

$$-\frac{2cx^3\left(1+\frac{ex^n}{d}\right)^{-q}(d+ex^n)^q\,\text{appellf}_1\left(\frac{3}{n},1,-q,\frac{n+3}{n},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}},-\frac{ex^n}{d}\right)}{3\left(-4ac+b^2+b\sqrt{-4ac+b^2}\right)}-\frac{2cx^3\left(1+\frac{ex^n}{d}\right)^{-q}(d+ex^n)^q\,\text{appellf}_1\left(\frac{3}{n},1,-q,\frac{n+3}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{ex^n}{d}\right)}{3\left(-4ac+b^2-b\sqrt{-4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] -2*c*x**3*(1 + e*x**n/d)**(-q)*(d + e*x**n)**q*appellf1(3/n, 1, -
q, (n + 3)/n, -2*c*x**n/(b + sqrt(-4*a*c + b**2)), -e*x**n/d)/(3*
(-4*a*c + b**2 + b*sqrt(-4*a*c + b**2))) - 2*c*x**3*(1 + e*x**n/d
)**(-q)*(d + e*x**n)**q*appellf1(3/n, 1, -q, (n + 3)/n, -2*c*x**n
/(b - sqrt(-4*a*c + b**2)), -e*x**n/d)/(3*(-4*a*c + b**2 - b*sqrt
(-4*a*c + b**2)))
```

Mathematica [A] time = 0.150858, size = 0, normalized size = 0.

$$\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Integrate[(x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]
```

Maple [F] time = 0.077, size = 0, normalized size = 0.

$$\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^q x^2}{cx^{2n} + bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^q*x^2/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^q*x^2/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^n + d)^q x^2}{cx^{2n} + bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^q*x^2/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)^q*x^2/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^q x^2}{cx^{2n} + bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^q*x^2/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^q*x^2/(c*x^(2*n) + b*x^n + a), x)
```

3.147 $\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}} dx$

Optimal. Leaf size=206

$$-\frac{cx^2\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{2}{n};1,-q;\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{cx^2\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{2}{n};1,-q;\frac{n+2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

```
[Out] -((c*x^2*(d + e*x^n)^q*AppellF1[2/n, 1, -q, (2 + n)/n, (-2*c*x^n)
/(b - Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b
^2 - 4*a*c])*(1 + (e*x^n)/d)^q)) - (c*x^2*(d + e*x^n)^q*AppellF1[
2/n, 1, -q, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x
^n)/d)])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(1 + (e*x^n)/d)^q)
```

Rubi [A] time = 0.862, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{cx^2\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{2}{n};1,-q;\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{cx^2\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{2}{n};1,-q;\frac{n+2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((c*x^2*(d + e*x^n)^q*AppellF1[2/n, 1, -q, (2 + n)/n, (-2*c*x^n)
/(b - Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b
^2 - 4*a*c])*(1 + (e*x^n)/d)^q)) - (c*x^2*(d + e*x^n)^q*AppellF1[
2/n, 1, -q, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x
^n)/d)])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(1 + (e*x^n)/d)^q)
```

Rubi in Sympy [A] time = 78.1556, size = 167, normalized size = 0.81

$$-\frac{cx^2\left(1+\frac{ex^n}{d}\right)^{-q}\left(d+ex^n\right)^q \operatorname{appellf}_1\left(\frac{2}{n},1,-q,\frac{n+2}{n},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}},-\frac{ex^n}{d}\right)}{-4ac+b^2+b\sqrt{-4ac+b^2}}-\frac{cx^2\left(1+\frac{ex^n}{d}\right)^{-q}\left(d+ex^n\right)^q \operatorname{appellf}_1\left(\frac{2}{n},1,-q,\frac{n+2}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{ex^n}{d}\right)}{-4ac+b^2-b\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] -c*x**2*(1 + e*x**n/d)**(-q)*(d + e*x**n)**q*appellf1(2/n, 1, -q,
 (n + 2)/n, -2*c*x**n/(b + sqrt(-4*a*c + b**2)), -e*x**n/d)/(-4*a
*c + b**2 + b*sqrt(-4*a*c + b**2)) - c*x**2*(1 + e*x**n/d)**(-q)*
(d + e*x**n)**q*appellf1(2/n, 1, -q, (n + 2)/n, -2*c*x**n/(b - sq
rt(-4*a*c + b**2)), -e*x**n/d)/(-4*a*c + b**2 - b*sqrt(-4*a*c + b
**2))
```

Mathematica [A] time = 0.136633, size = 0, normalized size = 0.

$$\int \frac{x\,(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Integrate[(x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int \frac{x\,(d+ex^n)^q}{a+bx^n+cx^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n+d)^q x}{cx^{2\,n}+bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^q*x/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")

[Out] integrate((e*x^n + d)^q*x/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^n + d)^q x}{cx^{2n} + bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^q*x/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")

[Out] integral((e*x^n + d)^q*x/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^q x}{cx^{2n} + bx^n + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^q*x/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")

[Out] integrate((e*x^n + d)^q*x/(c*x^(2*n) + b*x^n + a), x)
```

3.148 $\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=194

$$-\frac{2cx\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{1}{n};1,-q;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2cx\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{1}{n};1,-q;1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

```
[Out] (-2*c*x*(d + e*x^n)^q*AppellF1[n^(-1), 1, -q, 1 + n^(-1), (-2*c*x
^n)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqr
t[b^2 - 4*a*c])*(1 + (e*x^n)/d)^q) - (2*c*x*(d + e*x^n)^q*AppellF
1[n^(-1), 1, -q, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
-((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(1 + (e*x^n)/
d)^q)
```

Rubi [A] time = 0.620324, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$-\frac{2cx\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{1}{n};1,-q;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2cx\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{1}{n};1,-q;1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*c*x*(d + e*x^n)^q*AppellF1[n^(-1), 1, -q, 1 + n^(-1), (-2*c*x
^n)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqr
t[b^2 - 4*a*c])*(1 + (e*x^n)/d)^q) - (2*c*x*(d + e*x^n)^q*AppellF
1[n^(-1), 1, -q, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]),
-((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(1 + (e*x^n)/
d)^q)
```

Rubi in Sympy [A] time = 100.184, size = 167, normalized size = 0.86

$$-\frac{2cx\left(1+\frac{ex^n}{d}\right)^{-q}(d+ex^n)^q\operatorname{appellf}_1\left(\frac{1}{n},1,-q,1+\frac{1}{n},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}},-\frac{ex^n}{d}\right)}{-4ac+b^2+b\sqrt{-4ac+b^2}}-\frac{2cx\left(1+\frac{ex^n}{d}\right)^{-q}(d+ex^n)^q\operatorname{appellf}_1\left(\frac{1}{n},1,-q,1+\frac{1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{ex^n}{d}\right)}{-4ac+b^2-b\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] -2*c*x*(1 + e*x**n/d)**(-q)*(d + e*x**n)**q*appellf1(1/n, 1, -q,
1 + 1/n, -2*c*x**n/(b + sqrt(-4*a*c + b**2)), -e*x**n/d)/(-4*a*c
+ b**2 + b*sqrt(-4*a*c + b**2)) - 2*c*x*(1 + e*x**n/d)**(-q)*(d +
 e*x**n)**q*appellf1(1/n, 1, -q, 1 + 1/n, -2*c*x**n/(b - sqrt(-4*
a*c + b**2)), -e*x**n/d)/(-4*a*c + b**2 - b*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.0668307, size = 0, normalized size = 0.

$$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Integrate[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)), x]
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^q}{cx^{2n} + bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^q/(c*x^(2*n) + b*x^n + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^q/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^n + d)^q}{cx^{2n} + bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^q/(c*x^(2*n) + b*x^n + a),x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)^q/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^q}{cx^{2n} + bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^q/(c*x^(2*n) + b*x^n + a),x, algorithm="giac")

[Out] integrate((e*x^n + d)^q/(c*x^(2*n) + b*x^n + a), x)
```

3.149 $\int \frac{(d+ex^n)^q}{x(a+bx^n+cx^{2n})}\,dx$

Optimal. Leaf size=262

$$\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex^n)^{q+1}\ {}_2F_1\left(1,q+1;q+2;\frac{2c(ex^n+d)}{2cd-be+\sqrt{b^2-4ac}e}\right)}{an(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^n)^{q+1}\ {}_2F_1\left(1,q+1;q+2;\frac{2c(ex^n+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{an(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{(d+ex^n)^{q+1}\ {}_2F_1\left(1,q+1;q+2;\frac{ex^n}{d}+1\right)}{adn(q+1)}$$

```
[Out] (c*(1 + b/Sqrt[b^2 - 4*a*c])*(d + e*x^n)^(1 + q)*Hypergeometric2F
1[1, 1 + q, 2 + q, (2*c*(d + e*x^n))/(2*c*d - b*e + Sqrt[b^2 - 4*
a*c]*e)])/(a*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*n*(1 + q)) + (c*
(1 - b/Sqrt[b^2 - 4*a*c])*(d + e*x^n)^(1 + q)*Hypergeometric2F1[1
, 1 + q, 2 + q, (2*c*(d + e*x^n))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)])/(a*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*n*(1 + q)) - ((d +
e*x^n)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, 1 + (e*x^n)/d])
/(a*d*n*(1 + q))
```

Rubi [A] time = 1.4363, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172

$$\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex^n)^{q+1}\ {}_2F_1\left(1,q+1;q+2;\frac{2c(ex^n+d)}{2cd-be+\sqrt{b^2-4ac}e}\right)}{an(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^n)^{q+1}\ {}_2F_1\left(1,q+1;q+2;\frac{2c(ex^n+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{an(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{(d+ex^n)^{q+1}\ {}_2F_1\left(1,q+1;q+2;\frac{ex^n}{d}+1\right)}{adn(q+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (c*(1 + b/Sqrt[b^2 - 4*a*c])*(d + e*x^n)^(1 + q)*Hypergeometric2F
1[1, 1 + q, 2 + q, (2*c*(d + e*x^n))/(2*c*d - b*e + Sqrt[b^2 - 4*
```

```
a*c]*e)])/(a*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*n*(1 + q)) + (c*
(1 - b/Sqrt[b^2 - 4*a*c])*(d + e*x^n)^(1 + q)*Hypergeometric2F1[1
, 1 + q, 2 + q, (2*c*(d + e*x^n))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)])/(a*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*n*(1 + q)) - ((d +
e*x^n)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, 1 + (e*x^n)/d])
/(a*d*n*(1 + q))
```

Rubi in Sympy [A] time = 121.827, size = 243, normalized size = 0.93

$$\frac{c\left(b-\sqrt{-4ac+b^2}\right)(d+ex^n)^{q+1}\,{}_2F_1\left(\begin{matrix}1,q+1\\q+2\end{matrix}\middle|\frac{c(-2d-2ex^n)}{be-2cd+e\sqrt{-4ac+b^2}}\right)}{an(q+1)\sqrt{-4ac+b^2}\left(be-2cd+e\sqrt{-4ac+b^2}\right)}+\frac{c\left(b+\sqrt{-4ac+b^2}\right)(d+ex^n)^{q+1}\,{}_2F_1\left(\begin{matrix}1,q+1\\q+2\end{matrix}\middle|\frac{c(-2d-2ex^n)}{be-2cd-e\sqrt{-4ac+b^2}}\right)}{an(q+1)\sqrt{-4ac+b^2}\left(2cd-e\left(b-\sqrt{-4ac+b^2}\right)\right)}-\frac{(d+ex^n)^{q+1}\,{}_2F_1\left(\begin{matrix}1,q+1\\q+2\end{matrix}\middle|1+\frac{ex^n}{d}\right)}{adn(q+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)**q/x/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] c*(b - sqrt(-4*a*c + b**2))*(d + e*x**n)**(q + 1)*hyper((1, q + 1
), (q + 2,), c*(-2*d - 2*e*x**n)/(b*e - 2*c*d + e*sqrt(-4*a*c + b
**2)))/(a*n*(q + 1)*sqrt(-4*a*c + b**2)*(b*e - 2*c*d + e*sqrt(-4*
a*c + b**2))) + c*(b + sqrt(-4*a*c + b**2))*(d + e*x**n)**(q + 1)
*hyper((1, q + 1), (q + 2,), c*(-2*d - 2*e*x**n)/(b*e - 2*c*d - e
*sqrt(-4*a*c + b**2)))/(a*n*(q + 1)*sqrt(-4*a*c + b**2)*(2*c*d -
e*(b - sqrt(-4*a*c + b**2)))) - (d + e*x**n)**(q + 1)*hyper((1, q
 + 1), (q + 2,), 1 + e*x**n/d)/(a*d*n*(q + 1))
```

Mathematica [A] time = 0.0863113, size = 0, normalized size = 0.

$$\int \frac{(d+ex^n)^q}{x\left(a+bx^n+cx^{2n}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] Integrate[(d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))), x]
```

Maple [F] time = 0.108, size = 0, normalized size = 0.

$$\int \frac{(d+ex^n)^q}{x\,(a+bx^n+cx^{2n})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n+d)^q}{(cx^{2n}+bx^n+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x),x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^n+d)^q}{cxx^{2n}+bxx^n+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x),x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)^q/(c*x*x^(2*n) + b*x*x^n + a*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d+e*x**n)**q/x/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^q}{(cx^{2\,n} + bx^n + a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x), x)
```

3.150 $\int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})}\,dx$

Optimal. Leaf size=212

$$\frac{2c\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{1}{n};1,-q;-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{2c\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{1}{n};1,-q;-\frac{1-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[Out] (2*c*(d + e*x^n)^q*AppellF1[-n^(-1), 1, -q, -((1 - n)/n), (-2*c*x
^n)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqr
t[b^2 - 4*a*c])*x*(1 + (e*x^n)/d)^q) + (2*c*(d + e*x^n)^q*AppellF
1[-n^(-1), 1, -q, -((1 - n)/n), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]
), -((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*x*(1 + (e*
x^n)/d)^q)
```

Rubi [A] time = 1.07128, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{2c\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{1}{n};1,-q;-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{2c\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{1}{n};1,-q;-\frac{1-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (2*c*(d + e*x^n)^q*AppellF1[-n^(-1), 1, -q, -((1 - n)/n), (-2*c*x
^n)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqr
t[b^2 - 4*a*c])*x*(1 + (e*x^n)/d)^q) + (2*c*(d + e*x^n)^q*AppellF
1[-n^(-1), 1, -q, -((1 - n)/n), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]
), -((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*x*(1 + (e*
x^n)/d)^q)
```

Rubi in Sympy [A] time = 91.4398, size = 168, normalized size = 0.79

$$\frac{2c\left(1+\frac{ex^n}{d}\right)^{-q}(d+ex^n)^q\operatorname{appellf}_1\left(-\frac{1}{n},1,-q,\frac{n-1}{n},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}},-\frac{ex^n}{d}\right)}{x\left(-4ac+b^2+b\sqrt{-4ac+b^2}\right)}+\frac{2c\left(1+\frac{ex^n}{d}\right)^{-q}(d+ex^n)^q\operatorname{appellf}_1\left(-\frac{1}{n},1,-q,\frac{n-1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{ex^n}{d}\right)}{x\left(-4ac+b^2-b\sqrt{-4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)**q/x**2/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] 2*c*(1 + e*x**n/d)**(-q)*(d + e*x**n)**q*appellf1(-1/n, 1, -q, (n
 - 1)/n, -2*c*x**n/(b + sqrt(-4*a*c + b**2)), -e*x**n/d)/(x*(-4*a
*c + b**2 + b*sqrt(-4*a*c + b**2))) + 2*c*(1 + e*x**n/d)**(-q)*(d
 + e*x**n)**q*appellf1(-1/n, 1, -q, (n - 1)/n, -2*c*x**n/(b - sqr
t(-4*a*c + b**2)), -e*x**n/d)/(x*(-4*a*c + b**2 - b*sqrt(-4*a*c +
 b**2)))
```

Mathematica [A] time = 0.0921769, size = 0, normalized size = 0.

$$\int \frac{(d+ex^n)^q}{x^2\left(a+bx^n+cx^{2n}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] Integrate[(d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))), x]
```

Maple [F] time = 0.077, size = 0, normalized size = 0.

$$\int \frac{(d+ex^n)^q}{x^2\left(a+bx^n+cx^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^q}{(cx^{2n} + bx^n + a)x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^n + d)^q}{cx^2x^{2n} + bx^2x^n + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^2),x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)^q/(c*x^2*x^(2*n) + b*x^2*x^n + a*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**q/x**2/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^q}{(cx^{2n} + bx^n + a)x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^2),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

3.151 $\int \frac{(d+ex^n)^q}{x^3(a+bx^n+cx^{2n})}\,dx$

Optimal. Leaf size=210

$$\frac{c\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{2}{n};1,-q;-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x^2\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{c\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{2}{n};1,-q;-\frac{2-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x^2\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[Out] (c*(d + e*x^n)^q*AppellF1[-2/n, 1, -q, -((2 - n)/n), (-2*c*x^n)/(
b - Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2
 - 4*a*c])*x^2*(1 + (e*x^n)/d)^q) + (c*(d + e*x^n)^q*AppellF1[-2/
n, 1, -q, -((2 - n)/n), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*
x^n)/d)])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*x^2*(1 + (e*x^n)/d
)^q)
```

Rubi [A] time = 1.05331, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103

$$\frac{c\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{2}{n};1,-q;-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x^2\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{c\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{2}{n};1,-q;-\frac{2-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x^2\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (c*(d + e*x^n)^q*AppellF1[-2/n, 1, -q, -((2 - n)/n), (-2*c*x^n)/(
b - Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2
 - 4*a*c])*x^2*(1 + (e*x^n)/d)^q) + (c*(d + e*x^n)^q*AppellF1[-2/
n, 1, -q, -((2 - n)/n), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*
x^n)/d)])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*x^2*(1 + (e*x^n)/d
)^q)
```

Rubi in Sympy [A] time = 91.3952, size = 168, normalized size = 0.8

$$\frac{c\left(1+\frac{ex^n}{d}\right)^{-q}(d+ex^n)^q\operatorname{appellf}_1\left(-\frac{2}{n},1,-q,\frac{n-2}{n},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}},-\frac{ex^n}{d}\right)}{x^2\left(-4ac+b^2+b\sqrt{-4ac+b^2}\right)}+\frac{c\left(1+\frac{ex^n}{d}\right)^{-q}(d+ex^n)^q\operatorname{appellf}_1\left(-\frac{2}{n},1,-q,\frac{n-2}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{ex^n}{d}\right)}{x^2\left(-4ac+b^2-b\sqrt{-4ac+b^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d+e*x**n)**q/x**3/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] c*(1 + e*x**n/d)**(-q)*(d + e*x**n)**q*appellf1(-2/n, 1, -q, (n -
 2)/n, -2*c*x**n/(b + sqrt(-4*a*c + b**2)), -e*x**n/d)/(x**2*(-4*
a*c + b**2 + b*sqrt(-4*a*c + b**2))) + c*(1 + e*x**n/d)**(-q)*(d
+ e*x**n)**q*appellf1(-2/n, 1, -q, (n - 2)/n, -2*c*x**n/(b - sqrt
(-4*a*c + b**2)), -e*x**n/d)/(x**2*(-4*a*c + b**2 - b*sqrt(-4*a*c
 + b**2)))
```

Mathematica [A] time = 0.0906835, size = 0, normalized size = 0.

$$\int \frac{(d+ex^n)^q}{x^3\left(a+bx^n+cx^{2n}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] Integrate[(d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))), x]
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int \frac{(d+ex^n)^q}{x^3\left(a+bx^n+cx^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^q}{(cx^{2n} + bx^n + a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^3),x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^n + d)^q}{cx^3x^{2n} + bx^3x^n + ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^3),x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)^q/(c*x^3*x^(2*n) + b*x^3*x^n + a*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**q/x**3/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^q}{(cx^{2n} + bx^n + a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^3),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

3.152 $\int (fx)^m (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=498

$$\frac{d^2(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)}$$
$$+\frac{2dex^{n+1}(fx)^m\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+n+1}{n};-p,-p;\frac{m+2n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4a}}\right.}{m+n+1}$$
$$+\frac{e^2x^{2n+1}(fx)^m\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+2n+1}{n};-p,-p;\frac{m+3n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4}}\right.}{m+2n+1}$$

```
[Out] (d^2*(f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n,
-p, -p, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (
2*d*e*x^(1 + n)*(f*x)^m*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m
 + n)/n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*
c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m + n)*(1 + (2*c*
x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c]))^p) + (e^2*x^(1 + 2*n)*(f*x)^m*(a + b*x^n + c*x^(2*n))^p*Ap
pellF1[(1 + m + 2*n)/n, -p, -p, (1 + m + 3*n)/n, (-2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m
+ 2*n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 1.38747, antiderivative size = 498, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$

$$\frac{d^2(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)}$$
$$+\frac{2dex^{n+1}(fx)^m\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+n+1}{n};-p,-p;\frac{m+2n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4a}}\right.}{m+n+1}$$
$$+\frac{e^2x^{2n+1}(fx)^m\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+2n+1}{n};-p,-p;\frac{m+3n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4}}\right.}{m+2n+1}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (d^2*(f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n,
-p, -p, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*
```

```
x^n)/(b + Sqrt[b^2 - 4*a*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (
2*d*e*x^(1 + n)*(f*x)^m*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m
 + n)/n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*
c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m + n)*(1 + (2*c*
x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c]))^p) + (e^2*x^(1 + 2*n)*(f*x)^m*(a + b*x^n + c*x^(2*n))^p*Ap
pellF1[(1 + m + 2*n)/n, -p, -p, (1 + m + 3*n)/n, (-2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m
+ 2*n)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi in Sympy [A] time = 140.812, size = 442, normalized size = 0.89

$$\frac{d^2(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \operatorname{appellf}_1\left(\frac{m+1}{n},-p,-p,\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac}}\right.}{f(m+1)}$$
$$+\frac{2dex^n(fx)^{-n}(fx)^{m+n+1}\left(\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \operatorname{appellf}_1\left(\frac{m+n+1}{n},-p,-p,\frac{m+2n+1}{n},-\frac{}{b-\sqrt{}}\right.}{f(m+n+1)}$$
$$+\frac{e^2x^{2n}(fx)^{-2n}(fx)^{m+2n+1}\left(\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \operatorname{appellf}_1\left(\frac{m+2n+1}{n},-p,-p,\frac{m+3n+1}{n},-\frac{}{b}\right.}{f(m+2n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(d+e*x**n)**2*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] d**2*(f*x)**(m + 1)*(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)**(-p
)*(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**n + c*
x**(2*n))**p*appellf1((m + 1)/n, -p, -p, (m + n + 1)/n, -2*c*x**n
/(b - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*a*c + b**2)))/
(f*(m + 1)) + 2*d*e*x**n*(f*x)**(-n)*(f*x)**(m + n + 1)*(2*c*x**n
/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**n/(b + sqrt(-4*a*c
+ b**2)) + 1)**(-p)*(a + b*x**n + c*x**(2*n))**p*appellf1((m + n
+ 1)/n, -p, -p, (m + 2*n + 1)/n, -2*c*x**n/(b - sqrt(-4*a*c + b**
2)), -2*c*x**n/(b + sqrt(-4*a*c + b**2)))/(f*(m + n + 1)) + e**2*
x**(2*n)*(f*x)**(-2*n)*(f*x)**(m + 2*n + 1)*(2*c*x**n/(b - sqrt(-
4*a*c + b**2)) + 1)**(-p)*(2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1
)**(-p)*(a + b*x**n + c*x**(2*n))**p*appellf1((m + 2*n + 1)/n, -p
, -p, (m + 3*n + 1)/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2*c*
x**n/(b + sqrt(-4*a*c + b**2)))/(f*(m + 2*n + 1))
```

Mathematica [B] time = 17.6381, size = 1615, normalized size = 3.24

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] -((2^(-1 - p)*(b + Sqrt[b^2 - 4*a*c])*d^2*(1 + m + n)*x*(f*x)^m*(
-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x
^n)/c)^p*(-2*a + (-b + Sqrt[b^2 - 4*a*c])*x^n)^2*(a + x^n*(b + c*
x^n))^(-1 + p)*AppellF1[(1 + m)/n, -p, -p, (1 + m + n)/n, (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])
/((-b + Sqrt[b^2 - 4*a*c])*(1 + m)*((b - Sqrt[b^2 - 4*a*c])/(2*c)
 + x^n)^p*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(-2*a*(1 + m + n)*App
ellF1[(1 + m)/n, -p, -p, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*p*x^n*((-b + S
qrt[b^2 - 4*a*c])*AppellF1[(1 + m + n)/n, 1 - p, -p, (1 + m + 2*n
)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
 - 4*a*c])] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[(1 + m + n)/n, -p,
 1 - p, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))) + (c*(b + Sqrt[b^2 - 4*a*c])*
d*e*(1 + m + 2*n)*x^(1 + n)*(f*x)^m*((b - Sqrt[b^2 - 4*a*c] + 2*c
*x^n)/c)^(1 + p)*(-2*a + (-b + Sqrt[b^2 - 4*a*c])*x^n)^2*(a + x^n
*(b + c*x^n))^(-1 + p)*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2
*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b
^2 - 4*a*c])])/(2^p*(-b + Sqrt[b^2 - 4*a*c])*(1 + m + n)*((b - Sq
rt[b^2 - 4*a*c])/(2*c) + x^n)^p*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)
*(-2*a*(1 + m + 2*n)*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n
)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
 - 4*a*c])] + n*p*x^n*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[(1 + m +
 2*n)/n, 1 - p, -p, (1 + m + 3*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (b + Sqrt[b^2 - 4*a
*c])*AppellF1[(1 + m + 2*n)/n, -p, 1 - p, (1 + m + 3*n)/n, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]
))) + (2^(-1 - p)*c*(b + Sqrt[b^2 - 4*a*c])*e^2*(1 + m + 3*n)*x^(
1 + 2*n)*(f*x)^m*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/c)^(1 + p)*(-
2*a + (-b + Sqrt[b^2 - 4*a*c])*x^n)^2*(a + x^n*(b + c*x^n))^(-1 +
 p)*AppellF1[(1 + m + 2*n)/n, -p, -p, (1 + m + 3*n)/n, (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((
-b + Sqrt[b^2 - 4*a*c])*(1 + m + 2*n)*((b - Sqrt[b^2 - 4*a*c])/(2
*c) + x^n)^p*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(-2*a*(1 + m + 3*n
)*AppellF1[(1 + m + 2*n)/n, -p, -p, (1 + m + 3*n)/n, (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*p
*x^n*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[(1 + m + 3*n)/n, 1 - p, -
p, (1 + m + 4*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)
/(-b + Sqrt[b^2 - 4*a*c])] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[(1
+ m + 3*n)/n, -p, 1 - p, (1 + m + 4*n)/n, (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.09, size = 0, normalized size = 0.

$$\int (fx)^m (d + ex^n)^2 \left(a + bx^n + cx^{2n}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^n + d)^2 \left(cx^{2n} + bx^n + a\right)^p (fx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^2*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^2*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^{2n} + 2\,dex^n + d^2\right)\left(cx^{2n} + bx^n + a\right)^p (fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^2*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)*(c*x^(2*n) + b*x^n + a)^
p*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(d+e*x**n)**2*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)^2*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.153 $\int (fx)^m (d+ex^n) \left(a+bx^n+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=323

$$\frac{d(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)}+\frac{ex^{n+1}(fx)^m\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+n+1}{n};-p,-p;\frac{m+2n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{m+n+1}$$

```
[Out] (d*(f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p
, -p, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[
b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (e*
x^(1 + n)*(f*x)^m*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m + n)/
n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (
-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m + n)*(1 + (2*c*x^n)/(
b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))
^p)
```

Rubi [A] time = 0.824632, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$

$$\frac{d(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)}+\frac{ex^{n+1}(fx)^m\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+n+1}{n};-p,-p;\frac{m+2n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{m+n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```


```
[Out] (d*(f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p
, -p, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[
b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (e*
x^(1 + n)*(f*x)^m*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m + n)/
n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (
-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m + n)*(1 + (2*c*x^n)/(
b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))
^p)
```

Rubi in Sympy [A] time = 86.8058, size = 279, normalized size = 0.86

$$\frac{d\left(fx\right)^{m+1}\left(\frac{2cx^{n}}{b-\sqrt{-4ac+b^{2}}}+1\right)^{-p}\left(\frac{2cx^{n}}{b+\sqrt{-4ac+b^{2}}}+1\right)^{-p}\left(a+bx^{n}+cx^{2n}\right)^{p}\operatorname{appellf}_{1}\left(\frac{m+1}{n},-p,-p,\frac{m+n+1}{n},-\frac{2cx^{n}}{b-\sqrt{-4ac+b^{2}}},-\frac{2cx^{n}}{b+\sqrt{-4ac+b^{2}}}\right)}{f\left(m+1\right)}+\frac{ex^{n}\left(fx\right)^{-n}\left(fx\right)^{m+n+1}\left(\frac{2cx^{n}}{b-\sqrt{-4ac+b^{2}}}+1\right)^{-p}\left(\frac{2cx^{n}}{b+\sqrt{-4ac+b^{2}}}+1\right)^{-p}\left(a+bx^{n}+cx^{2n}\right)^{p}\operatorname{appellf}_{1}\left(\frac{m+n+1}{n},-p,-p,\frac{m+2n+1}{n},-\frac{2cx^{n}}{b-\sqrt{-4ac+b^{2}}},\dots\right)}{f\left(m+n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(d+e*x**n)*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] d*(f*x)**(m + 1)*(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(
2*c*x**n/(b + sqrt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**n + c*x**
(2*n))**p*appellf1((m + 1)/n, -p, -p, (m + n + 1)/n, -2*c*x**n/(b
 - sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*a*c + b**2)))/(f*
(m + 1)) + e*x**n*(f*x)**(-n)*(f*x)**(m + n + 1)*(2*c*x**n/(b - s
qrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**n/(b + sqrt(-4*a*c + b**2)
) + 1)**(-p)*(a + b*x**n + c*x**(2*n))**p*appellf1((m + n + 1)/n,
 -p, -p, (m + 2*n + 1)/n, -2*c*x**n/(b - sqrt(-4*a*c + b**2)), -2
*c*x**n/(b + sqrt(-4*a*c + b**2)))/(f*(m + n + 1))
```

Mathematica [B] time = 2.48676, size = 922, normalized size = 2.85

$$\frac{2^{-p-1}\left(b+\sqrt{b^{2}-4ac}\right)x(fx)^{m}\left(x^{n}+\frac{b-\sqrt{b^{2}-4ac}}{2c}\right)^{-p}\left(\frac{2cx^{n}+b-\sqrt{b^{2}-4ac}}{c}\right)^{p}\left(\left(\sqrt{b^{2}-4ac}-b\right)x^{n}-2a\right)^{2}\left(\left(cx^{n}+b\right)x^{n}+a\right)^{p-1}\left(\dots\right)}{\dots}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (2^(-1 - p)*(b + Sqrt[b^2 - 4*a*c])*x*(f*x)^m*((b - Sqrt[b^2 - 4*
a*c] + 2*c*x^n)/c)^p*(-2*a + (-b + Sqrt[b^2 - 4*a*c])*x^n)^2*(a +
 x^n*(b + c*x^n))^(-1 + p)*((d*(1 + m + n)^2*(-b + Sqrt[b^2 - 4*a
*c] - 2*c*x^n)*AppellF1[(1 + m)/n, -p, -p, (1 + m + n)/n, (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])
/((1 + m)*(2*a*(1 + m + n)*AppellF1[(1 + m)/n, -p, -p, (1 + m + n
)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
 - 4*a*c])] - n*p*x^n*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[(1 + m +
 n)/n, 1 - p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (b + Sqrt[b^2 - 4*a*c
])*AppellF1[(1 + m + n)/n, -p, 1 - p, (1 + m + 2*n)/n, (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))
```

```
+ (e*(1 + m + 2*n)*x^n*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)*AppellF1
[(1 + m + n)/n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(-2*a*(1 + m + 2
*n)*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*p
*x^n*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[(1 + m + 2*n)/n, 1 - p, -
p, (1 + m + 3*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)
/(-b + Sqrt[b^2 - 4*a*c])] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[(1
+ m + 2*n)/n, -p, 1 - p, (1 + m + 3*n)/n, (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))))/((-b + Sqrt[
b^2 - 4*a*c])*(1 + m + n)*((b - Sqrt[b^2 - 4*a*c])/(2*c) + x^n)^p
*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int (fx)^m (d + ex^n) \left(a + bx^n + cx^{2n}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x)

[Out] int((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^n + d)\left(cx^{2n} + bx^n + a\right)^p (fx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m,x, algorithm="maxima")

[Out] integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex^n + d)\left(cx^{2n} + bx^n + a\right)^p (fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m,x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^n + d)(cx^{2n} + bx^n + a)^p (fx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

3.154 $\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=158

$$\frac{(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)}$$

```
[Out] ((f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p,
-p, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^
2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.328257, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(f*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] ((f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p,
-p, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^
2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi in Sympy [A] time = 34.9082, size = 129, normalized size = 0.82

$$\frac{(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^n}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p \operatorname{appellf}_1\left(\frac{m+1}{n},-p,-p,\frac{m+n+1}{n},-\frac{2cx^n}{b-\sqrt{-4ac+b^2}},-\frac{2cx^n}{b+\sqrt{-4ac+b^2}}\right)}{f\left(m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] (f*x)**(m + 1)*(2*c*x**n/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*
c*x**n/(b + sqrt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**n + c*x**(2
*n))**p*appellf1((m + 1)/n, -p, -p, (m + n + 1)/n, -2*c*x**n/(b -
 sqrt(-4*a*c + b**2)), -2*c*x**n/(b + sqrt(-4*a*c + b**2)))/(f*(m
```

```
 + 1))
```

Mathematica [B] time = 0.920422, size = 534, normalized size = 3.38

$$-\frac{2^{-p-1}x(m+n+1)\left(\sqrt{b^2-4ac}+b\right)(fx)^m\left(\sqrt{b^2-4ac}-b-2cx^n\right)\left(x^n\left(\sqrt{b^2-4ac}-b\right)-\right.}{(m+1)\left(\sqrt{b^2-4ac}-b\right)\left(\sqrt{b^2-4ac}+b+2cx^n\right)\left(npx^n\left(\left(\sqrt{b^2-4ac}-b\right)F_1\left(\frac{m+n+1}{n};1-p,-p;\frac{m+2n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2c}{\sqrt{b^2-}}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(f*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] -((2^(-1 - p)*(b + Sqrt[b^2 - 4*a*c])*(1 + m + n)*x*(f*x)^m*(-b +
 Sqrt[b^2 - 4*a*c] - 2*c*x^n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/
c)^p*(-2*a + (-b + Sqrt[b^2 - 4*a*c])*x^n)^2*(a + x^n*(b + c*x^n)
)^(-1 + p)*AppellF1[(1 + m)/n, -p, -p, (1 + m + n)/n, (-2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((-
b + Sqrt[b^2 - 4*a*c])*(1 + m)*((b - Sqrt[b^2 - 4*a*c])/(2*c) + x
^n)^p*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)*(-2*a*(1 + m + n)*AppellF
1[(1 + m)/n, -p, -p, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + n*p*x^n*((-b + Sqrt[
b^2 - 4*a*c])*AppellF1[(1 + m + n)/n, 1 - p, -p, (1 + m + 2*n)/n,
 (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4
*a*c])] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[(1 + m + n)/n, -p, 1 -
 p, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n
)/(-b + Sqrt[b^2 - 4*a*c])]))))
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int (fx)^m\left(a+bx^n+cx^{2n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n}+bx^n+a\right)^p(fx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^{2\,n}+bx^{n}+a\right)^{p}\left(fx\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^{2\,n}+bx^{n}+a\right)^{p}\left(fx\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

3.155 $\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{d+ex^n}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{d+ex^n},x\right)$$

```
[Out] Unintegrable[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n), x]
```

Rubi [A] time = 0.0723834, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\text{Int}\left(\frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{d+ex^n},x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n),x]

[Out] Defer[Int][((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n), x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{d+ex^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p/(d+e*x**n),x)

[Out] Integral((f*x)**m*(a + b*x**n + c*x**(2*n))**p/(d + e*x**n), x)
```

Mathematica [A] time = 0.12649, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{d+ex^n}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n),x]

[Out] Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n), x]
```

Maple [A] time = 0.108, size = 0, normalized size = 0.

$$\int \frac{(fx)^m \left(a + bx^n + cx^{2n}\right)^p}{d + ex^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)

[Out] int((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n} + bx^n + a\right)^p (fx)^m}{ex^n + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d),x, algorithm="maxima")

[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^{2n} + bx^n + a\right)^p (fx)^m}{ex^n + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d),x, algorithm="fricas")

[Out] integral((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p/(d+e*x**n),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2\,n} + bx^n + a\right)^p (fx)^m}{ex^n + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d), x)
```

3.156 $\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{(d+ex^n)^2},x\right)$$

```
[Out] Unintegrable[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2, x
]
```

Rubi [A] time = 0.0704311, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\text{Int}\left(\frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{(d+ex^n)^2},x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2,x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2, x]
```

Rubi in Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p/(d+e*x**n)**2,x)
```

```
[Out] Integral((f*x)**m*(a + b*x**n + c*x**(2*n))**p/(d + e*x**n)**2, x
)
```

Mathematica [A] time = 0.26374, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2,x]
```

```
[Out] Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2, x]
```

Maple [A] time = 0.122, size = 0, normalized size = 0.

$$\int \frac{(fx)^m \left(a + bx^n + cx^{2n}\right)^p}{(d + ex^n)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

```
[Out] int((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n} + bx^n + a\right)^p (fx)^m}{(ex^n + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^{2n} + bx^n + a\right)^p (fx)^m}{e^2x^{2n} + 2\,dex^n + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d)^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e^2*x^(2*n) + 2*d*e*x
^n + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p/(d+e*x**n)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n} + bx^{n} + a\right)^{p} (fx)^{m}}{(ex^{n} + d)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d)^2, x)
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```